84. Sales The projected monthly sales S (in thousands of units) of lawn mowers are modeled by

$$S = 74 + 3t - 40 \cos \frac{\pi t}{6}$$

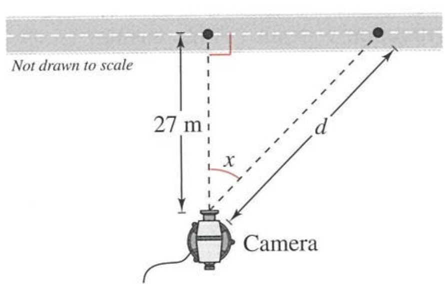
where t is the time (in months), with $t = 1$ corresponding to January.

(a) Graph the sales function over 1 year.

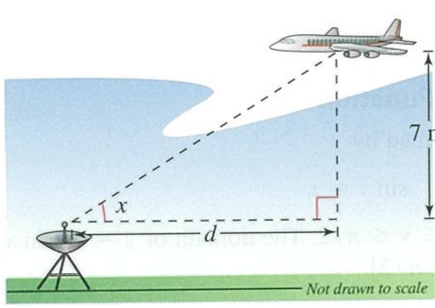
(b) What are the projected sales for June?

85. Television Coverage

A television camera is on a reviewing platform 27 meters from the street on which a parade passes from left to right (see figure). Write the distance d from the camera to a unit in the parade as a function of the angle x, and graph the function over the interval $-\pi/2 < x < \pi/2$. (Consider x as negative when a unit in the parade approaches from the left.)



86. Distance A plane flying at an altitude of 7 miles above a radar antenna passes directly over the radar antenna (see figure). Let d be the ground distance from the antenna to the point directly under the plane and let x be the angle of elevation to the plane from the antenna. (d is positive as the plane approaches the antenna.) Write d as a function of x and graph the function over the interval $0 < x < \pi$.



Irin-k/Shutterstock.com

Exploration

True or False? **In Exercises 87 and 88, determine whether the statement is true or false. Justify your answer.**

87. You can obtain the graph of $y = \csc x$ on a calculator by graphing the reciprocal of $y = \sin x$.

88. You can obtain the graph of $y = \sec x$ on a calculator by graphing a translation of the reciprocal of $y = \sin x$.

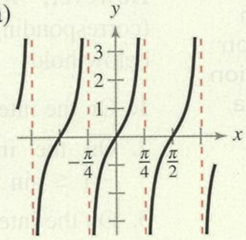
89. Think About It Consider the function $f(x) = x - \cos x$.

(a) Use a graphing utility to graph the function and verify that there exists a zero between 0 and 1. Use the graph to approximate the zero.

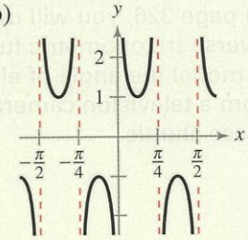
(b) Starting with $x_0 = 1$, generate a sequence $x_1, x_2, x_3, \ldots$, where $x_n = \cos(x_{n-1})$. For example, $x_0 = 1, x_1 = \cos(x_0), x_2 = \cos(x_1), x_3 = \cos(x_2), \ldots$. What value does the sequence approach?

90. HOW DO YOU SEE IT? Determine which function each graph represents. Do not use a calculator. Explain.

(a)



(b)



(a)	(b)
(i) $f(x) = \tan 2x$	(i) $f(x) = \sec 4x$
(ii) $f(x) = \tan(x/2)$	(ii) $f(x) = \csc 4x$
(iii) $f(x) = -\tan 2x$	(iii) $f(x) = \csc(x/4)$
(iv) $f(x) = -\tan(x/2)$	(iv) $f(x) = \sec(x/4)$

Graphical Reasoning **In Exercises 91 and 92, use a graphing utility to graph the function. Use the graph to determine the behavior of the function as $x \to c$. (*Note:* The notation $x \to c^+$ indicates that x approaches c from the right and $x \to c^-$ indicates that x approaches c from the left.)**

(a) $x \to 0^+$ **(b)** $x \to 0^-$ **(c)** $x \to \pi^+$ **(d)** $x \to \pi^-$

91. $f(x) = \cot x$ **92.** $f(x) = \csc x$

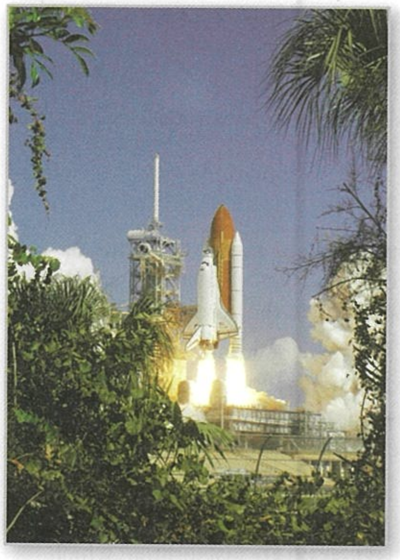
Graphical Reasoning **In Exercises 93 and 94, use a graphing utility to graph the function. Use the graph to determine the behavior of the function as $x \to c$.**

(a) $x \to (\pi/2)^+$ **(b)** $x \to (\pi/2)^-$

(c) $x \to (-\pi/2)^+$ **(d)** $x \to (-\pi/2)^-$

93. $f(x) = \tan x$ **94.** $f(x) = \sec x$

4.7 Inverse Trigonometric Functions



Inverse trigonometric functions have many applications in real life. For example, in Exercise 100 on page 326, you will use an inverse trigonometric function to model the angle of elevation from a television camera to a space shuttle.

- **Evaluate and graph the inverse sine function.**
- **Evaluate and graph other inverse trigonometric functions.**
- **Evaluate compositions with inverse trigonometric functions.**

Inverse Sine Function

Recall from Section 1.9 that for a function to have an inverse function, it must be one-to-one—that is, it must pass the Horizontal Line Test. Notice in Figure 4.47 that $y = \sin x$ does not pass the test because different values of x yield the same y-value.

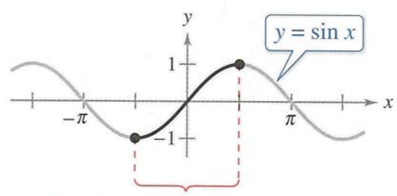


Figure 4.47

However, when you restrict the domain to the interval $-\pi/2 \le x \le \pi/2$ (corresponding to the black portion of the graph in Figure 4.47), the properties listed below hold.

1. On the interval $[-\pi/2, \pi/2]$, the function $y = \sin x$ is increasing.
2. On the interval $[-\pi/2, \pi/2]$, $y = \sin x$ takes on its full range of values, $-1 \le \sin x \le 1$.
3. On the interval $[-\pi/2, \pi/2]$, $y = \sin x$ is one-to-one.

So, on the restricted domain $-\pi/2 \le x \le \pi/2$, $y = \sin x$ has a unique inverse function called the **inverse sine function.** It is denoted by

$$y = \arcsin x \quad \text{or} \quad y = \sin^{-1} x.$$

The notation $\sin^{-1} x$ is consistent with the inverse function notation $f^{-1}(x)$. The $\arcsin x$ notation (read as "the arcsine of x") comes from the association of a central angle with its intercepted *arc length* on a unit circle. So, $\arcsin x$ means the angle (or arc) whose sine is x. Both notations, $\arcsin x$ and $\sin^{-1} x$ are commonly used in mathematics. You must remember that $\sin^{-1} x$ denotes the *inverse* sine function, *not* $1/\sin x$. The values of $\arcsin x$ lie in the interval

$$-\frac{\pi}{2} \le \arcsin x \le \frac{\pi}{2}.$$

Figure 4.48 on the next page shows the graph of $y = \arcsin x$.

REMARK When evaluating the inverse sine function, it helps to remember the phrase "the arcsine of x is the angle (or number) whose sine is x."

Definition of Inverse Sine Function

The **inverse sine function** is defined by

$$y = \arcsin x \quad \text{if and only if} \quad \sin y = x$$

where $-1 \le x \le 1$ and $-\pi/2 \le y \le \pi/2$. The domain of $y = \arcsin x$ is $[-1, 1]$, and the range is $[-\pi/2, \pi/2]$.

NASA

REMARK As with trigonometric functions, some of the work with inverse trigonometric functions can be done by *exact* calculations rather than by calculator approximations. Exact calculations help to increase your understanding of inverse functions by relating them to the right triangle definitions of trigonometric functions.

EXAMPLE 1 Evaluating the Inverse Sine Function

If possible, find the exact value of each expression.

a. $\arcsin\left(-\frac{1}{2}\right)$ **b.** $\sin^{-1}\frac{\sqrt{3}}{2}$ **c.** $\sin^{-1} 2$

Solution

a. You know that $\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$ and $-\frac{\pi}{6}$ lies in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, so

$$\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}.$$ Angle whose sine is $-\frac{1}{2}$

b. You know that $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ and $\frac{\pi}{3}$ lies in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, so

$$\sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}.$$ Angle whose sine is $\sqrt{3}/2$

c. It is not possible to evaluate $y = \sin^{-1} x$ when $x = 2$ because there is no angle whose sine is 2. Remember that the domain of the inverse sine function is $[-1, 1]$.

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If possible, find the exact value of each expression.

a. $\arcsin 1$ **b.** $\sin^{-1}(-2)$

EXAMPLE 2 Graphing the Arcsine Function

See LarsonPrecalculus.com for an interactive version of this type of example.

Sketch the graph of $y = \arcsin x$.

Solution

By definition, the equations $y = \arcsin x$ and $\sin y = x$ are equivalent for $-\pi/2 \le y \le \pi/2$. So, their graphs are the same. From the interval $[-\pi/2, \pi/2]$, assign values to y in the equation $\sin y = x$ to make a table of values.

y	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{2}$
$x = \sin y$	-1	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	1

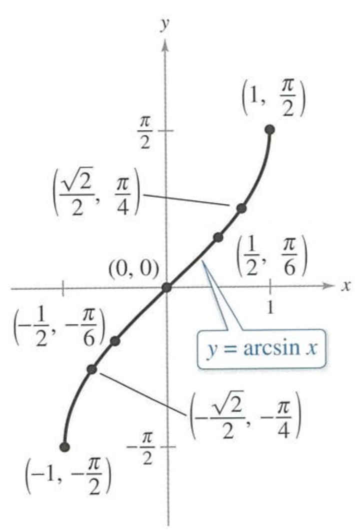

Figure 4.48

Then plot the points and connect them with a smooth curve. Figure 4.48 shows the graph of $y = \arcsin x$. Note that it is the reflection (in the line $y = x$) of the black portion of the graph in Figure 4.47. Be sure you see that Figure 4.48 shows the *entire* graph of the inverse sine function. Remember that the domain of $y = \arcsin x$ is the closed interval $[-1, 1]$ and the range is the closed interval $[-\pi/2, \pi/2]$.

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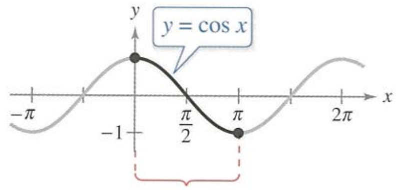
Use a graphing utility to graph $f(x) = \sin x$, $g(x) = \arcsin x$, and $y = x$ in the same viewing window to verify geometrically that g is the inverse function of f. (Be sure to restrict the domain of f properly.)

Other Inverse Trigonometric Functions

The cosine function is decreasing and one-to-one on the interval $0 \le x \le \pi$, as shown in the graph below.



Consequently, on this interval the cosine function has an inverse function—the **inverse cosine function**—denoted by

$$y = \arccos x \quad \text{or} \quad y = \cos^{-1} x.$$

Similarly, to define an **inverse tangent function,** restrict the domain of $y = \tan x$ to the interval $(-\pi/2, \pi/2)$. The inverse tangent function is denoted by

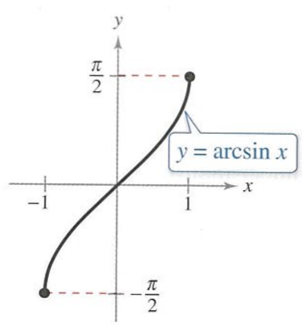
$$y = \arctan x \quad \text{or} \quad y = \tan^{-1} x.$$

The list below summarizes the definitions of the three most common inverse trigonometric functions. Definitions of the remaining three are explored in Exercises 111–113.

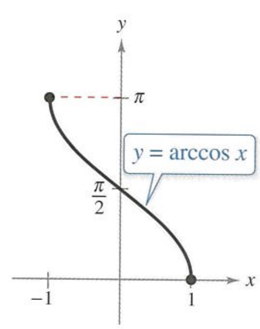
Definitions of the Inverse Trigonometric Functions

Function	Domain	Range
$y = \arcsin x$ if and only if $\sin y = x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \arccos x$ if and only if $\cos y = x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \arctan x$ if and only if $\tan y = x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

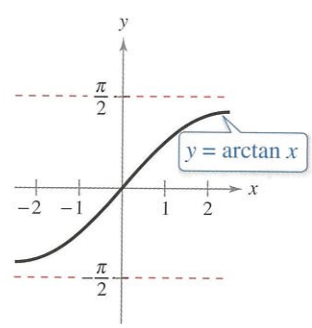
The graphs of these three inverse trigonometric functions are shown below.



Domain: $[-1, 1]$
Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
Intercept: $(0, 0)$
Symmetry: origin
Odd function



Domain: $[-1, 1]$
Range: $[0, \pi]$
y-intercept: $\left(0, \frac{\pi}{2}\right)$



Domain: $(-\infty, \infty)$
Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
Horizontal asymptotes: $y = \pm\frac{\pi}{2}$
Intercept: $(0, 0)$
Symmetry: origin
Odd function

EXAMPLE 3 Evaluating Inverse Trigonometric Functions

Find the exact value of each expression.

a. $\arccos \frac{\sqrt{2}}{2}$

b. $\arctan 0$

c. $\tan^{-1}(-1)$

Solution

a. You know that $\cos(\pi/4) = \sqrt{2}/2$ and $\pi/4$ lies in $[0, \pi]$, so

$$\arccos \frac{\sqrt{2}}{2} = \frac{\pi}{4}.$$ Angle whose cosine is $\sqrt{2}/2$

b. You know that $\tan 0 = 0$ and 0 lies in $(-\pi/2, \pi/2)$, so

$$\arctan 0 = 0.$$ Angle whose tangent is 0

c. You know that $\tan(-\pi/4) = -1$ and $-\pi/4$ lies in $(-\pi/2, \pi/2)$, so

$$\tan^{-1}(-1) = -\frac{\pi}{4}.$$ Angle whose tangent is -1

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Find the exact value of $\cos^{-1}(-1)$.

EXAMPLE 4 Calculators and Inverse Trigonometric Functions

Use a calculator to approximate the value of each expression, if possible.

a. $\arctan(-8.45)$

b. $\sin^{-1} 0.2447$

c. $\arccos 2$

Solution

Function	Mode	Calculator Keystrokes
a. $\arctan(-8.45)$	Radian	TAN⁻¹ ((−) 8.45) ENTER

From the display, it follows that $\arctan(-8.45) \approx -1.4530010$.

Function	Mode	Calculator Keystrokes
b. $\sin^{-1} 0.2447$	Radian	SIN⁻¹ (0.2447) ENTER

From the display, it follows that $\sin^{-1} 0.2447 \approx 0.2472103$.

Function	Mode	Calculator Keystrokes
c. $\arccos 2$	Radian	COS⁻¹ (2) ENTER

The calculator should display an *error message* because the domain of the inverse cosine function is $[-1, 1]$.

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Use a calculator to approximate the value of each expression, if possible.

a. $\arctan 4.84$

b. $\arcsin(-1.1)$

c. $\arccos(-0.349)$

In Example 4, had you set the calculator to *degree* mode, the displays would have been in degrees rather than in radians. This convention is peculiar to calculators. By definition, the values of inverse trigonometric functions are *always in radians*.

Compositions with Inverse Trigonometric Functions

▷ **ALGEBRA HELP** To review compositions of functions, see Section 1.8.

Recall from Section 1.9 that for all x in the domains of f and f^{-1}, inverse functions have the properties

$$f(f^{-1}(x)) = x \quad \text{and} \quad f^{-1}(f(x)) = x.$$

Inverse Properties of Trigonometric Functions

If $-1 \le x \le 1$ and $-\pi/2 \le y \le \pi/2$, then

$$\sin(\arcsin x) = x \quad \text{and} \quad \arcsin(\sin y) = y.$$

If $-1 \le x \le 1$ and $0 \le y \le \pi$, then

$$\cos(\arccos x) = x \quad \text{and} \quad \arccos(\cos y) = y.$$

If x is a real number and $-\pi/2 < y < \pi/2$, then

$$\tan(\arctan x) = x \quad \text{and} \quad \arctan(\tan y) = y.$$

Keep in mind that these inverse properties do not apply for arbitrary values of x and y. For example,

$$\arcsin\left(\sin \frac{3\pi}{2}\right) = \arcsin(-1) = -\frac{\pi}{2} \neq \frac{3\pi}{2}.$$

In other words, the property $\arcsin(\sin y) = y$ is not valid for values of y outside the interval $[-\pi/2, \pi/2]$.

EXAMPLE 5 Using Inverse Properties

If possible, find the exact value of each expression.

a. $\tan[\arctan(-5)]$ **b.** $\arcsin\left(\sin \frac{5\pi}{3}\right)$ **c.** $\cos(\cos^{-1} \pi)$

Solution

a. You know that -5 lies in the domain of the arctangent function, so the inverse property applies, and you have

$$\tan[\arctan(-5)] = -5.$$

b. In this case, $5\pi/3$ does not lie in the range of the arcsine function, $-\pi/2 \le y \le \pi/2$. However, $5\pi/3$ is coterminal with

$$\frac{5\pi}{3} - 2\pi = -\frac{\pi}{3}$$

which does lie in the range of the arcsine function, and you have

$$\arcsin\left(\sin \frac{5\pi}{3}\right) = \arcsin\left[\sin\left(-\frac{\pi}{3}\right)\right] = -\frac{\pi}{3}.$$

c. The expression $\cos(\cos^{-1} \pi)$ is not defined because $\cos^{-1} \pi$ is not defined. Remember that the domain of the inverse cosine function is $[-1, 1]$.

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If possible, find the exact value of each expression.

a. $\tan[\tan^{-1}(-14)]$ **b.** $\sin^{-1}\left(\sin \frac{7\pi}{4}\right)$ **c.** $\cos(\arccos 0.54)$

EXAMPLE 6 Evaluating Compositions of Functions

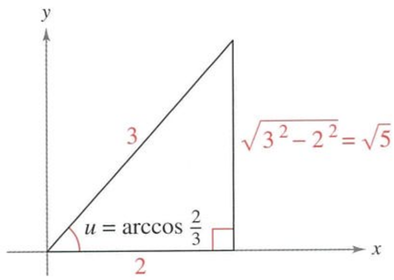
Find the exact value of each expression.

a. $\tan\left(\arccos \frac{2}{3}\right)$ **b.** $\cos\left[\arcsin\left(-\frac{3}{5}\right)\right]$

Solution

a. If you let $u = \arccos \frac{2}{3}$, then $\cos u = \frac{2}{3}$. The range of the inverse cosine function is $[0, \pi]$ and $\cos u$ is positive, so u is a *first*-quadrant angle. Sketch and label a right triangle with acute angle u, as shown in Figure 4.49. Consequently,

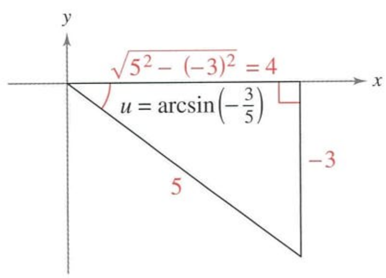
$$\tan\left(\arccos \frac{2}{3}\right) = \tan u = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{5}}{2}.$$



Angle whose cosine is $\frac{2}{3}$
Figure 4.49

b. If you let $u = \arcsin\left(-\frac{3}{5}\right)$, then $\sin u = -\frac{3}{5}$. The range of the inverse sine function is $[-\pi/2, \pi/2]$ and $\sin u$ is negative, so u is a *fourth*-quadrant angle. Sketch and label a right triangle with acute angle u, as shown in Figure 4.50. Consequently,

$$\cos\left[\arcsin\left(-\frac{3}{5}\right)\right] = \cos u = \frac{\text{adj}}{\text{hyp}} = \frac{4}{5}.$$



Angle whose sine is $-\frac{3}{5}$
Figure 4.50

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Find the exact value of $\cos\left[\arctan\left(-\frac{3}{4}\right)\right]$.

EXAMPLE 7 Some Problems from Calculus

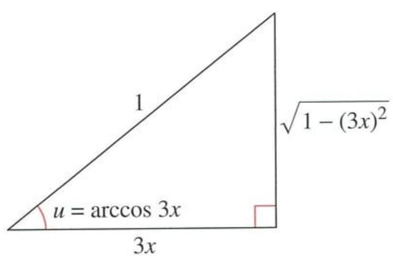
Write an algebraic expression that is equivalent to each expression.

a. $\sin(\arccos 3x), \quad 0 \le x \le \frac{1}{3}$ **b.** $\cot(\arccos 3x), \quad 0 \le x < \frac{1}{3}$

Solution

If you let $u = \arccos 3x$, then $\cos u = 3x$, where $-1 \le 3x \le 1$. Write

$$\cos u = \frac{\text{adj}}{\text{hyp}} = \frac{3x}{1}$$



Angle whose cosine is $3x$
Figure 4.51

and sketch a right triangle with acute angle u, as shown in Figure 4.51. From this triangle, convert each expression to algebraic form.

a. $\sin(\arccos 3x) = \sin u = \frac{\text{opp}}{\text{hyp}} = \sqrt{1 - 9x^2}, \quad 0 \le x \le \frac{1}{3}$

b. $\cot(\arccos 3x) = \cot u = \frac{\text{adj}}{\text{opp}} = \frac{3x}{\sqrt{1 - 9x^2}}, \quad 0 \le x < \frac{1}{3}$

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Write an algebraic expression that is equivalent to $\sec(\arctan x)$.

Summarize (Section 4.7)

1. State the definition of the inverse sine function *(page 318)*. For examples of evaluating and graphing the inverse sine function, see Examples 1 and 2.
2. State the definitions of the inverse cosine and inverse tangent functions *(page 320)*. For examples of evaluating inverse trigonometric functions, see Examples 3 and 4.
3. State the inverse properties of trigonometric functions *(page 322)*. For examples of finding compositions with inverse trigonometric functions, see Examples 5–7.

4.7 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: **Fill in the blanks.**

	Function	Alternative Notation	Domain	Range
1.	$y = \arcsin x$	__________	__________	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
2.	__________	$y = \cos^{-1} x$	$-1 \le x \le 1$	__________
3.	$y = \arctan x$	__________	__________	__________

4. A trigonometric function has an __________ function only when its domain is restricted.

Skills and Applications

Evaluating an Inverse Trigonometric Function **In Exercises 5–18, find the exact value of the expression, if possible.**

5. $\arcsin \frac{1}{2}$
6. $\arcsin 0$
7. $\arccos \frac{1}{2}$
8. $\arccos 0$
9. $\arctan \frac{\sqrt{3}}{3}$
10. $\arctan 1$
11. $\arcsin 3$
12. $\arctan \sqrt{3}$
13. $\tan^{-1}(-\sqrt{3})$
14. $\cos^{-1}(-2)$
15. $\arccos\left(-\frac{1}{2}\right)$
16. $\arcsin \frac{\sqrt{2}}{2}$
17. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
18. $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

Graphing an Inverse Trigonometric Function **In Exercises 19 and 20, use a graphing utility to graph f, g, and $y = x$ in the same viewing window to verify geometrically that g is the inverse function of f. (Be sure to restrict the domain of f properly.)**

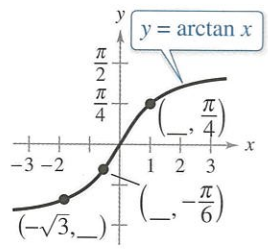
19. $f(x) = \cos x, \quad g(x) = \arccos x$
20. $f(x) = \tan x, \quad g(x) = \arctan x$

Calculators and Inverse Trigonometric Functions **In Exercises 21–36, use a calculator to approximate the value of the expression, if possible. Round your result to two decimal places.**

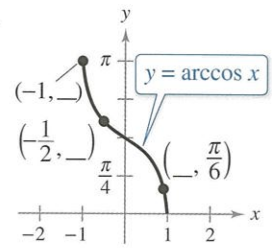
21. $\arccos 0.37$
22. $\arcsin 0.65$
23. $\arcsin(-0.75)$
24. $\arccos(-0.7)$
25. $\arctan(-3)$
26. $\arctan 25$
27. $\sin^{-1} 1.36$
28. $\cos^{-1} 0.26$
29. $\arccos(-0.41)$
30. $\arcsin(-0.125)$
31. $\arctan 0.92$
32. $\arctan 2.8$
33. $\arcsin \frac{7}{8}$
34. $\arccos\left(-\frac{4}{3}\right)$
35. $\tan^{-1}\left(-\frac{95}{7}\right)$
36. $\tan^{-1}(-\sqrt{372})$

Finding Missing Coordinates **In Exercises 37 and 38, determine the missing coordinates of the points on the graph of the function.**

37.

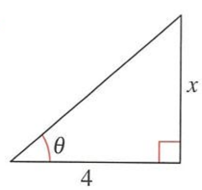


38.

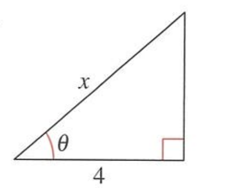


Using an Inverse Trigonometric Function **In Exercises 39–44, use an inverse trigonometric function to write θ as a function of x.**

39.



40.



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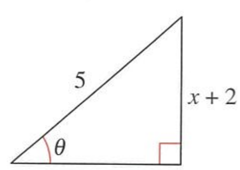

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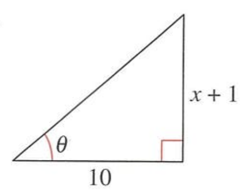

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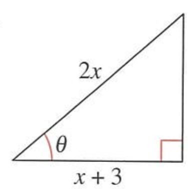

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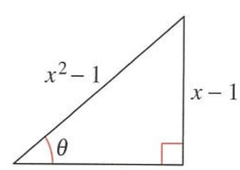


Using Inverse Properties **In Exercises 45–50, find the exact value of the expression, if possible.**

45. $\sin(\arcsin 0.3)$
46. $\tan(\arctan 45)$
47. $\cos[\arccos(-\sqrt{3})]$
48. $\sin[\arcsin(-0.2)]$
49. $\arcsin[\sin(9\pi/4)]$
50. $\arccos[\cos(-3\pi/2)]$

Evaluating a Composition of Functions **In Exercises 51–62, find the exact value of the expression, if possible.**

51. $\sin\left(\arctan \frac{3}{4}\right)$
52. $\cos\left(\arcsin \frac{4}{5}\right)$
53. $\cos(\tan^{-1} 2)$
54. $\sin\left(\cos^{-1} \sqrt{5}\right)$
55. $\sec\left(\arcsin \frac{5}{13}\right)$
56. $\csc\left[\arctan\left(-\frac{5}{12}\right)\right]$
57. $\cot\left[\arctan\left(-\frac{3}{5}\right)\right]$
58. $\sec\left[\arccos\left(-\frac{3}{4}\right)\right]$
59. $\tan\left[\arccos\left(-\frac{2}{3}\right)\right]$
60. $\cot\left(\arctan \frac{5}{8}\right)$
61. $\csc\left(\cos^{-1} \frac{\sqrt{3}}{2}\right)$
62. $\tan\left[\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right]$

Writing an Expression **In Exercises 63–72, write an algebraic expression that is equivalent to the given expression.**

63. $\cos(\arcsin 2x)$
64. $\sin(\arctan x)$
65. $\cot(\arctan x)$
66. $\sec(\arctan 3x)$
67. $\sin(\arccos x)$
68. $\csc[\arccos(x - 1)]$
69. $\tan\left(\arccos \frac{x}{3}\right)$
70. $\cot\left(\arctan \frac{1}{x}\right)$
71. $\csc\left(\arctan \frac{x}{a}\right)$
72. $\cos\left(\arcsin \frac{x - h}{r}\right)$

Using Technology **In Exercises 73 and 74, use a graphing utility to graph f and g in the same viewing window to verify that the two functions are equal. Explain why they are equal. Identify any asymptotes of the graphs.**

73. $f(x) = \sin(\arctan 2x), \quad g(x) = \dfrac{2x}{\sqrt{1 + 4x^2}}$

74. $f(x) = \tan\left(\arccos \frac{x}{2}\right), \quad g(x) = \dfrac{\sqrt{4 - x^2}}{x}$

Completing an Equation **In Exercises 75–78, complete the equation.**

75. $\arctan \frac{9}{x} = \arcsin(\quad), \quad x > 0$

76. $\arcsin \frac{\sqrt{36 - x^2}}{6} = \arccos(\quad), \quad 0 \le x \le 6$

77. $\arccos \frac{3}{\sqrt{x^2 - 2x + 10}} = \arcsin(\quad)$

78. $\arccos \frac{x - 2}{2} = \arctan(\quad), \quad 2 < x < 4$

Sketching the Graph of a Function **In Exercises 79–84, sketch the graph of the function and compare the graph to the graph of the parent inverse trigonometric function.**

79. $y = 2 \arcsin x$
80. $f(x) = \arctan 2x$
81. $f(x) = \frac{\pi}{2} + \arctan x$
82. $g(t) = \arccos(t + 2)$
83. $h(v) = \arccos \frac{v}{2}$
84. $f(x) = \arcsin \frac{x}{4}$

Graphing an Inverse Trigonometric Function **In Exercises 85–90, use a graphing utility to graph the function.**

85. $f(x) = 2 \arccos 2x$
86. $f(x) = \pi \arcsin 4x$
87. $f(x) = \arctan(2x - 3)$
88. $f(x) = -3 + \arctan \pi x$
89. $f(x) = \pi - \sin^{-1} \frac{2}{3}$
90. $f(x) = \frac{\pi}{2} + \cos^{-1} \frac{1}{\pi}$

Using a Trigonometric Identity **In Exercises 91 and 92, write the function in terms of the sine function by using the identity**

$$A \cos \omega t + B \sin \omega t = \sqrt{A^2 + B^2} \sin\left(\omega t + \arctan \frac{A}{B}\right).$$

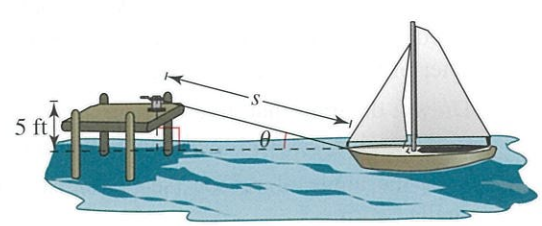
Use a graphing utility to graph both forms of the function. What does the graph imply?

91. $f(t) = 3 \cos 2t + 3 \sin 2t$
92. $f(t) = 4 \cos \pi t + 3 \sin \pi t$

Behavior of an Inverse Trigonometric Function **In Exercises 93–98, fill in the blank. If not possible, state the reason. (*Note:* The notation $x \to c^+$ indicates that x approaches c from the right and $x \to c^-$ indicates that x approaches c from the left.)**

93. As $x \to 1^-$, the value of $\arcsin x \to$ ▭.
94. As $x \to 1^-$, the value of $\arccos x \to$ ▭.
95. As $x \to \infty$, the value of $\arctan x \to$ ▭.
96. As $x \to -1^+$, the value of $\arcsin x \to$ ▭.
97. As $x \to -1^+$, the value of $\arccos x \to$ ▭.
98. As $x \to -\infty$, the value of $\arctan x \to$ ▭.

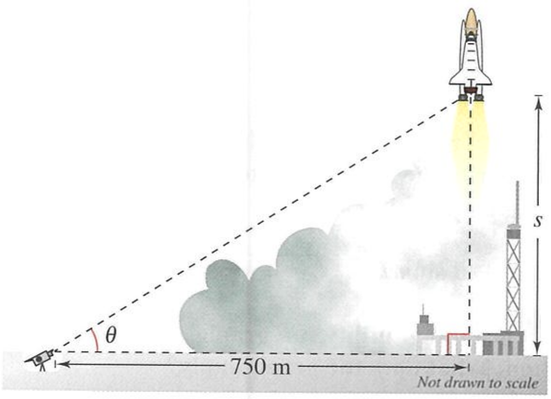
99. **Docking a Boat** A boat is pulled in by means of a winch located on a dock 5 feet above the deck of the boat (see figure). Let θ be the angle of elevation from the boat to the winch and let s be the length of the rope from the winch to the boat.



(a) Write θ as a function of s.

(b) Find θ when $s = 40$ feet and $s = 20$ feet.

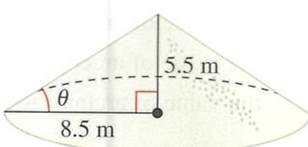
100. Videography A television camera at ground level films the lift-off of a space shuttle at a point 750 meters from the launch pad (see figure). Let θ be the angle of elevation to the shuttle and let s be the height of the shuttle.



(a) Write θ as a function of s.

(b) Find θ when $s = 300$ meters and $s = 1200$ meters.

101. Granular Angle of Repose Different types of granular substances naturally settle at different angles when stored in cone-shaped piles. This angle θ is called the *angle of repose* (see figure). When rock salt is stored in a cone-shaped pile 5.5 meters high, the diameter of the pile's base is about 17 meters.



(a) Find the angle of repose for rock salt.

(b) How tall is a pile of rock salt that has a base diameter of 20 meters?

102. Granular Angle of Repose When shelled corn is stored in a cone-shaped pile 20 feet high, the diameter of the pile's base is about 94 feet.

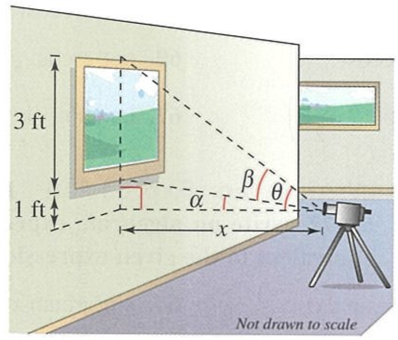
(a) Draw a diagram that gives a visual representation of the problem. Label the known quantities.

(b) Find the angle of repose (see Exercise 101) for shelled corn.

(c) How tall is a pile of shelled corn that has a base diameter of 60 feet?

103. Photography A photographer takes a picture of a three-foot-tall painting hanging in an art gallery. The camera lens is 1 foot below the lower edge of the painting (see figure). The angle β subtended by the camera lens x feet from the painting is given by

$$\beta = \arctan \frac{3x}{x^2 + 4}, \quad x > 0.$$

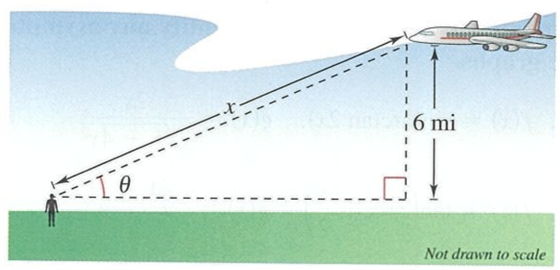


(a) Use a graphing utility to graph β as a function of x.

(b) Use the graph to approximate the distance from the picture when β is maximum.

(c) Identify the asymptote of the graph and interpret its meaning in the context of the problem.

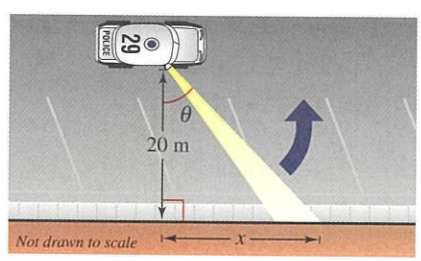
104. Angle of Elevation An airplane flies at an altitude of 6 miles toward a point directly over an observer. Consider θ and x as shown in the figure.



(a) Write θ as a function of x.

(b) Find θ when $x = 12$ miles and $x = 7$ miles.

105. Police Patrol A police car with its spotlight on is parked 20 meters from a warehouse. Consider θ and x as shown in the figure.



(a) Write θ as a function of x.

(b) Find θ when $x = 5$ meters and $x = 12$ meters.

NASA

Exploration

True or False? **In Exercises 106–109, determine whether the statement is true or false. Justify your answer.**

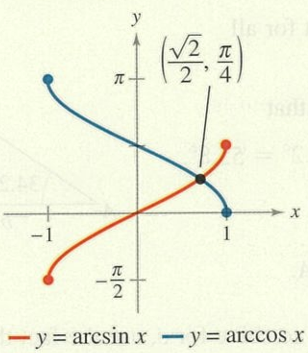
106. $\sin \frac{5\pi}{6} = \frac{1}{2} \Rightarrow \arcsin \frac{1}{2} = \frac{5\pi}{6}$

107. $\tan\left(-\frac{\pi}{4}\right) = -1 \Rightarrow \arctan(-1) = -\frac{\pi}{4}$

108. $\arctan x = \frac{\arcsin x}{\arccos x}$

109. $\sin^{-1} x = \frac{1}{\sin x}$

110. HOW DO YOU SEE IT? Use the figure below to determine the value(s) of x for which each statement is true.



(a) $\arcsin x < \arccos x$

(b) $\arcsin x = \arccos x$

(c) $\arcsin x > \arccos x$

111. Inverse Cotangent Function Define the inverse cotangent function by restricting the domain of the cotangent function to the interval $(0, \pi)$, and sketch the graph of the inverse trigonometric function.

112. Inverse Secant Function Define the inverse secant function by restricting the domain of the secant function to the intervals $[0, \pi/2)$ and $(\pi/2, \pi]$, and sketch the graph of the inverse trigonometric function.

113. Inverse Cosecant Function Define the inverse cosecant function by restricting the domain of the cosecant function to the intervals $[-\pi/2, 0)$ and $(0, \pi/2]$, and sketch the graph of the inverse trigonometric function.

114. Writing Use the results of Exercises 111–113 to explain how to graph (a) the inverse cotangent function, (b) the inverse secant function, and (c) the inverse cosecant function on a graphing utility.

Evaluating an Inverse Trigonometric Function **In Exercises 115–120, use the results of Exercises 111–113 to find the exact value of the expression.**

115. $\operatorname{arcsec} \sqrt{2}$

116. $\operatorname{arcsec} 1$

117. $\operatorname{arccot}(-1)$

118. $\operatorname{arccot}(-\sqrt{3})$

119. $\operatorname{arccsc}(-1)$

120. $\operatorname{arccsc} \frac{2\sqrt{3}}{3}$

Calculators and Inverse Trigonometric Functions **In Exercises 121–126, use the results of Exercises 111–113 and a calculator to approximate the value of the expression. Round your result to two decimal places.**

121. $\operatorname{arcsec} 2.54$

122. $\operatorname{arcsec}(-1.52)$

123. $\operatorname{arccsc}\left(-\frac{25}{3}\right)$

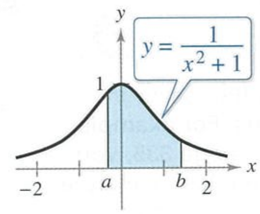
124. $\operatorname{arccsc}(-12)$

125. $\operatorname{arccot} 5.25$

126. $\operatorname{arccot}\left(-\frac{16}{7}\right)$

127. Area In calculus, it is shown that the area of the region bounded by the graphs of $y = 0$, $y = 1/(x^2 + 1)$, $x = a$, and $x = b$ (see figure) is given by

Area $= \arctan b - \arctan a$.



Find the area for each value of a and b.

(a) $a = 0, b = 1$

(b) $a = -1, b = 1$

(c) $a = 0, b = 3$

(d) $a = -1, b = 3$

128. Think About It Use a graphing utility to graph the functions $f(x) = \sqrt{x}$ and $g(x) = 6 \arctan x$. For $x > 0$, it appears that $g > f$. Explain how you know that there exists a positive real number a such that $g < f$ for $x > a$. Approximate the number a.

129. Think About It Consider the functions

$f(x) = \sin x$ and $f^{-1}(x) = \arcsin x$.

(a) Use a graphing utility to graph the composite functions $f \circ f^{-1}$ and $f^{-1} \circ f$.

(b) Explain why the graphs in part (a) are not the graph of the line $y = x$. Why do the graphs of $f \circ f^{-1}$ and $f^{-1} \circ f$ differ?

130. Proof Prove each identity.

(a) $\arcsin(-x) = -\arcsin x$

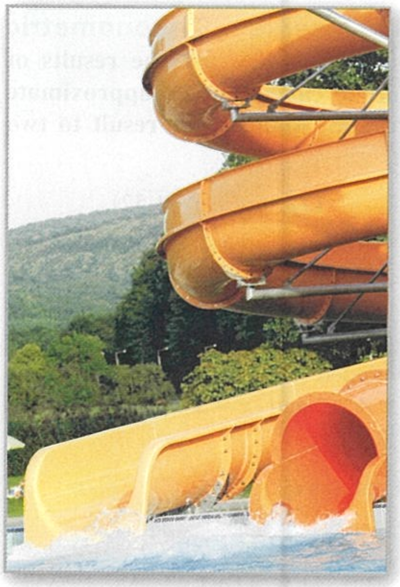
(b) $\arctan(-x) = -\arctan x$

(c) $\arctan x + \arctan \frac{1}{x} = \frac{\pi}{2}, \quad x > 0$

(d) $\arcsin x + \arccos x = \frac{\pi}{2}$

(e) $\arcsin x = \arctan \frac{x}{\sqrt{1 - x^2}}$

4.8 Applications and Models



Right triangles often occur in real-life situations. For example, in Exercise 30 on page 335, you will use right triangles to analyze the design of a new slide at a water park.

- Solve real-life problems involving right triangles.
- Solve real-life problems involving directional bearings.
- Solve real-life problems involving harmonic motion.

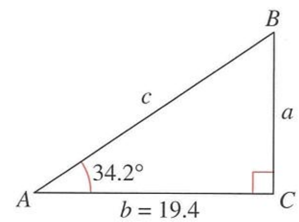
Applications Involving Right Triangles

In this section, the three angles of a right triangle are denoted by A, B, and C (where C is the right angle), and the lengths of the sides opposite these angles are denoted by a, b, and c, respectively (where c is the hypotenuse).

EXAMPLE 1 Solving a Right Triangle

See LarsonPrecalculus.com for an interactive version of this type of example.

Solve the right triangle shown at the right for all unknown sides and angles.



Solution Because $C = 90°$, it follows that

$$A + B = 90° \quad \text{and} \quad B = 90° - 34.2° = 55.8°.$$

To solve for a, use the fact that

$$\tan A = \frac{\text{opp}}{\text{adj}} = \frac{a}{b} \quad \Longrightarrow \quad a = b \tan A.$$

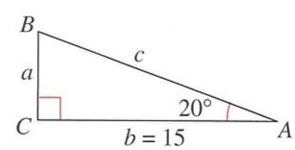
So, $a = 19.4 \tan 34.2° \approx 13.2$. Similarly, to solve for c, use the fact that

$$\cos A = \frac{\text{adj}}{\text{hyp}} = \frac{b}{c} \quad \Longrightarrow \quad c = \frac{b}{\cos A}.$$

So, $c = \dfrac{19.4}{\cos 34.2°} \approx 23.5$.

✓ Checkpoint Audio-video solution in English & Spanish at LarsonPrecalculus.com

Solve the right triangle shown at the right for all unknown sides and angles.



EXAMPLE 2 Finding a Side of a Right Triangle

The height of a mountain is 5000 feet. The distance between its peak and that of an adjacent mountain is 25,000 feet. The angle of elevation between the two peaks is 27°. (See Figure 4.52.) What is the height of the adjacent mountain?

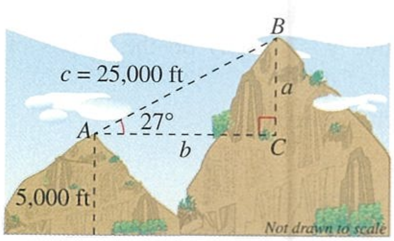


Figure 4.52

Solution From the figure, $\sin A = a/c$, so

$$a = c \sin A = 25{,}000 \sin 27° \approx 11{,}350.$$

The height of the adjacent mountain is about $11{,}350 + 5000 = 16{,}350$ feet.

✓ Checkpoint Audio-video solution in English & Spanish at LarsonPrecalculus.com

A ladder that is 16 feet long leans against the side of a house. The angle of elevation of the ladder is 80°. Find the height from the top of the ladder to the ground.

István Csak | Dreamstime

EXAMPLE 3 Finding a Side of a Right Triangle

At a point 200 feet from the base of a building, the angle of elevation to the *bottom* of a smokestack is 35°, whereas the angle of elevation to the *top* is 53°, as shown in Figure 4.53. Find the height s of the smokestack alone.

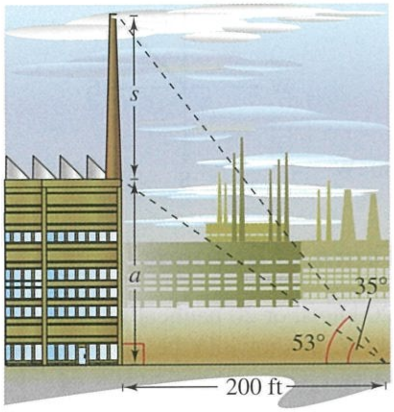


Figure 4.53

Solution

This problem involves two right triangles. For the smaller right triangle, use the fact that

$$\tan 35° = \frac{a}{200}$$

to find that the height of the building is

$$a = 200 \tan 35°.$$

For the larger right triangle, use the equation

$$\tan 53° = \frac{a + s}{200}$$

to find that

$$a + s = 200 \tan 53°.$$

So, the height of the smokestack is

$$\begin{aligned} s &= 200 \tan 53° - a \\ &= 200 \tan 53° - 200 \tan 35° \\ &\approx 125.4 \text{ feet.} \end{aligned}$$

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

At a point 65 feet from the base of a church, the angles of elevation to the bottom of the steeple and the top of the steeple are 35° and 43°, respectively. Find the height of the steeple.

EXAMPLE 4 Finding an Angle of Depression

A swimming pool is 20 meters long and 12 meters wide. The bottom of the pool is slanted so that the water depth is 1.3 meters at the shallow end and 4 meters at the deep end, as shown in Figure 4.54. Find the angle of depression (in degrees) of the bottom of the pool.

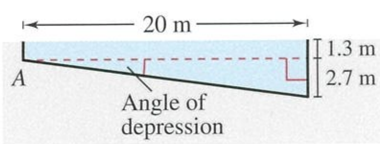


Figure 4.54

Solution Using the tangent function,

$$\begin{aligned} \tan A &= \frac{\text{opp}}{\text{adj}} \\ &= \frac{2.7}{20} \\ &= 0.135. \end{aligned}$$

So, the angle of depression is

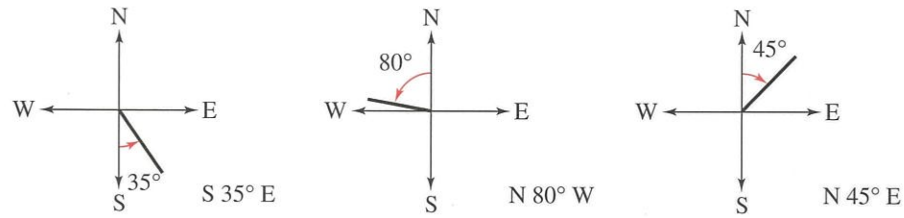
$$A = \arctan 0.135 \approx 0.13419 \text{ radian} \approx 7.69°.$$

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

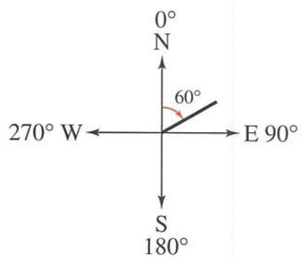
From the time a small airplane is 100 feet high and 1600 ground feet from its landing runway, the plane descends in a straight line to the runway. Determine the angle of descent (in degrees) of the plane.

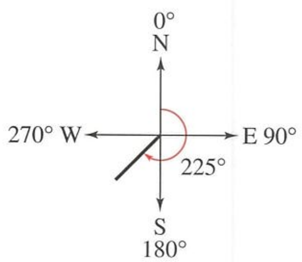
Trigonometry and Bearings

In surveying and navigation, directions can be given in terms of **bearings.** A bearing measures the acute angle that a path or line of sight makes with a fixed north-south line. For example, in the figures below, the bearing S 35° E means 35 degrees east of south, N 80° W means 80 degrees west of north, and N 45° E means 45 degrees east of north.



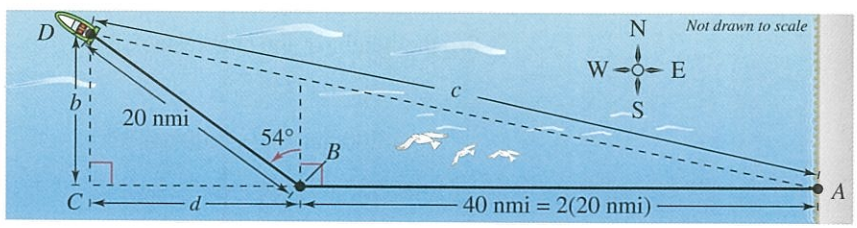
REMARK In *air navigation*, bearings are measured in degrees *clockwise* from north. The figures below illustrate examples of air navigation bearings





EXAMPLE 5 Finding Directions in Terms of Bearings

A ship leaves port at noon and heads due west at 20 knots, or 20 nautical miles (nmi) per hour. At 2 P.M. the ship changes course to N 54° W, as shown in the figure below. Find the ship's bearing and distance from port at 3 P.M.



Solution

For triangle BCD, you have

$$B = 90° - 54° = 36°.$$

The two sides of this triangle are

$$b = 20 \sin 36° \quad \text{and} \quad d = 20 \cos 36°.$$

For triangle ACD, find angle A.

$$\tan A = \frac{b}{d + 40} = \frac{20 \sin 36°}{20 \cos 36° + 40} \approx 0.209$$

$$A \approx \arctan 0.209 \approx 0.20603 \text{ radian} \approx 11.80°$$

The angle with the north-south line is $90° - 11.80° = 78.20°$. So, the bearing of the ship is N 78.20° W. Finally, from triangle ACD, you have

$$\sin A = \frac{b}{c}$$

which yields

$$c = \frac{b}{\sin A} = \frac{20 \sin 36°}{\sin 11.80°} \approx 57.5 \text{ nautical miles.} \qquad \text{Distance from port}$$

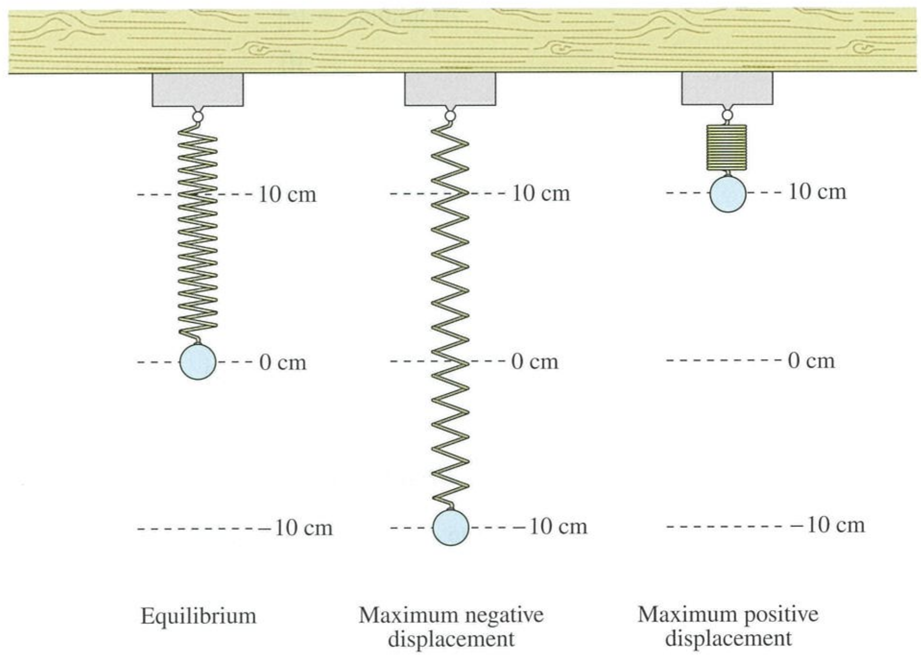
Checkpoint Audio-video solution in English & Spanish at LarsonPrecalculus.com

A sailboat leaves a pier heading due west at 8 knots. After 15 minutes, the sailboat changes course to N 16° W at 10 knots. Find the sailboat's bearing and distance from the pier after 12 minutes on this course.

Harmonic Motion

The periodic nature of the trigonometric functions is useful for describing the motion of a point on an object that vibrates, oscillates, rotates, or is moved by wave motion.

For example, consider a ball that is bobbing up and down on the end of a spring. Assume that the maximum distance the ball moves vertically upward or downward from its equilibrium (at rest) position is 10 centimeters (see figure). Assume further that the time it takes for the ball to move from its maximum displacement above zero to its maximum displacement below zero and back again is $t = 4$ seconds. With the ideal conditions of perfect elasticity and no friction or air resistance, the ball would continue to move up and down in a uniform and regular manner.



The period (time for one complete cycle) of the motion is

Period $= 4$ seconds

the amplitude (maximum displacement from equilibrium) is

Amplitude $= 10$ centimeters

and the **frequency** (number of cycles per second) is

Frequency $= \frac{1}{4}$ cycle per second.

Motion of this nature can be described by a sine or cosine function and is called **simple harmonic motion.**

Definition of Simple Harmonic Motion

A point that moves on a coordinate line is in **simple harmonic motion** when its distance d from the origin at time t is given by either

$$d = a \sin \omega t \quad \text{or} \quad d = a \cos \omega t$$

where a and ω are real numbers such that $\omega > 0$. The motion has amplitude $|a|$, period $\frac{2\pi}{\omega}$, and frequency $\frac{\omega}{2\pi}$.

EXAMPLE 6 Simple Harmonic Motion

Write an equation for the simple harmonic motion of the ball described on the preceding page.

Solution

The spring is at equilibrium ($d = 0$) when $t = 0$, so use the equation

$$d = a \sin \omega t.$$

Moreover, the maximum displacement from zero is 10 and the period is 4. Using this information, you have

$$\text{Amplitude} = |a|$$
$$= 10$$

$$\text{Period} = \frac{2\pi}{\omega} = 4 \quad \Longrightarrow \quad \omega = \frac{\pi}{2}.$$

Consequently, an equation of motion is

$$d = 10 \sin \frac{\pi}{2} t.$$

Note that the choice of

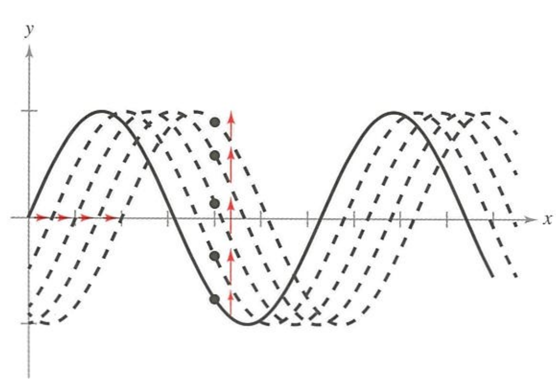
$$a = 10 \quad \text{or} \quad a = -10$$

depends on whether the ball initially moves up or down.

✓ Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Write an equation for simple harmonic motion for which $d = 0$ when $t = 0$, the amplitude is 6 centimeters, and the period is 3 seconds.

One illustration of the relationship between sine waves and harmonic motion is in the wave motion that results when you drop a stone into a calm pool of water. The waves move outward in roughly the shape of sine (or cosine) waves, as shown at the right. Now suppose you are fishing in the same pool of water and your fishing bobber does not move horizontally. As the waves move outward from the dropped stone, the fishing bobber moves up and down in simple harmonic motion, as shown below.



EXAMPLE 7 Simple Harmonic Motion

Consider the equation for simple harmonic motion $d = 6 \cos \frac{3\pi}{4} t$. Find (a) the maximum displacement, (b) the frequency, (c) the value of d when $t = 4$, and (d) the least positive value of t for which $d = 0$.

Algebraic Solution

The equation has the form $d = a \cos \omega t$, with $a = 6$ and $\omega = 3\pi/4$.

a. The maximum displacement (from the point of equilibrium) is the amplitude. So, the maximum displacement is 6.

b.
$$\text{Frequency} = \frac{\omega}{2\pi} = \frac{3\pi/4}{2\pi} = \frac{3}{8} \text{ cycle per unit of time}$$

c. $d = 6 \cos\left[\frac{3\pi}{4}(4)\right] = 6 \cos 3\pi = 6(-1) = -6$

d. To find the least positive value of t for which $d = 0$, solve

$$6 \cos \frac{3\pi}{4} t = 0.$$

First divide each side by 6 to obtain

$$\cos \frac{3\pi}{4} t = 0.$$

This equation is satisfied when

$$\frac{3\pi}{4} t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots.$$

Multiply these values by $4/(3\pi)$ to obtain

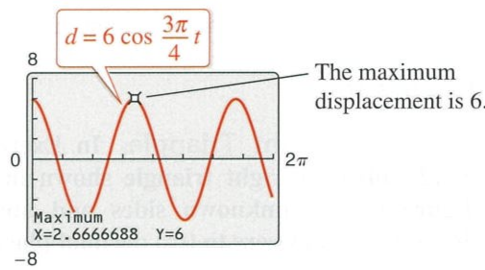
$$t = \frac{2}{3}, 2, \frac{10}{3}, \ldots.$$

So, the least positive value of t is $t = \frac{2}{3}$.

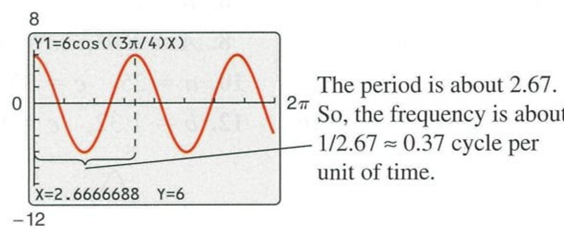
Graphical Solution

Use a graphing utility set in *radian* mode.

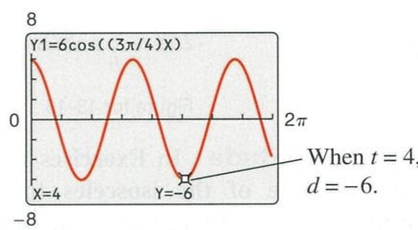
a.



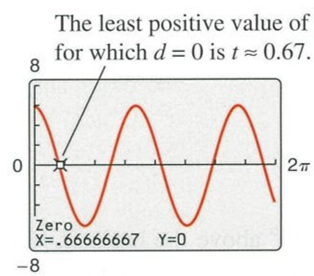
b.



c.



d.



✓ Checkpoint Audio-video solution in English & Spanish at LarsonPrecalculus.com

Rework Example 7 for the equation $d = 4 \cos 6\pi t$.

Summarize (Section 4.8)

1. Describe real-life applications of right triangles *(pages 328 and 329, Examples 1–4)*.
2. Describe a real-life application of a directional bearing *(page 330, Example 5)*.
3. Describe real-life applications of simple harmonic motion *(pages 332 and 333, Examples 6 and 7)*.

4.8 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

1. A ________ measures the acute angle that a path or line of sight makes with a fixed north-south line.
2. A point that moves on a coordinate line is in simple ________ ________ when its distance d from the origin at time t is given by either $d = a \sin \omega t$ or $d = a \cos \omega t$.
3. The time for one complete cycle of a point in simple harmonic motion is its ________.
4. The number of cycles per second of a point in simple harmonic motion is its ________.

Skills and Applications

Solving a Right Triangle **In Exercises 5–12, solve the right triangle shown in the figure for all unknown sides and angles. Round your answers to two decimal places.**

5. $A = 60°$, $c = 12$
6. $B = 25°$, $b = 4$
7. $B = 72.8°$, $a = 4.4$
8. $A = 8.4°$, $a = 40.5$
9. $a = 3$, $b = 4$
10. $a = 25$, $c = 35$
11. $b = 15.70$, $c = 55.16$
12. $b = 1.32$, $c = 9.45$

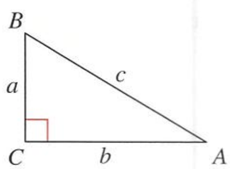


Figure for 5–12

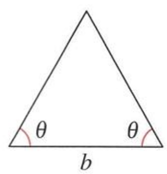
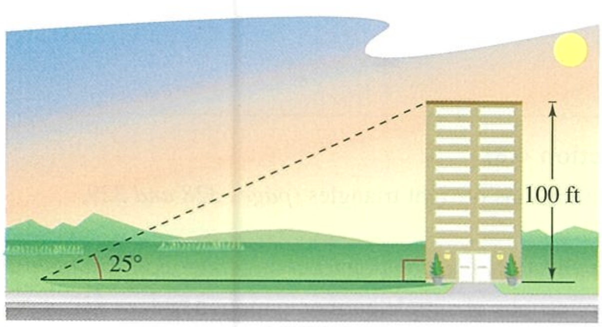


Figure for 13–16

Finding an Altitude **In Exercises 13–16, find the altitude of the isosceles triangle shown in the figure. Round your answers to two decimal places.**

13. $\theta = 45°$, $b = 6$
14. $\theta = 22°$, $b = 14$
15. $\theta = 32°$, $b = 8$
16. $\theta = 27°$, $b = 11$

17. **Length** The sun is 25° above the horizon. Find the length of a shadow cast by a building that is 100 feet tall (see figure).



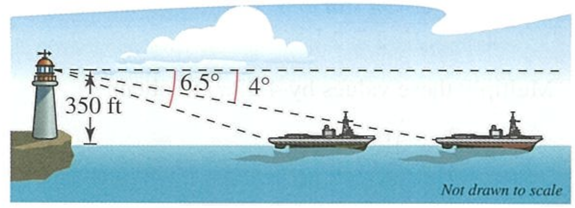
18. **Length** The sun is 20° above the horizon. Find the length of a shadow cast by a park statue that is 12 feet tall.

19. **Height** A ladder that is 20 feet long leans against the side of a house. The angle of elevation of the ladder is 80°. Find the height from the top of the ladder to the ground.

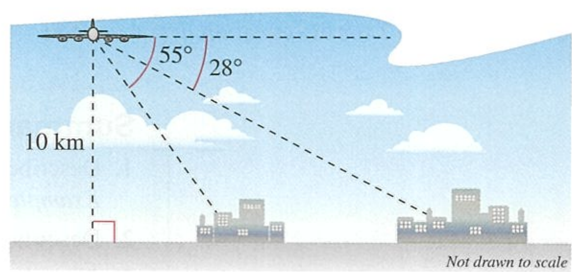
20. **Height** The length of a shadow of a tree is 125 feet when the angle of elevation of the sun is 33°. Approximate the height of the tree.

21. **Height** At a point 50 feet from the base of a church, the angles of elevation to the bottom of the steeple and the top of the steeple are 35° and 48°, respectively. Find the height of the steeple.

22. **Distance** An observer in a lighthouse 350 feet above sea level observes two ships directly offshore. The angles of depression to the ships are 4° and 6.5° (see figure). How far apart are the ships?



23. **Distance** A passenger in an airplane at an altitude of 10 kilometers sees two towns directly to the east of the plane. The angles of depression to the towns are 28° and 55° (see figure). How far apart are the towns?



24. **Angle of Elevation** The height of an outdoor basketball backboard is $12\frac{1}{2}$ feet, and the backboard casts a shadow 17 feet long.

(a) Draw a right triangle that gives a visual representation of the problem. Label the known and unknown quantities.

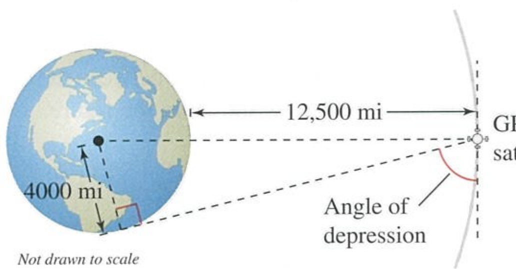
(b) Use a trigonometric function to write an equation involving the unknown angle of elevation.

(c) Find the angle of elevation.

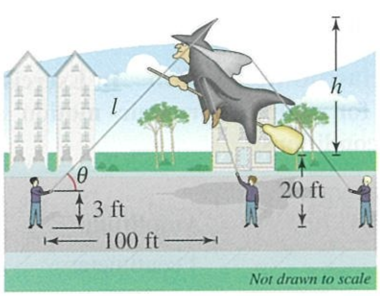
25. **Angle of Elevation** An engineer designs a 75-foot cellular telephone tower. Find the angle of elevation to the top of the tower at a point on level ground 50 feet from its base.

26. **Angle of Depression** A cellular telephone tower that is 120 feet tall is placed on top of a mountain that is 1200 feet above sea level. What is the angle of depression from the top of the tower to a cell phone user who is 5 horizontal miles away and 400 feet above sea level?

27. **Angle of Depression** A Global Positioning System satellite orbits 12,500 miles above Earth's surface (see figure). Find the angle of depression from the satellite to the horizon. Assume the radius of Earth is 4000 miles.



28. **Height** You are holding one of the tethers attached to the top of a giant character balloon that is floating approximately 20 feet above ground level. You are standing approximately 100 feet ahead of the balloon (see figure).



(a) Find an equation for the length l of the tether you are holding in terms of h, the height of the balloon from top to bottom.

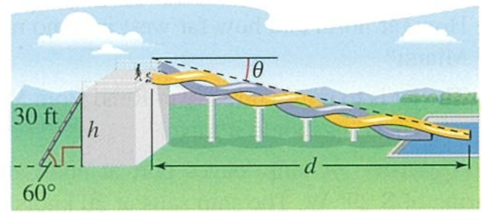
(b) Find an equation for the angle of elevation θ from you to the top of the balloon.

(c) The angle of elevation to the top of the balloon is 35°. Find the height h of the balloon.

29. **Altitude** You observe a plane approaching overhead and assume that its speed is 550 miles per hour. The angle of elevation of the plane is 16° at one time and 57° one minute later. Approximate the altitude of the plane.

30. **Waterslide Design**

The designers of a water park have sketched a preliminary drawing of a new slide (see figure).

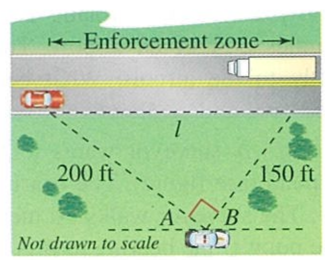


(a) Find the height h of the slide.

(b) Find the angle of depression θ from the top of the slide to the end of the slide at the ground in terms of the horizontal distance d a rider travels.

(c) Safety restrictions require the angle of depression to be no less than 25° and no more than 30°. Find an interval for how far a rider travels horizontally.

31. **Speed Enforcement** A police department has set up a speed enforcement zone on a straight length of highway. A patrol car is parked parallel to the zone, 200 feet from one end and 150 feet from the other end (see figure).



(a) Find the length l of the zone and the measures of angles A and B (in degrees).

(b) Find the minimum amount of time (in seconds) it takes for a vehicle to pass through the zone without exceeding the posted speed limit of 35 miles per hour.

32. **Airplane Ascent** During takeoff, an airplane's angle of ascent is 18° and its speed is 260 feet per second.

(a) Find the plane's altitude after 1 minute.

(b) How long will it take for the plane to climb to an altitude of 10,000 feet?

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33. Air Navigation An airplane flying at 550 miles per hour has a bearing of 52°. After flying for 1.5 hours, how far north and how far east will the plane have traveled from its point of departure?

34. Air Navigation A jet leaves Reno, Nevada, and heads toward Miami, Florida, at a bearing of 100°. The distance between the two cities is approximately 2472 miles.

(a) How far north and how far west is Reno relative to Miami?

(b) The jet is to return directly to Reno from Miami. At what bearing should it travel?

35. Navigation A ship leaves port at noon and has a bearing of S 29° W. The ship sails at 20 knots.

(a) How many nautical miles south and how many nautical miles west will the ship have traveled by 6:00 P.M.?

(b) At 6:00 P.M., the ship changes course to due west. Find the ship's bearing and distance from port at 7:00 P.M.

36. Navigation A privately owned yacht leaves a dock in Myrtle Beach, South Carolina, and heads toward Freeport in the Bahamas at a bearing of S 1.4° E. The yacht averages a speed of 20 knots over the 428-nautical-mile trip.

(a) How long will it take the yacht to make the trip?

(b) How far east and south is the yacht after 12 hours?

(c) A plane leaves Myrtle Beach to fly to Freeport. At what bearing should it travel?

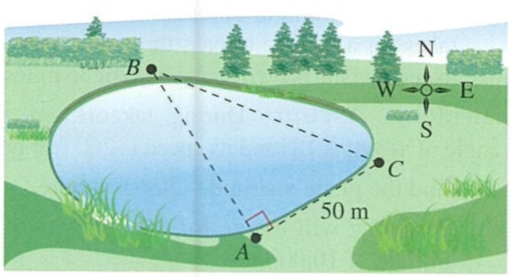
37. Navigation A ship is 45 miles east and 30 miles south of port. The captain wants to sail directly to port. What bearing should the captain take?

38. Air Navigation An airplane is 160 miles north and 85 miles east of an airport. The pilot wants to fly directly to the airport. What bearing should the pilot take?

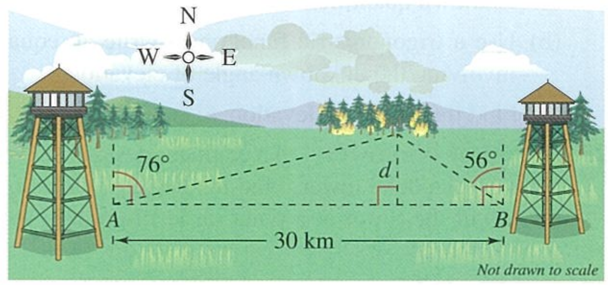
39. Surveying A surveyor wants to find the distance across a pond (see figure). The bearing from A to B is N 32° W. The surveyor walks 50 meters from A to C, and at the point C the bearing to B is N 68° W.

(a) Find the bearing from A to C.

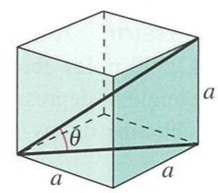
(b) Find the distance from A to B.



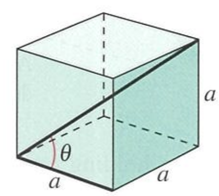
40. Location of a Fire Fire tower A is 30 kilometers due west of fire tower B. A fire is spotted from the towers, and the bearings from A and B are N 76° E and N 56° W, respectively (see figure). Find the distance d of the fire from the line segment AB.



41. Geometry Determine the angle between the diagonal of a cube and the diagonal of its base, as shown in the figure.



42. Geometry Determine the angle between the diagonal of a cube and its edge, as shown in the figure.



43. Geometry Find the length of the sides of a regular pentagon inscribed in a circle of radius 25 inches.

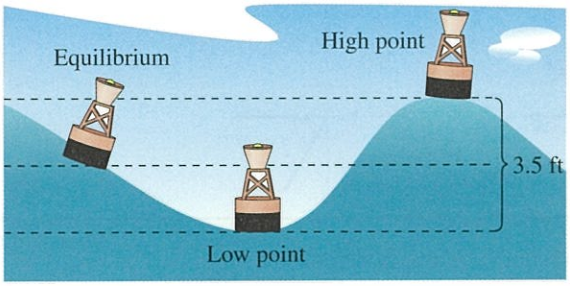
44. Geometry Find the length of the sides of a regular hexagon inscribed in a circle of radius 25 inches.

Simple Harmonic Motion **In Exercises 45–48, find a model for simple harmonic motion satisfying the specified conditions.**

	Displacement ($t = 0$)	Amplitude	Period
45.	0	4 centimeters	2 seconds
46.	0	3 meters	6 seconds
47.	3 inches	3 inches	1.5 seconds
48.	2 feet	2 feet	10 seconds

49. Tuning Fork A point on the end of a tuning fork moves in simple harmonic motion described by $d = a \sin \omega t$. Find ω given that the tuning fork for middle C has a frequency of 262 vibrations per second.

50. Wave Motion A buoy oscillates in simple harmonic motion as waves go past. The buoy moves a total of 3.5 feet from its low point to its high point (see figure), and it returns to its high point every 10 seconds. Write an equation that describes the motion of the buoy where the high point corresponds to the time $t = 0$.



Simple Harmonic Motion In Exercises 51–54, for the simple harmonic motion described by the trigonometric function, find (a) the maximum displacement, (b) the frequency, (c) the value of d when $t = 5$, and (d) the least positive value of t for which $d = 0$. Use a graphing utility to verify your results.

51. $d = 9 \cos \frac{6\pi}{5}t$

52. $d = \frac{1}{2} \cos 20\pi t$

53. $d = \frac{1}{4} \sin 6\pi t$

54. $d = \frac{1}{64} \sin 792\pi t$

55. Oscillation of a Spring A ball that is bobbing up and down on the end of a spring has a maximum displacement of 3 inches. Its motion (in ideal conditions) is modeled by $y = \frac{1}{4} \cos 16t$, $t > 0$, where y is measured in feet and t is the time in seconds.

(a) Graph the function.

(b) What is the period of the oscillations?

(c) Determine the first time the weight passes the point of equilibrium $(y = 0)$.

56. Hours of Daylight The numbers of hours H of daylight in Denver, Colorado, on the 15th of each month starting with January are: 9.68, 10.72, 11.92, 13.25, 14.35, 14.97, 14.72, 13.73, 12.47, 11.18, 10.00, and 9.37. A model for the data is

$$H(t) = 12.13 + 2.77 \sin\left(\frac{\pi t}{6} - 1.60\right)$$

where t represents the month, with $t = 1$ corresponding to January. *(Source: United States Navy)*

(a) Use a graphing utility to graph the data and the model in the same viewing window.

(b) What is the period of the model? Is it what you expected? Explain.

(c) What is the amplitude of the model? What does it represent in the context of the problem?

57. Sales The table shows the average sales S (in millions of dollars) of an outerwear manufacturer for each month t, where $t = 1$ corresponds to January.

Time, t	1	2	3	4	5	6	7	8	9	10	11	12
Sales, S	13.46	11.15	8.00	4.85	2.54	1.70	2.54	4.85	8.00	11.15	13.46	14.30

(a) Create a scatter plot of the data.

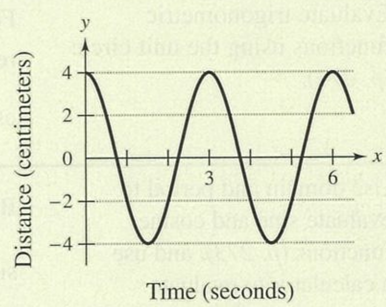
(b) Find a trigonometric model that fits the data. Graph the model with your scatter plot. How well does the model fit the data?

(c) What is the period of the model? Do you think it is reasonable given the context? Explain.

(d) Interpret the meaning of the model's amplitude in the context of the problem.

Exploration

58. HOW DO YOU SEE IT? The graph below shows the displacement of an object in simple harmonic motion.



(a) What is the amplitude?

(b) What is the period?

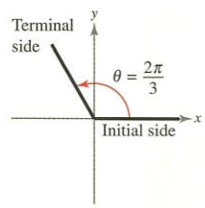
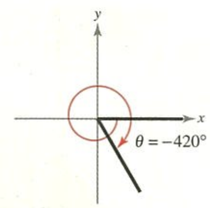
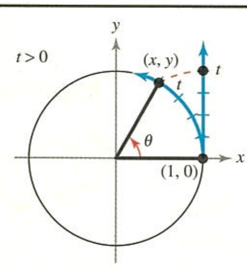
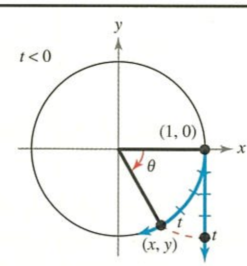
(c) Is the equation of the simple harmonic motion of the form $d = a \sin \omega t$ or $d = a \cos \omega t$?

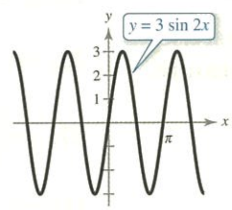
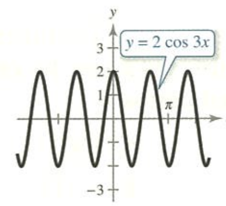
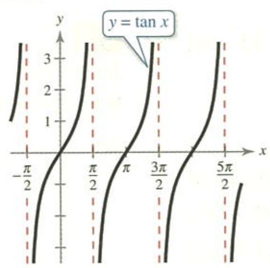
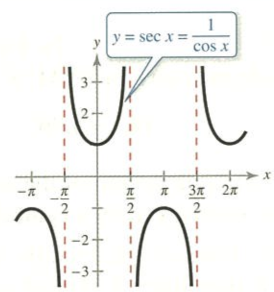
True or False? In Exercises 59 and 60, determine whether the statement is true or false. Justify your answer.

59. The Leaning Tower of Pisa is not vertical, but when you know the angle of elevation θ to the top of the tower as you stand d feet away from it, its height h can be found using the formula $h = d \tan \theta$.

60. The bearing N 24° E means 24 degrees north of east.

Chapter Summary

	What Did You Learn?	Explanation/Examples	Review Exercises
Section 4.1	Describe angles *(p. 260).*	 	1–4
	Use radian measure *(p. 261)* and degree measure *(p. 263).*	To convert degrees to radians, multiply degrees by $\frac{\pi \text{ rad}}{180°}$. To convert radians to degrees, multiply radians by $\frac{180°}{\pi \text{ rad}}$.	5–14
	Use angles and their measure to model and solve real-life problems *(p. 264).*	Angles and their measure can be used to find arc length and the area of a sector of a circle. (See Examples 5 and 8.)	15–18
Section 4.2	Identify a unit circle and describe its relationship to real numbers *(p. 270).*	 	19–22
	Evaluate trigonometric functions using the unit circle *(p. 271).*	For the point (x, y) on the unit circle corresponding to a real number t: $\sin t = y$; $\cos t = x$; $\tan t = \frac{y}{x}$, $x \neq 0$; $\csc t = \frac{1}{y}$, $y \neq 0$; $\sec t = \frac{1}{x}$, $x \neq 0$; and $\cot t = \frac{x}{y}$, $y \neq 0$.	23, 24
	Use domain and period to evaluate sine and cosine functions *(p. 273),* and use a calculator to evaluate trigonometric functions *(p. 274).*	Because $\frac{13\pi}{6} = 2\pi + \frac{\pi}{6}$, $\sin \frac{13\pi}{6} = \sin \frac{\pi}{6} = \frac{1}{2}$. $\sin \frac{3\pi}{8} \approx 0.9239$, $\cot(-1.2) \approx -0.3888$	25–32
Section 4.3	Evaluate trigonometric functions of acute angles *(p. 277).*	$\sin \theta = \frac{\text{opp}}{\text{hyp}}$, $\cos \theta = \frac{\text{adj}}{\text{hyp}}$, $\tan \theta = \frac{\text{opp}}{\text{adj}}$ $\csc \theta = \frac{\text{hyp}}{\text{opp}}$, $\sec \theta = \frac{\text{hyp}}{\text{adj}}$, $\cot \theta = \frac{\text{adj}}{\text{opp}}$ $\csc 29° \, 15' = 1/\sin 29.25° \approx 2.0466$	33–38
	Use fundamental trigonometric identities *(p. 280).*	$\sin \theta = \frac{1}{\csc \theta}$, $\tan \theta = \frac{\sin \theta}{\cos \theta}$, $\sin^2 \theta + \cos^2 \theta = 1$	39, 40
	Use trigonometric functions to model and solve real-life problems *(p. 282).*	Trigonometric functions can be used to find the height of a monument, the angle between two paths, and the length and height of a ramp. (See Examples 8–10.)	41, 42

	What Did You Learn?	Explanation/Examples	Review Exercises
Section 4.4	Evaluate trigonometric functions of any angle *(p. 288)*.	Let (3, 4) be a point on the terminal side of θ. Then $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, and $\tan\theta = \frac{4}{3}$.	43–50
	Find reference angles *(p. 290)*.	Let θ be an angle in standard position. Its reference angle is the acute angle θ' formed by the terminal side of θ and the horizontal axis.	51–54
	Evaluate trigonometric functions of real numbers *(p. 291)*.	$\cos\frac{7\pi}{3} = \frac{1}{2}$ because $\theta' = \frac{7\pi}{3} - 2\pi = \frac{\pi}{3}$ and $\cos\frac{\pi}{3} = \frac{1}{2}$.	55–62
Section 4.5	Sketch the graphs of sine and cosine functions using amplitude and period *(p. 299)*.	 	63, 64
	Sketch translations of the graphs of sine and cosine functions *(p. 301)*.	For $y = d + a\sin(bx - c)$ and $y = d + a\cos(bx - c)$, the constant c results in horizontal translations and the constant d results in vertical translations. (See Examples 4–6.)	65–68
	Use sine and cosine functions to model real-life data *(p. 303)*.	A cosine function can be used to model the depth of the water at the end of a dock. (See Example 7.)	69, 70
Section 4.6	Sketch the graphs of tangent *(p. 308)*, cotangent *(p. 310)*, secant *(p. 311)*, and cosecant functions *(p. 311)*.	 	71–74
	Sketch the graphs of damped trigonometric functions *(p. 313)*.	In $f(x) = x\cos 2x$, the factor x is called the damping factor.	75, 76
Section 4.7	Evaluate and graph inverse trigonometric functions *(p. 318)*.	$\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$, $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$, $\tan^{-1}\sqrt{3} = \frac{\pi}{3}$	77–86
	Evaluate compositions with inverse trigonometric functions *(p. 322)*.	$\sin(\sin^{-1} 0.4) = 0.4$, $\cos\left(\arctan\frac{5}{12}\right) = \frac{12}{13}$	87–92
Section 4.8	Solve real-life problems involving right triangles *(p. 328)*.	A trigonometric function can be used to find the height of a smokestack on top of a building. (See Example 3.)	93, 94
	Solve real-life problems involving directional bearings *(p. 330)*.	Trigonometric functions can be used to find a ship's bearing and distance from a port at a given time. (See Example 5.)	95
	Solve real-life problems involving harmonic motion *(p. 331)*.	Trigonometric functions can be used to describe the motion of a point on an object that vibrates, oscillates, rotates, or is moved by wave motion. (See Examples 6 and 7.)	96

Review Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

4.1 Using Radian or Degree Measure In Exercises 1–4, (a) sketch the angle in standard position, (b) determine the quadrant in which the angle lies, and (c) determine two coterminal angles (one positive and one negative).

1. $\frac{15\pi}{4}$
2. $-\frac{4\pi}{3}$
3. $-110°$
4. $280°$

Converting from Degrees to Radians In Exercises 5–8, convert the degree measure to radian measure. Round to three decimal places.

5. $450°$
6. $190°$
7. $-16°$
8. $-112°$

Converting from Radians to Degrees In Exercises 9–12, convert the radian measure to degree measure. Round to three decimal places, if necessary.

9. $\frac{3\pi}{10}$
10. $-\frac{11\pi}{6}$
11. -3.5
12. 5.7

Converting to D° M′ S″ Form In Exercises 13 and 14, convert the angle measure to D° M′ S″ form.

13. $198.4°$
14. $-5.96°$

15. **Arc Length** Find the length of the arc on a circle of radius 20 inches intercepted by a central angle of 138°.

16. **Phonograph** Phonograph records are vinyl discs that rotate on a turntable. A typical record album is 12 inches in diameter and plays at $33\frac{1}{3}$ revolutions per minute.
    (a) Find the angular speed of a record album.
    (b) Find the linear speed (in inches per minute) of the outer edge of a record album.

Area of a Sector of a Circle In Exercises 17 and 18, find the area of the sector of a circle of radius r and central angle θ.

	Radius r	Central Angle θ
17.	20 inches	150°
18.	7.5 millimeters	$2\pi/3$ radians

4.2 Finding a Point on the Unit Circle In Exercises 19–22, find the point (x, y) on the unit circle that corresponds to the real number t.

19. $t = 2\pi/3$
20. $t = 7\pi/4$
21. $t = 7\pi/6$
22. $t = -4\pi/3$

Evaluating Trigonometric Functions In Exercises 23 and 24, evaluate (if possible) the six trigonometric functions at the real number.

23. $t = \frac{3\pi}{4}$
24. $t = -\frac{2\pi}{3}$

Using Period to Evaluate Sine and Cosine In Exercises 25–28, evaluate the trigonometric function using its period as an aid.

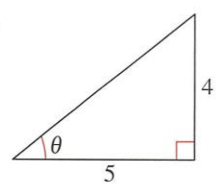
25. $\sin \frac{11\pi}{4}$
26. $\cos 4\pi$
27. $\cos\left(-\frac{17\pi}{6}\right)$
28. $\sin\left(-\frac{13\pi}{3}\right)$

Using a Calculator In Exercises 29–32, use a calculator to evaluate the trigonometric function. Round your answer to four decimal places. (Be sure the calculator is in the correct mode.)

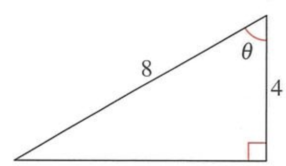
29. $\sec \frac{12\pi}{5}$
30. $\sin\left(-\frac{\pi}{9}\right)$
31. $\tan 33$
32. $\csc 10.5$

4.3 Evaluating Trigonometric Functions In Exercises 33 and 34, find the exact values of the six trigonometric functions of the angle θ.

33.



34.



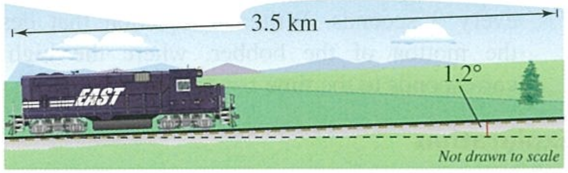
Using a Calculator In Exercises 35–38, use a calculator to evaluate the trigonometric function. Round your answer to four decimal places. (Be sure the calculator is in the correct mode.)

35. $\tan 33°$
36. $\sec 79.3°$
37. $\cot 15° 14'$
38. $\cos 78° 11' 58''$

Applying Trigonometric Identities In Exercises 39 and 40, use the given function value and the trigonometric identities to find the exact value of each indicated trigonometric function.

39. $\sin \theta = \frac{1}{3}$
    (a) $\csc \theta$ (b) $\cos \theta$
    (c) $\sec \theta$ (d) $\tan \theta$
40. $\csc \theta = 5$
    (a) $\sin \theta$ (b) $\cot \theta$
    (c) $\tan \theta$ (d) $\sec(90° - \theta)$

41. **Railroad Grade** A train travels 3.5 kilometers on a straight track with a grade of 1.2° (see figure). What is the vertical rise of the train in that distance?



42. **Guy Wire** A guy wire runs from the ground to the top of a 25-foot telephone pole. The angle formed between the wire and the ground is 52°. How far from the base of the pole is the guy wire anchored to the ground? Assume the pole is perpendicular to the ground.

4.4 Evaluating Trigonometric Functions **In Exercises 43–46, the point is on the terminal side of an angle in standard position. Find the exact values of the six trigonometric functions of the angle.**

43. (12, 16)
44. (3, −4)
45. (0.3, 0.4)
46. $\left(-\frac{10}{3}, -\frac{2}{3}\right)$

Evaluating Trigonometric Functions **In Exercises 47–50, find the exact values of the remaining five trigonometric functions of θ satisfying the given conditions.**

47. $\sec\theta = \frac{6}{5}$, $\tan\theta < 0$
48. $\csc\theta = \frac{3}{2}$, $\cos\theta < 0$
49. $\cos\theta = -\frac{2}{5}$, $\sin\theta > 0$
50. $\sin\theta = -\frac{1}{2}$, $\cos\theta > 0$

Finding a Reference Angle **In Exercises 51–54, find the reference angle θ'. Sketch θ in standard position and label θ'.**

51. $\theta = 264°$
52. $\theta = 635°$
53. $\theta = -6\pi/5$
54. $\theta = 17\pi/3$

Using a Reference Angle **In Exercises 55–58, evaluate the sine, cosine, and tangent of the angle without using a calculator.**

55. −150°
56. 495°
57. $\pi/3$
58. $-5\pi/4$

Using a Calculator **In Exercises 59–62, use a calculator to evaluate the trigonometric function. Round your answer to four decimal places. (Be sure the calculator is in the correct mode.)**

59. sin 106°
60. tan 37°
61. $\tan(-17\pi/15)$
62. $\cos(-25\pi/7)$

4.5 Sketching the Graph of a Sine or Cosine Function **In Exercises 63–68, sketch the graph of the function. (Include two full periods.)**

63. $y = \sin 6x$
64. $f(x) = -\cos 3x$
65. $y = 5 + \sin \pi x$
66. $y = -4 - \cos \pi x$
67. $g(t) = \frac{5}{2}\sin(t - \pi)$
68. $g(t) = 3\cos(t + \pi)$

69. **Sound Waves** Sound waves can be modeled using sine functions of the form $y = a \sin bx$, where x is measured in seconds.
    (a) Write an equation of a sound wave whose amplitude is 2 and whose period is $\frac{1}{264}$ second.
    (b) What is the frequency of the sound wave described in part (a)?

70. **Meteorology** The times S of sunset (Greenwich Mean Time) at 40° north latitude on the 15th of each month starting with January are: 16:59, 17:35, 18:06, 18:38, 19:08, 19:30, 19:28, 18:57, 18:10, 17:21, 16:44, and 16:36. A model (in which minutes have been converted to the decimal parts of an hour) for the data is

$$S(t) = 18.10 - 1.41 \sin\left(\frac{\pi t}{6} + 1.55\right)$$

where t represents the month, with $t = 1$ corresponding to January. *(Source: NOAA)*
    (a) Use a graphing utility to graph the data and the model in the same viewing window.
    (b) What is the period of the model? Is it what you expected? Explain.
    (c) What is the amplitude of the model? What does it represent in the context of the problem?

4.6 Sketching the Graph of a Trigonometric Function **In Exercises 71–74, sketch the graph of the function. (Include two full periods.)**

71. $f(t) = \tan\left(t + \frac{\pi}{2}\right)$
72. $f(x) = \frac{1}{2}\cot x$
73. $f(x) = \frac{1}{2}\csc \frac{x}{2}$
74. $h(t) = \sec\left(t - \frac{\pi}{4}\right)$

Analyzing a Damped Trigonometric Graph **In Exercises 75 and 76, use a graphing utility to graph the function and the damping factor of the function in the same viewing window. Describe the behavior of the function as x increases without bound.**

75. $f(x) = x \cos x$
76. $g(x) = e^x \cos x$

4.7 Evaluating an Inverse Trigonometric Function **In Exercises 77–80, find the exact value of the expression.**

77. $\arcsin(-1)$
78. $\cos^{-1} 1$
79. $\text{arccot}\ \sqrt{3}$
80. $\text{arcsec}(-\sqrt{2})$

Calculators and Inverse Trigonometric Functions **In Exercises 81–84, use a calculator to approximate the value of the expression, if possible. Round your result to two decimal places.**

81. $\tan^{-1}(-1.3)$
82. $\arccos 0.372$
83. $\text{arccot}\ 15.5$
84. $\text{arccsc}(-4.03)$

Graphing an Inverse Trigonometric Function **In Exercises 85 and 86, use a graphing utility to graph the function.**

85. $f(x) = \arctan(x/2)$
86. $f(x) = -\arcsin 2x$

Evaluating a Composition of Functions **In Exercises 87–90, find the exact value of the expression.**

87. $\cos\left(\arctan \frac{3}{4}\right)$
88. $\tan\left(\arccos \frac{3}{5}\right)$
89. $\sec\left(\arctan \frac{12}{5}\right)$
90. $\cot\left[\arcsin\left(-\frac{12}{13}\right)\right]$

Writing an Expression **In Exercises 91 and 92, write an algebraic expression that is equivalent to the given expression.**

91. $\tan[\arccos(x/2)]$
92. $\sec[\arcsin(x-1)]$

4.8

93. **Angle of Elevation** The height of a radio transmission tower is 70 meters, and it casts a shadow of length 30 meters. Draw a right triangle that gives a visual representation of the problem. Label the known and unknown quantities. Then find the angle of elevation.

94. **Height** A football lands at the edge of the roof of your school building. When you are 25 feet from the base of the building, the angle of elevation to the football is 21°. How high off the ground is the football?

95. **Air Navigation** From city A to city B, a plane flies 650 miles at a bearing of 48°. From city B to city C, the plane flies 810 miles at a bearing of 115°. Find the distance from city A to city C and the bearing from city A to city C.

96. **Wave Motion** A fishing bobber oscillates in simple harmonic motion because of the waves in a lake. The bobber moves a total of 1.5 inches from its low point to its high point and returns to its high point every 3 seconds. Write an equation that describes the motion of the bobber, where the high point corresponds to the time $t = 0$.

Exploration

True or False? **In Exercises 97 and 98, determine whether the statement is true or false. Justify your answer.**

97. $y = \sin\theta$ is not a function because $\sin 30° = \sin 150°$.
98. Because $\tan(3\pi/4) = -1$, $\arctan(-1) = 3\pi/4$.

99. **Writing** Describe the behavior of $f(\theta) = \sec\theta$ at the zeros of $g(\theta) = \cos\theta$. Explain.

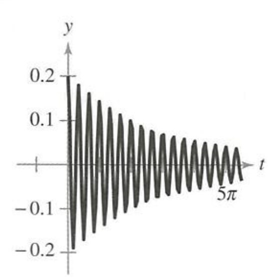
100. **Conjecture**
(a) Use a graphing utility to complete the table.

θ	0.1	0.4	0.7	1.0	1.3
$\tan\left(\theta - \frac{\pi}{2}\right)$					
$-\cot\theta$					

(b) Make a conjecture about the relationship between $\tan[\theta - (\pi/2)]$ and $-\cot\theta$.

101. **Writing** When graphing the sine and cosine functions, determining the amplitude is part of the analysis. Explain why this is not true for the other four trigonometric functions.

102. **Oscillation of a Spring** A weight is suspended from a ceiling by a steel spring. The weight is lifted (positive direction) from the equilibrium position and released. The resulting motion of the weight is modeled by $y = Ae^{-kt}\cos bt = \frac{1}{5}e^{-t/10}\cos 6t$, where y is the distance (in feet) from equilibrium and t is the time (in seconds). The figure shows the graph of the function. For each of the following, describe the change in the graph without graphing the resulting function.
(a) A is changed from $\frac{1}{5}$ to $\frac{1}{3}$.
(b) k is changed from $\frac{1}{10}$ to $\frac{1}{3}$.
(c) b is changed from 6 to 9.



Chapter Test

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Take this test as you would take a test in class. When you are finished, check your work against the answers given in the back of the book.

1. Consider an angle that measures $\frac{5\pi}{4}$ radians.

(a) Sketch the angle in standard position.

(b) Determine two coterminal angles (one positive and one negative).

(c) Convert the radian measure to degree measure.

2. A truck is moving at a rate of 105 kilometers per hour, and the diameter of each of its wheels is 1 meter. Find the angular speed of the wheels in radians per minute.

3. A water sprinkler sprays water on a lawn over a distance of 25 feet and rotates through an angle of 130°. Find the area of the lawn watered by the sprinkler.

4. Given that θ is an acute angle and $\tan\theta = \frac{3}{2}$, find the exact values of the other five trigonometric functions of θ.

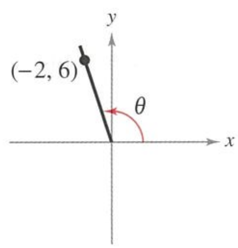


Figure for 5

5. Find the exact values of the six trigonometric functions of the angle θ shown in the figure.

6. Find the reference angle θ' of the angle $\theta = 205°$. Sketch θ in standard position and label θ'.

7. Determine the quadrant in which θ lies when $\sec\theta < 0$ and $\tan\theta > 0$.

8. Find two exact values of θ in degrees $(0° \le \theta < 360°)$ for which $\cos\theta = -\sqrt{3}/2$. Do not use a calculator.

In Exercises 9 and 10, find the exact values of the remaining five trigonometric functions of θ satisfying the given conditions.

9. $\cos\theta = \frac{3}{5}, \quad \tan\theta < 0$

10. $\sec\theta = -\frac{29}{20}, \quad \sin\theta > 0$

In Exercises 11–13, sketch the graph of the function. (Include two full periods.)

11. $g(x) = -2\sin\left(x - \frac{\pi}{4}\right)$

12. $f(t) = \cos\left(t + \frac{\pi}{2}\right) - 1$

13. $f(x) = \frac{1}{2}\tan 2x$

In Exercises 14 and 15, use a graphing utility to graph the function. If the function is periodic, find its period. If not, describe the behavior of the function as x increases without bound.

14. $y = \sin 2\pi x + 2\cos\pi x$

15. $y = 6e^{-0.12x}\cos(0.25x)$

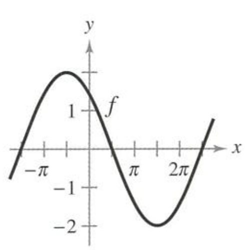


Figure for 16

16. Find a, b, and c for the function $f(x) = a\sin(bx + c)$ such that the graph of f matches the figure.

17. Find the exact value of $\cot\left(\arcsin\frac{3}{8}\right)$.

18. Sketch the graph of the function $f(x) = 2\arcsin\left(\frac{1}{2}x\right)$.

19. An airplane is 90 miles south and 110 miles east of an airport. What bearing should the pilot take to fly directly to the airport?

20. A ball on a spring starts at its lowest point of 6 inches below equilibrium, bounces to its maximum height of 6 inches above equilibrium, and returns to its lowest point in a total of 2 seconds. Write an equation for the simple harmonic motion of the ball.

Proofs in Mathematics

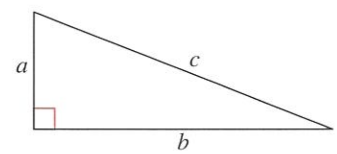
The Pythagorean Theorem

The Pythagorean Theorem is one of the most famous theorems in mathematics. More than 350 different proofs now exist. James A. Garfield, the twentieth president of the United States, developed a proof of the Pythagorean Theorem in 1876. His proof, shown below, involves the fact that two congruent right triangles and an isosceles right triangle can form a trapezoid.

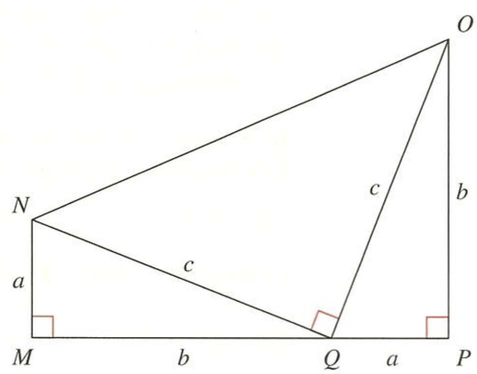
The Pythagorean Theorem

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse, where a and b are the lengths of the legs and c is the length of the hypotenuse.

$$a^2 + b^2 = c^2$$



Proof



$$\text{Area of trapezoid } MNOP = \text{Area of } \triangle MNQ + \text{Area of } \triangle PQO + \text{Area of } \triangle NOQ$$

$$\frac{1}{2}(a + b)(a + b) = \frac{1}{2}ab + \frac{1}{2}ab + \frac{1}{2}c^2$$

$$\frac{1}{2}(a + b)(a + b) = ab + \frac{1}{2}c^2$$

$$(a + b)(a + b) = 2ab + c^2$$

$$a^2 + 2ab + b^2 = 2ab + c^2$$

$$a^2 + b^2 = c^2$$

P.S. Problem Solving

1. Angle of Rotation The restaurant at the top of the Space Needle in Seattle, Washington, is circular and has a radius of 47.25 feet. The dining part of the restaurant revolves, making about one complete revolution every 48 minutes. A dinner party, seated at the edge of the revolving restaurant at 6:45 P.M., finishes at 8:57 P.M.

(a) Find the angle through which the dinner party rotated.

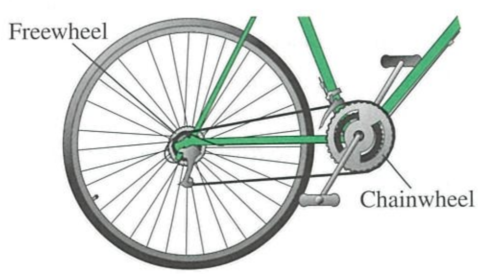
(b) Find the distance the party traveled during dinner.

2. Bicycle Gears A bicycle's gear ratio is the number of times the freewheel turns for every one turn of the chainwheel (see figure). The table shows the numbers of teeth in the freewheel and chainwheel for the first five gears of an 18-speed touring bicycle. The chainwheel completes one rotation for each gear. Find the angle through which the freewheel turns for each gear. Give your answers in both degrees and radians.

DATA

Spreadsheet at LarsonPrecalculus.com

Gear Number	Number of Teeth in Freewheel	Number of Teeth in Chainwheel
1	32	24
2	26	24
3	22	24
4	32	40
5	19	24



3. Height of a Ferris Wheel Car A model for the height h (in feet) of a Ferris wheel car is

$$h = 50 + 50 \sin 8\pi t$$

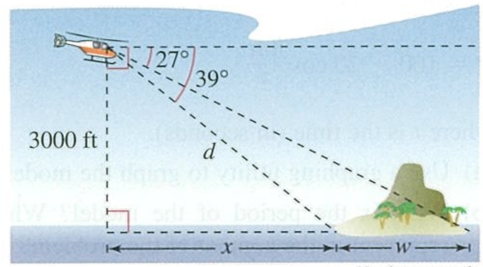
where t is the time (in minutes). (The Ferris wheel has a radius of 50 feet.) This model yields a height of 50 feet when $t = 0$. Alter the model so that the height of the car is 1 foot when $t = 0$.

4. Periodic Function The function f is periodic, with period c. So, $f(t + c) = f(t)$. Determine whether each statement is true or false. Explain.

(a) $f(t - 2c) = f(t)$ (b) $f\left(t + \frac{1}{2}c\right) = f\left(\frac{1}{2}t\right)$

(c) $f\left(\frac{1}{2}[t + c]\right) = f\left(\frac{1}{2}t\right)$ (d) $f\left(\frac{1}{2}[t + 4c]\right) = f\left(\frac{1}{2}t\right)$

5. Surveying A surveyor in a helicopter is determining the width of an island, as shown in the figure.



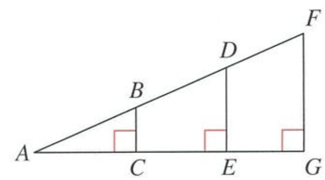
Not drawn to scale

(a) What is the shortest distance d the helicopter must travel to land on the island?

(b) What is the horizontal distance x the helicopter must travel before it is directly over the nearer end of the island?

(c) Find the width w of the island. Explain how you found your answer.

6. Similar Triangles and Trigonometric Functions Use the figure below.



(a) Explain why $\triangle ABC$, $\triangle ADE$, and $\triangle AFG$ are similar triangles.

(b) What does similarity imply about the ratios

$$\frac{BC}{AB}, \quad \frac{DE}{AD}, \quad \text{and} \quad \frac{FG}{AF}?$$

(c) Does the value of $\sin A$ depend on which triangle from part (a) is used to calculate it? Does the value of $\sin A$ change when you use a different right triangle similar to the three given triangles?

(d) Do your conclusions from part (c) apply to the other five trigonometric functions? Explain.

7. Using Technology Use a graphing utility to graph h, and use the graph to determine whether h is even, odd, or neither.

(a) $h(x) = \cos^2 x$ (b) $h(x) = \sin^2 x$

8. Squares of Even and Odd Functions Given that f is an even function and g is an odd function, use the results of Exercise 7 to make a conjecture about each function h.

(a) $h(x) = [f(x)]^2$ (b) $h(x) = [g(x)]^2$

9. **Blood Pressure** The pressure P (in millimeters of mercury) against the walls of the blood vessels of a patient is modeled by

$$P = 100 - 20\cos\frac{8\pi t}{3}$$

where t is the time (in seconds).

(a) Use a graphing utility to graph the model.

(b) What is the period of the model? What does it represent in the context of the problem?

(c) What is the amplitude of the model? What does it represent in the context of the problem?

(d) If one cycle of this model is equivalent to one heartbeat, what is the pulse of the patient?

(e) A physician wants the patient's pulse rate to be 64 beats per minute or less. What should the period be? What should the coefficient of t be?

10. **Biorhythms** A popular theory that attempts to explain the ups and downs of everyday life states that each person has three cycles, called biorhythms, which begin at birth. These three cycles can be modeled by the sine functions below, where t is the number of days since birth.

Physical (23 days): $P = \sin\frac{2\pi t}{23},\quad t \geq 0$

Emotional (28 days): $E = \sin\frac{2\pi t}{28},\quad t \geq 0$

Intellectual (33 days): $I = \sin\frac{2\pi t}{33},\quad t \geq 0$

Consider a person who was born on July 20, 1995.

(a) Use a graphing utility to graph the three models in the same viewing window for $7300 \leq t \leq 7380$.

(b) Describe the person's biorhythms during the month of September 2015.

(c) Calculate the person's three energy levels on September 22, 2015.

11. **Graphical Reasoning**

(a) Use a graphing utility to graph the functions

$f(x) = 2\cos 2x + 3\sin 3x$

and

$g(x) = 2\cos 2x + 3\sin 4x.$

(b) Use the graphs from part (a) to find the period of each function.

(c) Is the function $h(x) = A\cos\alpha x + B\sin\beta x$, where α and β are positive integers, periodic? Explain.

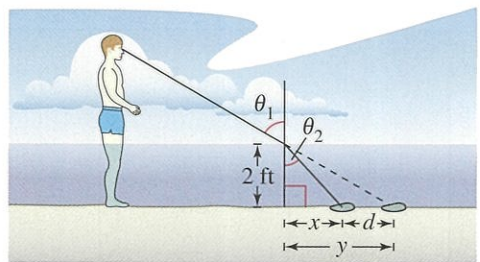
12. **Analyzing Trigonometric Functions** Two trigonometric functions f and g have periods of 2, and their graphs intersect at $x = 5.35$.

(a) Give one positive value of x less than 5.35 and one value of x greater than 5.35 at which the functions have the same value.

(b) Determine one negative value of x at which the graphs intersect.

(c) Is it true that $f(13.35) = g(-4.65)$? Explain.

13. **Refraction** When you stand in shallow water and look at an object below the surface of the water, the object will look farther away from you than it really is. This is because when light rays pass between air and water, the water refracts, or bends, the light rays. The index of refraction for water is 1.333. This is the ratio of the sine of θ_1 and the sine of θ_2 (see figure).



(a) While standing in water that is 2 feet deep, you look at a rock at angle $\theta_1 = 60°$ (measured from a line perpendicular to the surface of the water). Find θ_2.

(b) Find the distances x and y.

(c) Find the distance d between where the rock is and where it appears to be.

(d) What happens to d as you move closer to the rock? Explain.

14. **Polynomial Approximation** Using calculus, it can be shown that the arctangent function can be approximated by the polynomial

$$\arctan x \approx x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7}$$

where x is in radians.

(a) Use a graphing utility to graph the arctangent function and its polynomial approximation in the same viewing window. How do the graphs compare?

(b) Study the pattern in the polynomial approximation of the arctangent function and predict the next term. Then repeat part (a). How does the accuracy of the approximation change when an additional term is added?