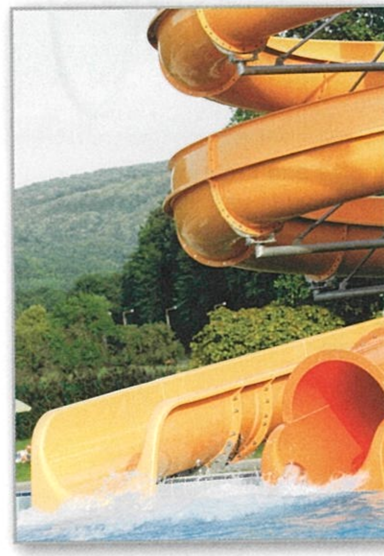
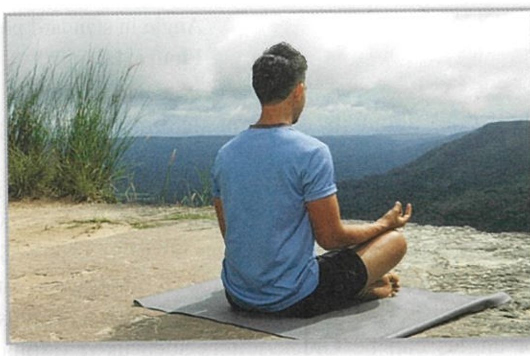
4 Trigonometry



Waterslide Design (Exercise 30, page 335)

Television Coverage (Exercise 85, page 317)



Respiratory Cycle (Exercise 80, page 306)

Temperature of a City (Exercise 99, page 296)

Skateboard Ramp (Example 10, page 283)

Clockwise from top left, Irin-k/Shutterstock.com; István Csak | Dreamstime; iStockphoto.com/DenisTangneyJr; iStockphoto.com/lzf; Neo2620/Shutterstock.com

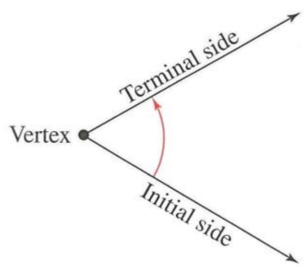
4.1 Radian and Degree Measure

Angles and their measure have a wide variety of real-life applications. For example, in Exercise 68 on page 269, you will use angles and their measure to model the distance a cyclist travels.

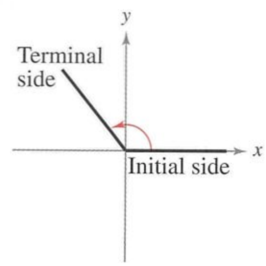
- Describe angles.
- Use radian measure.
- Use degree measure.
- Use angles and their measure to model and solve real-life problems.

Angles

As derived from the Greek language, the word **trigonometry** means "measurement of triangles." Originally, trigonometry dealt with relationships among the sides and angles of triangles and was instrumental in the development of astronomy, navigation, and surveying. With the development of calculus and the physical sciences in the 17th century, a different perspective arose—one that viewed the classic trigonometric relationships as *functions* with the set of real numbers as their domains. Consequently, the applications of trigonometry expanded to include a vast number of physical phenomena, such as sound waves, planetary orbits, vibrating strings, pendulums, and orbits of atomic particles. This text incorporates *both* perspectives, starting with angles and their measure.



Angle
Figure 4.1



Angle in standard position
Figure 4.2

Rotating a ray (half-line) about its endpoint determines an **angle.** The starting position of the ray is the **initial side** of the angle, and the position after rotation is the **terminal side,** as shown in Figure 4.1. The endpoint of the ray is the **vertex** of the angle. This perception of an angle fits a coordinate system in which the origin is the vertex and the initial side coincides with the positive x-axis. Such an angle is in **standard position,** as shown in Figure 4.2. Counterclockwise rotation generates **positive angles** and clockwise rotation generates **negative angles,** as shown in Figure 4.3. Labels for angles can be Greek letters such as α (alpha), β (beta), and θ (theta) or uppercase letters such as A, B, and C. In Figure 4.4, note that angles α and β have the same initial and terminal sides. Such angles are **coterminal.**

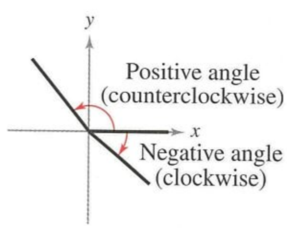
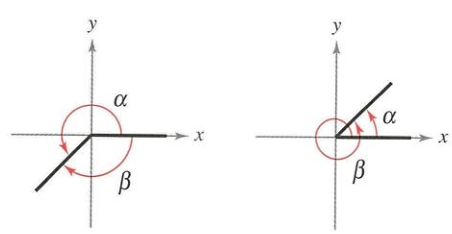


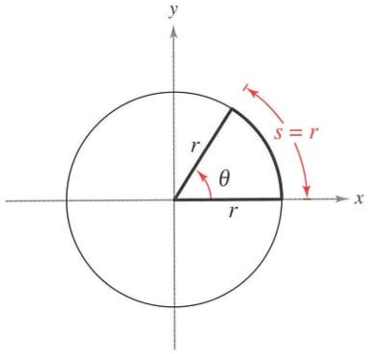
Figure 4.3



Coterminal angles
Figure 4.4

Radian Measure

The amount of rotation from the initial side to the terminal side determines the **measure of an angle.** One way to measure angles is in *radians.* This type of measure is especially useful in calculus. To define a radian, use a **central angle** of a circle, which is an angle whose vertex is the center of the circle, as shown in Figure 4.5.



Arc length = radius when $\theta = 1$ radian.
Figure 4.5

> **Definition of a Radian**
>
> One **radian** (rad) is the measure of a central angle θ that intercepts an arc s equal in length to the radius r of the circle. (See Figure 4.5.) Algebraically, this means that
>
> $$\theta = \frac{s}{r}$$
>
> where θ is measured in radians. (Note that $\theta = 1$ when $s = r$.)

The circumference of a circle is $2\pi r$ units, so it follows that a central angle of one full revolution (counterclockwise) corresponds to an arc length of $s = 2\pi r$. Moreover, $2\pi \approx 6.28$, so there are just over six radius lengths in a full circle, as shown in Figure 4.6. The units of measure for s and r are the same, so the ratio s/r has no units—it is a real number.

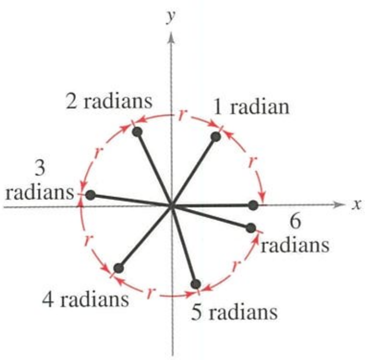


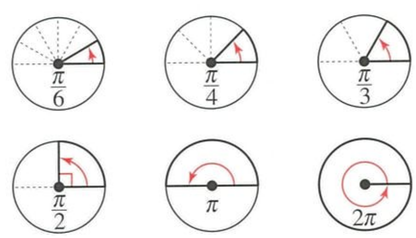
Figure 4.6

The measure of an angle of one full revolution is $s/r = 2\pi r/r = 2\pi$ radians, so you can obtain the following.

$$\frac{1}{2} \text{ revolution} = \frac{2\pi}{2} = \pi \text{ radians} \qquad \frac{1}{4} \text{ revolution} = \frac{2\pi}{4} = \frac{\pi}{2} \text{ radians}$$

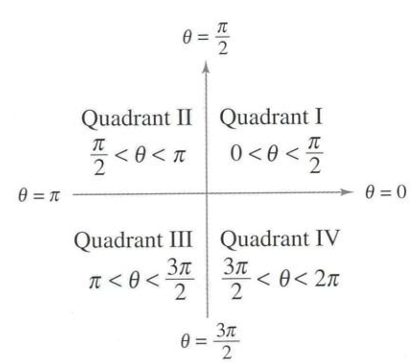
$$\frac{1}{6} \text{ revolution} = \frac{2\pi}{6} = \frac{\pi}{3} \text{ radians}$$

These and other common angles are shown below.



• • **REMARK** The phrase "θ lies in a quadrant" is an abbreviation for the phrase "the terminal side of θ lies in a quadrant." The terminal sides of the "quadrantal angles" 0, $\pi/2$, π, and $3\pi/2$ do not lie within quadrants.

Recall that the four quadrants in a coordinate system are numbered I, II, III, and IV. The figure below shows which angles between 0 and 2π lie in each of the four quadrants. Note that angles between 0 and $\pi/2$ are **acute** angles and angles between $\pi/2$ and π are **obtuse** angles.



Two angles are coterminal when they have the same initial and terminal sides. For example, the angles 0 and 2π are coterminal, as are the angles $\pi/6$ and $13\pi/6$. To find an angle that is coterminal to a given angle θ, add or subtract 2π (one revolution), as demonstrated in Example 1. A given angle θ has infinitely many coterminal angles. For example, $\theta = \pi/6$ is coterminal with $(\pi/6) + 2n\pi$, where n is an integer.

EXAMPLE 1 Finding Coterminal Angles

See LarsonPrecalculus.com for an interactive version of this type of example.

ALGEBRA HELP To review operations involving fractions, see Appendix A.1.

a. For the positive angle $13\pi/6$, subtract 2π to obtain a coterminal angle.

$$\frac{13\pi}{6} - 2\pi = \frac{\pi}{6}$$ See Figure 4.7.

b. For the negative angle $-2\pi/3$, add 2π to obtain a coterminal angle.

$$-\frac{2\pi}{3} + 2\pi = \frac{4\pi}{3}$$ See Figure 4.8.

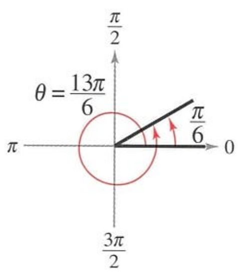


Figure 4.7

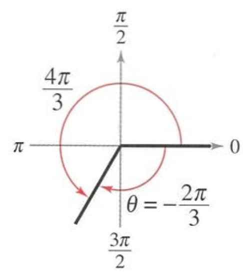


Figure 4.8

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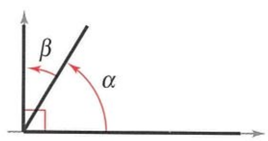
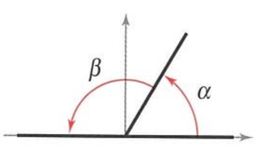
Determine two coterminal angles (one positive and one negative) for each angle.

a. $\theta = \dfrac{9\pi}{4}$ **b.** $\theta = -\dfrac{\pi}{3}$



Complementary angles



Supplementary angles

Figure 4.9

Two positive angles α and β are **complementary** (complements of each other) when their sum is $\pi/2$. Two positive angles are **supplementary** (supplements of each other) when their sum is π. (See Figure 4.9.)

EXAMPLE 2 Complementary and Supplementary Angles

a. The complement of $\dfrac{2\pi}{5}$ is $\dfrac{\pi}{2} - \dfrac{2\pi}{5} = \dfrac{5\pi}{10} - \dfrac{4\pi}{10} = \dfrac{\pi}{10}$.

The supplement of $\dfrac{2\pi}{5}$ is $\pi - \dfrac{2\pi}{5} = \dfrac{5\pi}{5} - \dfrac{2\pi}{5} = \dfrac{3\pi}{5}$.

b. There is no complement of $4\pi/5$ because $4\pi/5$ is greater than $\pi/2$. (Remember that complements are *positive* angles.) The supplement of $4\pi/5$ is

$$\pi - \frac{4\pi}{5} = \frac{5\pi}{5} - \frac{4\pi}{5} = \frac{\pi}{5}.$$

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Find (if possible) the complement and supplement of (a) $\pi/6$ and (b) $5\pi/6$.

Degree Measure

Another way to measure angles is in **degrees,** denoted by the symbol °. A measure of one degree (1°) is equivalent to a rotation of $\frac{1}{360}$ of a complete revolution about the vertex. To measure angles, it is convenient to mark degrees on the circumference of a circle, as shown in Figure 4.10. So, a full revolution (counterclockwise) corresponds to 360°, a half revolution corresponds to 180°, a quarter revolution corresponds to 90°, and so on.

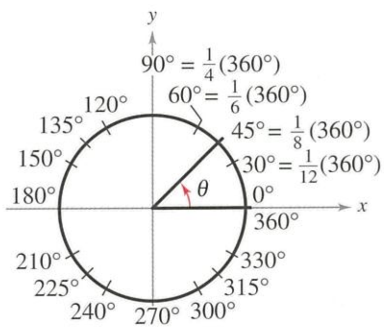

Figure 4.10

One complete revolution corresponds to 2π radians, so degrees and radians are related by the equations

$$360° = 2\pi \text{ rad} \quad \text{and} \quad 180° = \pi \text{ rad}.$$

From these equations, you obtain

$$1° = \frac{\pi}{180} \text{ rad} \quad \text{and} \quad 1 \text{ rad} = \left(\frac{180}{\pi}\right)^{\circ}$$

which lead to the conversion rules below.

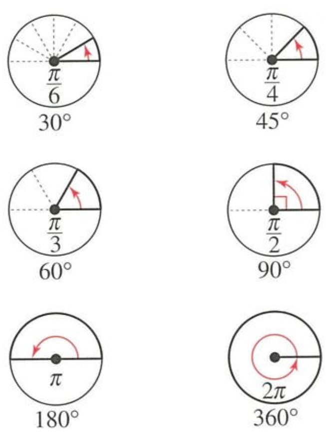

Figure 4.11

Conversions Between Degrees and Radians

1. To convert degrees to radians, multiply degrees by $\frac{\pi \text{ rad}}{180°}$.

2. To convert radians to degrees, multiply radians by $\frac{180°}{\pi \text{ rad}}$.

To apply these two conversion rules, use the basic relationship π rad = 180°. (See Figure 4.11.)

When no units of angle measure are specified, *radian measure is implied.* For example, $\theta = 2$ implies that $\theta = 2$ radians.

EXAMPLE 3 **Converting from Degrees to Radians**

a. $135° = (135 \text{ deg})\left(\frac{\pi \text{ rad}}{180 \text{ deg}}\right) = \frac{3\pi}{4} \text{ radians}$ Multiply by $\frac{\pi \text{ rad}}{180°}$.

b. $540° = (540 \text{ deg})\left(\frac{\pi \text{ rad}}{180 \text{ deg}}\right) = 3\pi \text{ radians}$ Multiply by $\frac{\pi \text{ rad}}{180°}$.

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Convert each degree measure to radian measure as a multiple of π. Do not use a calculator.

a. 60° **b.** 320°

EXAMPLE 4 **Converting from Radians to Degrees**

a. $-\frac{\pi}{2} \text{ rad} = \left(-\frac{\pi}{2} \text{ rad}\right)\left(\frac{180 \text{ deg}}{\pi \text{ rad}}\right) = -90°$ Multiply by $\frac{180°}{\pi \text{ rad}}$.

b. $2 \text{ rad} = (2 \text{ rad})\left(\frac{180 \text{ deg}}{\pi \text{ rad}}\right) = \frac{360°}{\pi} \approx 114.59°$ Multiply by $\frac{180°}{\pi \text{ rad}}$.

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Convert each radian measure to degree measure. Do not use a calculator.

a. $\pi/6$ **b.** $5\pi/3$

TECHNOLOGY With calculators, it is convenient to use *decimal* degrees to denote fractional parts of degrees. Historically, however, fractional parts of degrees were expressed in *minutes* and *seconds*, using the prime (′) and double prime (″) notations, respectively. That is,

$$1' = \text{one minute} = \frac{1}{60}(1°)$$

$$1'' = \text{one second} = \frac{1}{3600}(1°).$$

For example, you would write an angle θ of 64 degrees, 32 minutes, and 47 seconds as $\theta = 64° \, 32' \, 47''$.

Many calculators have special keys for converting an angle in degrees, minutes, and seconds (D° M′ S″) to decimal degree form and vice versa.

Applications

To measure arc length along a circle, use the radian measure formula, $\theta = s/r$.

Arc Length

For a circle of radius r, a central angle θ intercepts an arc of length s given by

$s = r\theta$ — Length of circular arc

where θ is measured in radians. Note that if $r = 1$, then $s = \theta$, and the radian measure of θ equals the arc length.

EXAMPLE 5 **Finding Arc Length**

A circle has a radius of 4 inches. Find the length of the arc intercepted by a central angle of 240°, as shown in Figure 4.12.

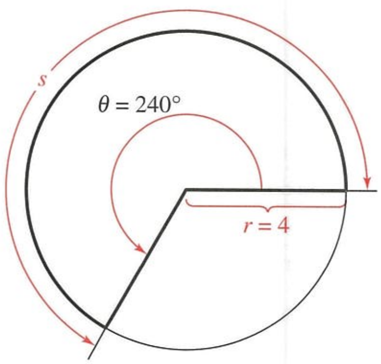

Figure 4.12

Solution To use the formula $s = r\theta$, first convert 240° to radian measure.

$$240° = (240 \text{ deg})\left(\frac{\pi \text{ rad}}{180 \text{ deg}}\right)$$

$$= \frac{4\pi}{3} \text{ radians}$$

Then, using a radius of $r = 4$ inches, find the arc length.

$$s = r\theta \qquad \text{Length of circular arc}$$

$$= 4\left(\frac{4\pi}{3}\right) \qquad \text{Substitute for } r \text{ and } \theta.$$

$$\approx 16.76 \text{ inches} \qquad \text{Use a calculator.}$$

Note that the units for r determine the units for $r\theta$ because θ is in radian measure, which has no units.

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A circle has a radius of 27 inches. Find the length of the arc intercepted by a central angle of 160°.

REMARK Linear speed measures how fast the particle moves, and angular speed measures how fast the angle changes. To establish a relationship between linear speed v and angular speed ω, divide each side of the formula for arc length by t, as shown.

$$s = r\theta$$

$$\frac{s}{t} = \frac{r\theta}{t}$$

$$v = r\omega$$

The formula for the length of a circular arc can be used to analyze the motion of a particle moving at a *constant speed* along a circular path.

Linear and Angular Speeds

Consider a particle moving at a constant speed along a circular arc of radius r. If s is the length of the arc traveled in time t, then the **linear speed** v of the particle is

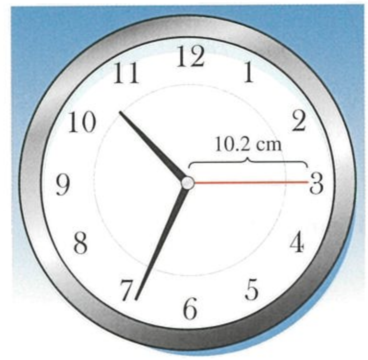
$$\text{Linear speed } v = \frac{\text{arc length}}{\text{time}} = \frac{s}{t}.$$

Moreover, if θ is the angle (in radian measure) corresponding to the arc length s, then the **angular speed** ω (the lowercase Greek letter omega) of the particle is

$$\text{Angular speed } \omega = \frac{\text{central angle}}{\text{time}} = \frac{\theta}{t}.$$

EXAMPLE 6 Finding Linear Speed

The second hand of a clock is 10.2 centimeters long, as shown at the right. Find the linear speed of the tip of the second hand as it passes around the clock face.



Solution In one revolution, the arc length traveled is

$$s = 2\pi r$$
$$= 2\pi(10.2) \quad \text{Substitute for } r.$$
$$= 20.4\pi \text{ centimeters.}$$

The time required for the second hand to travel this distance is

$$t = 1 \text{ minute} = 60 \text{ seconds.}$$

So, the linear speed of the tip of the second hand is

$$v = \frac{s}{t}$$
$$= \frac{20.4\pi \text{ centimeters}}{60 \text{ seconds}}$$
$$\approx 1.07 \text{ centimeters per second.}$$

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The second hand of a clock is 8 centimeters long. Find the linear speed of the tip of the second hand as it passes around the clock face.

Figure 4.13

EXAMPLE 7 Finding Angular and Linear Speeds

The blades of a wind turbine are 116 feet long (see Figure 4.13). The propeller rotates at 15 revolutions per minute.

a. Find the angular speed of the propeller in radians per minute.

b. Find the linear speed of the tips of the blades.

Solution

a. Each revolution corresponds to 2π radians, so the propeller turns $15(2\pi) = 30\pi$ radians per minute. In other words, the angular speed is

$$\omega = \frac{\theta}{t} = \frac{30\pi \text{ radians}}{1 \text{ minute}} = 30\pi \text{ radians per minute.}$$

b. The linear speed is

$$v = \frac{s}{t} = \frac{r\theta}{t} = \frac{116(30\pi) \text{ feet}}{1 \text{ minute}} \approx 10{,}933 \text{ feet per minute.}$$

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The circular blade on a saw has a radius of 4 inches and it rotates at 2400 revolutions per minute.

a. Find the angular speed of the blade in radians per minute.

b. Find the linear speed of the edge of the blade.

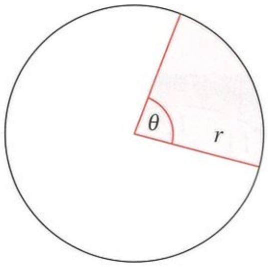


Figure 4.14

A **sector** of a circle is the region bounded by two radii of the circle and their intercepted arc (see Figure 4.14).

Area of a Sector of a Circle

For a circle of radius r, the area A of a sector of the circle with central angle θ is

$$A = \frac{1}{2}r^2\theta$$

where θ is measured in radians.

EXAMPLE 8 Area of a Sector of a Circle

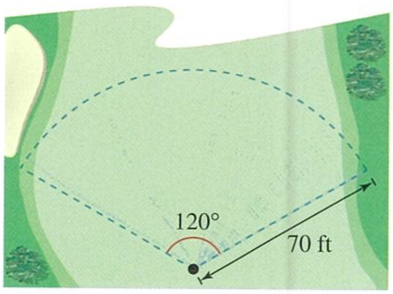


Figure 4.15

A sprinkler on a golf course fairway sprays water over a distance of 70 feet and rotates through an angle of 120° (see Figure 4.15). Find the area of the fairway watered by the sprinkler.

Solution

First convert 120° to radian measure.

$$\theta = 120°$$

$$= (120 \text{ deg})\left(\frac{\pi \text{ rad}}{180 \text{ deg}}\right) \qquad \text{Multiply by } \frac{\pi \text{ rad}}{180°}.$$

$$= \frac{2\pi}{3} \text{ radians}$$

Then, using $\theta = 2\pi/3$ and $r = 70$, the area is

$$A = \frac{1}{2}r^2\theta \qquad \text{Formula for the area of a sector of a circle}$$

$$= \frac{1}{2}(70)^2\left(\frac{2\pi}{3}\right) \qquad \text{Substitute for } r \text{ and } \theta.$$

$$= \frac{4900\pi}{3} \qquad \text{Multiply.}$$

$$\approx 5131 \text{ square feet.} \qquad \text{Use a calculator.}$$

✓ **Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

A sprinkler sprays water over a distance of 40 feet and rotates through an angle of 80°. Find the area watered by the sprinkler.

Summarize (Section 4.1)

1. Describe an angle *(page 260)*.
2. Explain how to use radian measure *(page 261)*. For examples involving radian measure, see Examples 1 and 2.
3. Explain how to use degree measure *(page 263)*. For examples involving degree measure, see Examples 3 and 4.
4. Describe real-life applications involving angles and their measure *(pages 264–266, Examples 5–8)*.

4.1 Exercises

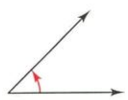
See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

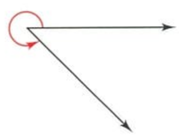
Vocabulary: Fill in the blanks.

1. Two angles that have the same initial and terminal sides are ________.
2. One ________ is the measure of a central angle that intercepts an arc equal in length to the radius of the circle.
3. Two positive angles that have a sum of $\pi/2$ are ________ angles, and two positive angles that have a sum of π are ________ angles.
4. The angle measure that is equivalent to a rotation of $\frac{1}{360}$ of a complete revolution about an angle's vertex is one ________.
5. The ________ speed of a particle is the ratio of the arc length traveled to the elapsed time, and the ________ speed of a particle is the ratio of the change in the central angle to the elapsed time.
6. The area A of a sector of a circle with radius r and central angle θ, where θ is measured in radians, is given by the formula ________.

Skills and Applications

Estimating an Angle In Exercises 7–10, estimate the angle to the nearest one-half radian.

7. 

8. 

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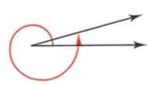
10. 

Determining Quadrants In Exercises 11 and 12, determine the quadrant in which each angle lies.

11. (a) $\frac{\pi}{4}$ (b) $-\frac{5\pi}{4}$ **12.** (a) $-\frac{\pi}{6}$ (b) $\frac{11\pi}{9}$

Sketching Angles In Exercises 13 and 14, sketch each angle in standard position.

13. (a) $\frac{\pi}{3}$ (b) $-\frac{2\pi}{3}$ **14.** (a) $\frac{5\pi}{2}$ (b) 4

Finding Coterminal Angles In Exercises 15 and 16, determine two coterminal angles (one positive and one negative) for each angle. Give your answers in radians.

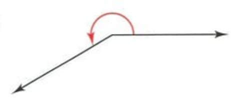
15. (a) $\frac{\pi}{6}$ (b) $-\frac{5\pi}{6}$ **16.** (a) $\frac{2\pi}{3}$ (b) $-\frac{9\pi}{4}$

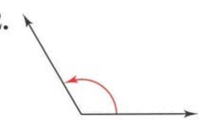
Complementary and Supplementary Angles In Exercises 17–20, find (if possible) the complement and supplement of each angle.

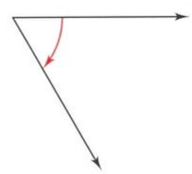
17. (a) $\frac{\pi}{12}$ (b) $\frac{11\pi}{12}$ **18.** (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$

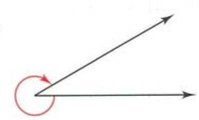
19. (a) 1 (b) 2 **20.** (a) 3 (b) 1.5

Estimating an Angle In Exercises 21–24, estimate the number of degrees in the angle.

21. 

22. 

23. 

24. 

Determining Quadrants In Exercises 25 and 26, determine the quadrant in which each angle lies.

25. (a) 130° (b) -8.3°

26. (a) $-132^\circ\,50'$ (b) 3.4°

Sketching Angles In Exercises 27 and 28, sketch each angle in standard position.

27. (a) 270° (b) -120° **28.** (a) 135° (b) -750°

Finding Coterminal Angles In Exercises 29 and 30, determine two coterminal angles (one positive and one negative) for each angle. Give your answers in degrees.

29. (a) 120° (b) -210° **30.** (a) 45° (b) -420°

Complementary and Supplementary Angles In Exercises 31–34, find (if possible) the complement and supplement of each angle.

31. (a) 18° (b) 85° **32.** (a) 46° (b) 93°

33. (a) 24° (b) 126° **34.** (a) 130° (b) 170°

Converting from Degrees to Radians **In Exercises 35 and 36, convert each degree measure to radian measure as a multiple of π. Do not use a calculator.**

35. (a) $120°$ (b) $-20°$

36. (a) $-60°$ (b) $144°$

Converting from Radians to Degrees **In Exercises 37 and 38, convert each radian measure to degree measure. Do not use a calculator.**

37. (a) $\frac{3\pi}{2}$ (b) $-\frac{7\pi}{6}$

38. (a) $-\frac{7\pi}{12}$ (b) $\frac{5\pi}{4}$

Converting from Degrees to Radians **In Exercises 39–42, convert the degree measure to radian measure. Round to three decimal places.**

39. $45°$

40. $-48.27°$

41. $-0.54°$

42. $345°$

Converting from Radians to Degrees **In Exercises 43–46, convert the radian measure to degree measure. Round to three decimal places, if necessary.**

43. $\frac{5\pi}{11}$

44. $\frac{15\pi}{8}$

45. -4.2π

46. -0.57

Converting to Decimal Degree Form **In Exercises 47 and 48, convert each angle measure to decimal degree form.**

47. (a) $54°\,45'$ (b) $-128°\,30'$

48. (a) $135°\,10'\,36''$ (b) $-408°\,16'\,20''$

Converting to D° M′ S″ Form **In Exercises 49 and 50, convert each angle measure to D° M′ S″ form.**

49. (a) $240.6°$ (b) $-145.8°$

50. (a) $345.12°$ (b) $-3.58°$

Finding Arc Length **In Exercises 51 and 52, find the length of the arc on a circle of radius r intercepted by a central angle θ.**

51. $r = 15$ inches, $\theta = 120°$

52. $r = 3$ meters, $\theta = 150°$

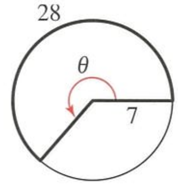
Finding the Central Angle **In Exercises 53 and 54, find the radian measure of the central angle of a circle of radius r that intercepts an arc of length s.**

53. $r = 80$ kilometers, $s = 150$ kilometers

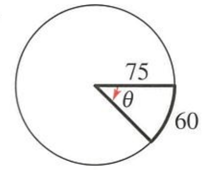
54. $r = 14$ feet, $s = 8$ feet

Finding the Central Angle **In Exercises 55 and 56, find the radian measure of the central angle.**

55.



56.



Area of a Sector of a Circle **In Exercises 57 and 58, find the area of the sector of a circle of radius r and central angle θ.**

57. $r = 6$ inches, $\theta = \frac{\pi}{3}$

58. $r = 2.5$ feet, $\theta = 225°$

Error Analysis **In Exercises 59 and 60, describe the error.**

59. $20° = (20 \text{ deg})\left(\frac{180 \text{ rad}}{\pi \text{ deg}}\right) = \frac{3600}{\pi} \text{ rad}$ ✗

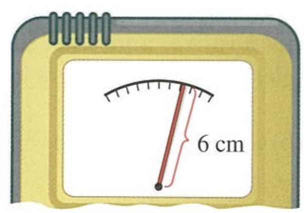
60. A circle has a radius of 6 millimeters. The length of the arc intercepted by a central angle of $72°$ is

$$s = r\theta$$
$$= 6(72)$$
$$= 432 \text{ millimeters.}$$ ✗

Earth-Space Science **In Exercises 61 and 62, find the distance between the cities. Assume that Earth is a sphere of radius 4000 miles and that the cities are on the same longitude (one city is due north of the other).**

	City	Latitude
61.	Dallas, Texas	$32°\,47'\,9''$ N
	Omaha, Nebraska	$41°\,15'\,50''$ N
62.	San Francisco, California	$37°\,47'\,36''$ N
	Seattle, Washington	$47°\,37'\,18''$ N

63. **Instrumentation** The pointer on a voltmeter is 6 centimeters in length (see figure). Find the number of degrees through which the pointer rotates when it moves 2.5 centimeters on the scale.



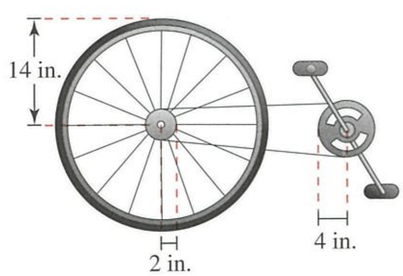
64. **Linear and Angular Speed** A $7\frac{1}{4}$-inch circular power saw blade rotates at 5200 revolutions per minute.
    (a) Find the angular speed of the saw blade in radians per minute.
    (b) Find the linear speed (in feet per minute) of the saw teeth as they contact the wood being cut.

65. **Linear and Angular Speed** A carousel with a 50-foot diameter makes 4 revolutions per minute.
    (a) Find the angular speed of the carousel in radians per minute.
    (b) Find the linear speed (in feet per minute) of the platform rim of the carousel.

66. **Linear and Angular Speed** A Blu-ray disc is approximately 12 centimeters in diameter. The drive motor of a Blu-ray player is able to rotate up to 10,000 revolutions per minute.
    (a) Find the maximum angular speed (in radians per second) of a Blu-ray disc as it rotates.
    (b) Find the maximum linear speed (in meters per second) of a point on the outermost track as the disc rotates.

67. **Linear and Angular Speed** A computerized spin balance machine rotates a 25-inch-diameter tire at 480 revolutions per minute.
    (a) Find the road speed (in miles per hour) at which the tire is being balanced.
    (b) At what rate should the spin balance machine be set so that the tire is being tested for 55 miles per hour?

68. **Speed of a Bicycle**
The radii of the pedal sprocket, the wheel sprocket, and the wheel of the bicycle in the figure are 4 inches, 2 inches, and 14 inches, respectively. A cyclist pedals at a rate of 1 revolution per second.



(a) Find the speed of the bicycle in feet per second and miles per hour.
(b) Use your result from part (a) to write a function for the distance d (in miles) a cyclist travels in terms of the number n of revolutions of the pedal sprocket.

(c) Write a function for the distance d (in miles) a cyclist travels in terms of the time t (in seconds). Compare this function with the function from part (b).

69. **Area** A sprinkler on a golf green is set to spray water over a distance of 15 meters and to rotate through an angle of 150°. Draw a diagram that shows the region that can be irrigated with the sprinkler. Find the area of the region.

70. **Area** A car's rear windshield wiper rotates 125°. The total length of the wiper mechanism is 25 inches and the length of the wiper blade is 14 inches. Find the area wiped by the wiper blade.

Exploration

True or False? **In Exercises 71–74, determine whether the statement is true or false. Justify your answer.**

71. An angle measure containing π must be in radian measure.

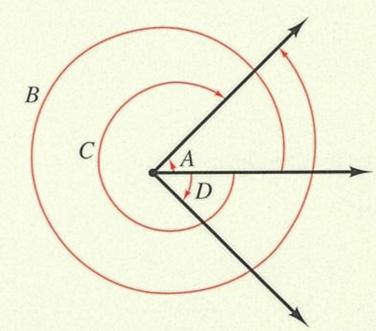
72. A measurement of 4 radians corresponds to two complete revolutions from the initial side to the terminal side of an angle.

73. The difference between the measures of two coterminal angles is always a multiple of 360° when expressed in degrees and is always a multiple of 2π radians when expressed in radians.

74. An angle that measures $-1260°$ lies in Quadrant III.

75. **Writing** When the radius of a circle increases and the magnitude of a central angle is held constant, how does the length of the intercepted arc change? Explain.

76. **HOW DO YOU SEE IT?** Determine which angles in the figure are coterminal angles with angle A. Explain.



77. **Think About It** A fan motor turns at a given angular speed. How does the speed of the tips of the blades change when a fan of greater diameter is installed on the motor? Explain.

78. **Think About It** Is a degree or a radian the larger unit of measure? Explain.

79. **Proof** Prove that the area of a circular sector of radius r with central angle θ is $A = \frac{1}{2}\theta r^2$, where θ is measured in radians.

4.2 Trigonometric Functions: The Unit Circle

Trigonometric functions can help you analyze the movement of an oscillating weight. For example, in Exercise 50 on page 276, you will analyze the displacement of an oscillating weight suspended by a spring using a model that is the product of a trigonometric function and an exponential function.

- **Identify a unit circle and describe its relationship to real numbers.**
- **Evaluate trigonometric functions using the unit circle.**
- **Use domain and period to evaluate sine and cosine functions, and use a calculator to evaluate trigonometric functions.**

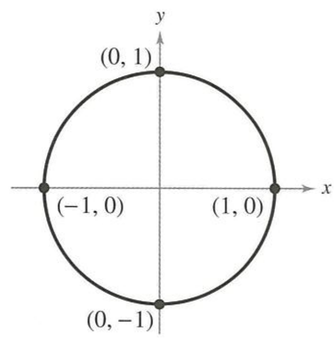
The Unit Circle

The two historical perspectives of trigonometry incorporate different methods for introducing the trigonometric functions. One such perspective is based on the unit circle.

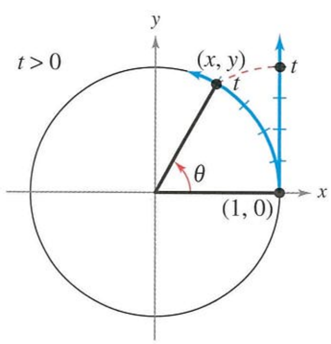
Consider the **unit circle** given by

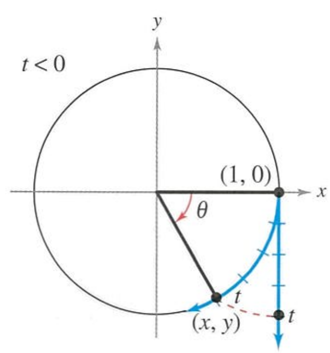
$$x^2 + y^2 = 1 \quad \text{Unit circle}$$

as shown in the figure below.



Imagine wrapping the real number line around this circle, with positive numbers corresponding to a counterclockwise wrapping and negative numbers corresponding to a clockwise wrapping, as shown in the figures below.





As the real number line wraps around the unit circle, each real number t corresponds to a point (x, y) on the circle. For example, the real number 0 corresponds to the point $(1, 0)$. Moreover, the unit circle has a circumference of 2π, so the real number 2π also corresponds to the point $(1, 0)$.

Each real number t also corresponds to a central angle θ (in standard position) whose radian measure is t. With this interpretation of t, the arc length formula

$$s = r\theta \quad (\text{with } r = 1)$$

indicates that the real number t is the (directional) length of the arc intercepted by the angle θ, given in radians.

The Trigonometric Functions

From the preceding discussion, the coordinates x and y are two functions of the real variable t. These coordinates are used to define the six trigonometric functions of a real number t.

sine **cosecant** **cosine** **secant** **tangent** **cotangent**

Abbreviations for these six functions are sin, csc, cos, sec, tan, and cot, respectively.

REMARK Note that the functions in the second row are the *reciprocals* of the corresponding functions in the first row.

Definitions of Trigonometric Functions

Let t be a real number and let (x, y) be the point on the unit circle corresponding to t.

$$\sin t = y \qquad \cos t = x \qquad \tan t = \frac{y}{x},\quad x \neq 0$$

$$\csc t = \frac{1}{y},\quad y \neq 0 \qquad \sec t = \frac{1}{x},\quad x \neq 0 \qquad \cot t = \frac{x}{y},\quad y \neq 0$$

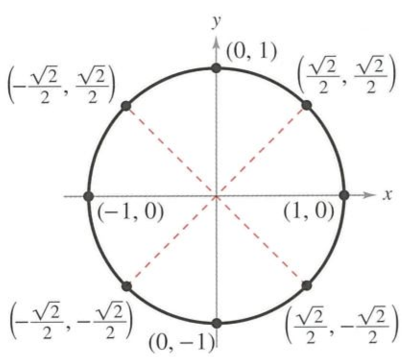


Figure 4.16

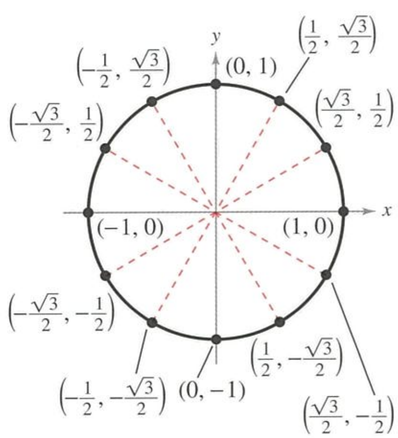


Figure 4.17

In the definitions of the trigonometric functions, note that the tangent and secant are not defined when $x = 0$. For example, $t = \pi/2$ corresponds to $(x, y) = (0, 1)$, so $\tan(\pi/2)$ and $\sec(\pi/2)$ are *undefined*. Similarly, the cotangent and cosecant are not defined when $y = 0$. For example, $t = 0$ corresponds to $(x, y) = (1, 0)$, so cot 0 and csc 0 are *undefined*.

In Figure 4.16, the unit circle is divided into eight equal arcs, corresponding to t-values of

$$0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}, \text{ and } 2\pi.$$

Similarly, in Figure 4.17, the unit circle is divided into 12 equal arcs, corresponding to t-values of

$$0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}, \frac{11\pi}{6}, \text{ and } 2\pi.$$

To verify the points on the unit circle in Figure 4.16, note that

$$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

lies on the line $y = x$. So, substituting x for y in the equation of the unit circle produces the following

$$x^2 + x^2 = 1 \implies 2x^2 = 1 \implies x^2 = \frac{1}{2} \implies x = \pm\frac{\sqrt{2}}{2}$$

Because the point is in the first quadrant and $y = x$, you have

$$x = \frac{\sqrt{2}}{2} \quad \text{and} \quad y = \frac{\sqrt{2}}{2}.$$

Similar reasoning can be used to verify the rest of the points in Figure 4.16 and the points in Figure 4.17.

Using the (x, y) coordinates in Figures 4.16 and 4.17, you can evaluate the trigonometric functions for these common t-values. Examples 1 and 2 demonstrate this procedure. You should study and learn these exact function values for common t-values because they will help you perform calculations in later sections.

EXAMPLE 1 Evaluating Trigonometric Functions

See LarsonPrecalculus.com for an interactive version of this type of example.

ALGEBRA HELP To review dividing fractions and rationalizing denominators, see Appendix A.1 and Appendix A.2, respectively.

Evaluate the six trigonometric functions at each real number.

a. $t = \frac{\pi}{6}$ **b.** $t = \frac{5\pi}{4}$ **c.** $t = \pi$ **d.** $t = -\frac{\pi}{3}$

Solution For each t-value, begin by finding the corresponding point (x, y) on the unit circle. Then use the definitions of trigonometric functions listed on page 271.

a. $t = \pi/6$ corresponds to the point $(x, y) = \left(\sqrt{3}/2, 1/2\right)$.

$$\sin\frac{\pi}{6} = y = \frac{1}{2} \qquad \csc\frac{\pi}{6} = \frac{1}{y} = \frac{1}{1/2} = 2$$

$$\cos\frac{\pi}{6} = x = \frac{\sqrt{3}}{2} \qquad \sec\frac{\pi}{6} = \frac{1}{x} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\tan\frac{\pi}{6} = \frac{y}{x} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \qquad \cot\frac{\pi}{6} = \frac{x}{y} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$$

b. $t = 5\pi/4$ corresponds to the point $(x, y) = \left(-\sqrt{2}/2, -\sqrt{2}/2\right)$.

$$\sin\frac{5\pi}{4} = y = -\frac{\sqrt{2}}{2} \qquad \csc\frac{5\pi}{4} = \frac{1}{y} = -\frac{2}{\sqrt{2}} = -\sqrt{2}$$

$$\cos\frac{5\pi}{4} = x = -\frac{\sqrt{2}}{2} \qquad \sec\frac{5\pi}{4} = \frac{1}{x} = -\frac{2}{\sqrt{2}} = -\sqrt{2}$$

$$\tan\frac{5\pi}{4} = \frac{y}{x} = \frac{-\sqrt{2}/2}{-\sqrt{2}/2} = 1 \qquad \cot\frac{5\pi}{4} = \frac{x}{y} = \frac{-\sqrt{2}/2}{-\sqrt{2}/2} = 1$$

c. $t = \pi$ corresponds to the point $(x, y) = (-1, 0)$.

$$\sin\pi = y = 0 \qquad \csc\pi = \frac{1}{y} \text{ is undefined.}$$

$$\cos\pi = x = -1 \qquad \sec\pi = \frac{1}{x} = \frac{1}{-1} = -1$$

$$\tan\pi = \frac{y}{x} = \frac{0}{-1} = 0 \qquad \cot\pi = \frac{x}{y} \text{ is undefined.}$$

d. Moving *clockwise* around the unit circle, $t = -\pi/3$ corresponds to the point $(x, y) = \left(1/2, -\sqrt{3}/2\right)$.

$$\sin\left(-\frac{\pi}{3}\right) = y = -\frac{\sqrt{3}}{2} \qquad \csc\left(-\frac{\pi}{3}\right) = \frac{1}{y} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

$$\cos\left(-\frac{\pi}{3}\right) = x = \frac{1}{2} \qquad \sec\left(-\frac{\pi}{3}\right) = \frac{1}{x} = \frac{1}{1/2} = 2$$

$$\tan\left(-\frac{\pi}{3}\right) = \frac{y}{x} = \frac{-\sqrt{3}/2}{1/2} = -\sqrt{3}$$

$$\cot\left(-\frac{\pi}{3}\right) = \frac{x}{y} = \frac{1/2}{-\sqrt{3}/2} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

✓ **Checkpoint** Audio-video solution in English & Spanish at LarsonPrecalculus.com

Evaluate the six trigonometric functions at each real number.

a. $t = \pi/2$ **b.** $t = 0$ **c.** $t = -5\pi/6$ **d.** $t = -3\pi/4$

Domain and Period of Sine and Cosine

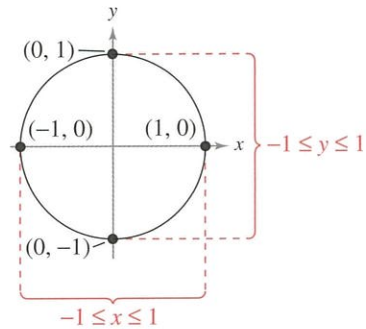
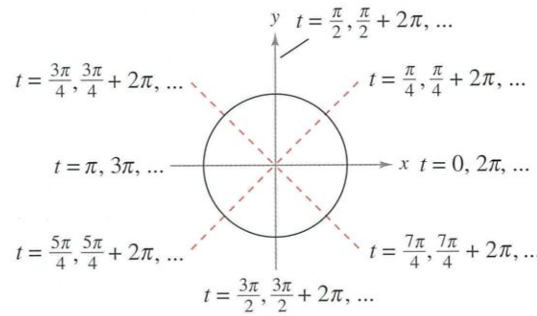


Figure 4.18

The *domain* of the sine and cosine functions is the set of all real numbers. To determine the *range* of these two functions, consider the unit circle shown in Figure 4.18. You know that $\sin t = y$ and $\cos t = x$. Moreover, (x, y) is on the unit circle, so you also know that $-1 \le y \le 1$ and $-1 \le x \le 1$. This means that the values of sine and cosine also range between -1 and 1.

$$-1 \le y \le 1 \qquad \text{and} \qquad -1 \le x \le 1$$
$$-1 \le \sin t \le 1 \qquad\qquad -1 \le \cos t \le 1$$

Adding 2π to each value of t in the interval $[0, 2\pi]$ results in a revolution around the unit circle, as shown in the figure below.



The values of $\sin(t + 2\pi)$ and $\cos(t + 2\pi)$ correspond to those of $\sin t$ and $\cos t$. Repeated revolutions (positive or negative) on the unit circle yield similar results. This leads to the general result

$$\sin(t + 2\pi n) = \sin t \quad \text{and} \quad \cos(t + 2\pi n) = \cos t$$

for any integer n and real number t. Functions that behave in such a repetitive (or cyclic) manner are **periodic.**

REMARK From this definition, it follows that the sine and cosine functions are periodic and have a period of 2π. The other four trigonometric functions are also periodic and will be discussed further in Section 4.6.

Definition of Periodic Function

A function f is **periodic** when there exists a positive real number c such that

$$f(t + c) = f(t)$$

for all t in the domain of f. The smallest number c for which f is periodic is the **period** of f.

Recall from Section 1.5 that a function f is *even* when $f(-t) = f(t)$ and is *odd* when $f(-t) = -f(t)$.

Even and Odd Trigonometric Functions

The cosine and secant functions are *even.*

$$\cos(-t) = \cos t \qquad \sec(-t) = \sec t$$

The sine, cosecant, tangent, and cotangent functions are *odd.*

$$\sin(-t) = -\sin t \qquad \csc(-t) = -\csc t$$

$$\tan(-t) = -\tan t \qquad \cot(-t) = -\cot t$$

EXAMPLE 2 Evaluating Sine and Cosine

a. Because $\frac{13\pi}{6} = 2\pi + \frac{\pi}{6}$, you have $\sin\frac{13\pi}{6} = \sin\left(2\pi + \frac{\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$.

b. Because $-\frac{7\pi}{2} = -4\pi + \frac{\pi}{2}$, you have

$$\cos\left(-\frac{7\pi}{2}\right) = \cos\left(-4\pi + \frac{\pi}{2}\right) = \cos\frac{\pi}{2} = 0.$$

c. For $\sin t = \frac{4}{5}$, $\sin(-t) = -\frac{4}{5}$ because the sine function is odd.

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a. Use the period of the cosine function to evaluate $\cos(9\pi/2)$.

b. Use the period of the sine function to evaluate $\sin(-7\pi/3)$.

c. Evaluate $\cos t$ given that $\cos(-t) = 0.3$.

When evaluating a trigonometric function with a calculator, set the calculator to the desired *mode* of measurement (*degree* or *radian*). Most calculators do not have keys for the cosecant, secant, and cotangent functions. To evaluate these functions, you can use the [x⁻¹] key with their respective reciprocal functions: sine, cosine, and tangent. For example, to evaluate $\csc(\pi/8)$, use the fact that

$$\csc\frac{\pi}{8} = \frac{1}{\sin(\pi/8)}$$

and enter the keystroke sequence below in *radian* mode.

[(] [SIN] [(] [π] [÷] 8 [)] [)] [x⁻¹] [ENTER]   Display 2.6131259

TECHNOLOGY When evaluating trigonometric functions with a calculator, remember to enclose all fractional angle measures in parentheses. For example, to evaluate $\sin t$ for $t = \pi/6$, enter

[SIN] [(] [π] [÷] 6 [)] [ENTER].

These keystrokes yield the correct value of 0.5. Note that some calculators automatically place a left parenthesis after trigonometric functions.

EXAMPLE 3 Using a Calculator

Function	Mode	Calculator Keystrokes	Display
a. $\sin\frac{2\pi}{3}$	Radian	[SIN] [(] 2 [π] [÷] 3 [)] [ENTER]	0.8660254
b. cot 1.5	Radian	[(] [TAN] [(] 1.5 [)] [)] [x⁻¹] [ENTER]	0.0709148

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Use a calculator to evaluate (a) $\sin(5\pi/7)$ and (b) csc 2.0.

Summarize (Section 4.2)

1. Explain how to identify a unit circle and describe its relationship to real numbers *(page 270)*.
2. State the unit circle definitions of trigonometric functions *(page 271)*. For an example of evaluating trigonometric functions using the unit circle, see Example 1.
3. Explain how to use domain and period to evaluate sine and cosine functions *(page 273)*, and describe how to use a calculator to evaluate trigonometric functions *(page 274)*. For an example of using domain and period to evaluate sine and cosine functions, see Example 2. For an example of using a calculator to evaluate trigonometric functions, see Example 3.

4.2 Exercises

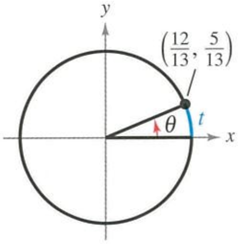
See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

1. Each real number t corresponds to a point (x, y) on the ________ ________.
2. A function f is ________ when there exists a positive real number c such that $f(t + c) = f(t)$ for all t in the domain of f.
3. The smallest number c for which a function f is periodic is the ________ of f.
4. A function f is ________ when $f(-t) = -f(t)$ and ________ when $f(-t) = f(t)$.

Skills and Applications

Evaluating Trigonometric Functions In Exercises 5–8, find the exact values of the six trigonometric functions of the real number t.

5.


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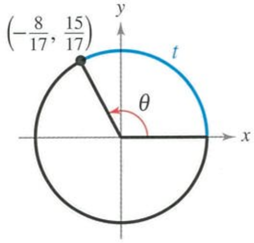
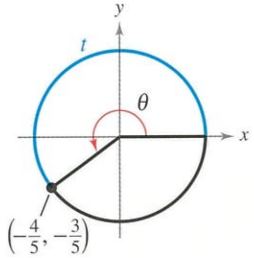


7.


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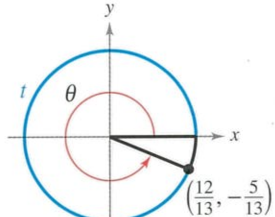


Finding a Point on the Unit Circle In Exercises 9–12, find the point (x, y) on the unit circle that corresponds to the real number t.

9. $t = \pi/2$
10. $t = \pi/4$
11. $t = 5\pi/6$
12. $t = 4\pi/3$

Evaluating Sine, Cosine, and Tangent In Exercises 13–22, evaluate (if possible) the sine, cosine, and tangent at the real number.

13. $t = \dfrac{\pi}{4}$
14. $t = \dfrac{\pi}{3}$
15. $t = -\dfrac{\pi}{6}$
16. $t = -\dfrac{\pi}{4}$
17. $t = -\dfrac{7\pi}{4}$
18. $t = -\dfrac{4\pi}{3}$
19. $t = \dfrac{11\pi}{6}$
20. $t = \dfrac{5\pi}{3}$
21. $t = -\dfrac{3\pi}{2}$
22. $t = -2\pi$

Evaluating Trigonometric Functions In Exercises 23–30, evaluate (if possible) the six trigonometric functions at the real number.

23. $t = 2\pi/3$
24. $t = 5\pi/6$
25. $t = 4\pi/3$
26. $t = 7\pi/4$
27. $t = -5\pi/3$
28. $t = -3\pi/2$
29. $t = -\pi/2$
30. $t = -\pi$

Using Period to Evaluate Sine and Cosine In Exercises 31–36, evaluate the trigonometric function using its period as an aid.

31. $\sin 4\pi$
32. $\cos 3\pi$
33. $\cos(7\pi/3)$
34. $\sin(9\pi/4)$
35. $\sin(19\pi/6)$
36. $\sin(-8\pi/3)$

Using the Value of a Function In Exercises 37–42, use the given value to evaluate each function.

37. $\sin t = \frac{1}{2}$
 (a) $\sin(-t)$
 (b) $\csc(-t)$
38. $\sin(-t) = \frac{3}{8}$
 (a) $\sin t$
 (b) $\csc t$
39. $\cos(-t) = -\frac{1}{5}$
 (a) $\cos t$
 (b) $\sec(-t)$
40. $\cos t = -\frac{3}{4}$
 (a) $\cos(-t)$
 (b) $\sec(-t)$
41. $\sin t = \frac{4}{5}$
 (a) $\sin(\pi - t)$
 (b) $\sin(t + \pi)$
42. $\cos t = \frac{4}{5}$
 (a) $\cos(\pi - t)$
 (b) $\cos(t + \pi)$

Using a Calculator In Exercises 43–48, use a calculator to evaluate the trigonometric function. Round your answer to four decimal places. (Be sure the calculator is in the correct mode.)

43. $\sin 0.6$
44. $\cos(-2.8)$
45. $\tan(\pi/8)$
46. $\tan(5\pi/7)$
47. $\sec 3.1$
48. $\cot(-1.1)$

49. Harmonic Motion The displacement from equilibrium of an oscillating weight suspended by a spring is given by

$$y(t) = \frac{1}{2}\cos 6t$$

where y is the displacement in feet and t is the time in seconds. Find the displacement when (a) $t = 0$, (b) $t = \frac{1}{4}$, and (c) $t = \frac{1}{2}$.

50. Harmonic Motion

The displacement from equilibrium of an oscillating weight suspended by a spring and subject to the damping effect of friction is given by

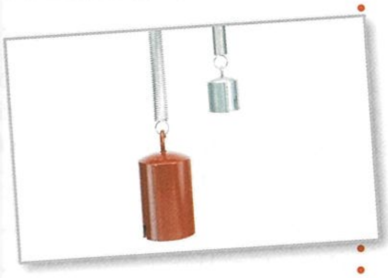
$$y(t) = \frac{1}{2}e^{-t}\cos 6t$$

where y is the displacement in feet and t is the time in seconds.

(a) Complete the table

t	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
y					

(b) Use the *table* feature of a graphing utility to approximate the time when the weight reaches equilibrium.



(c) What appears to happen to the displacement as t increases?

Exploration

True or False? **In Exercises 51–54, determine whether the statement is true or false. Justify your answer.**

51. Because $\sin(-t) = -\sin t$, the sine of a negative angle is a negative number.

52. The real number 0 corresponds to the point $(0, 1)$ on the unit circle.

53. $\tan a = \tan(a - 6\pi)$

54. $\cos\left(-\frac{7\pi}{2}\right) = \cos\left(\pi + \frac{\pi}{2}\right)$

55. Conjecture Let (x_1, y_1) and (x_2, y_2) be points on the unit circle corresponding to $t = t_1$ and $t = \pi - t_1$, respectively.

(a) Identify the symmetry of the points (x_1, y_1) and (x_2, y_2).

(b) Make a conjecture about any relationship between $\sin t_1$ and $\sin(\pi - t_1)$.

(c) Make a conjecture about any relationship between $\cos t_1$ and $\cos(\pi - t_1)$.

Richard Megna/Fundamental Photographs

56. Using the Unit Circle Use the unit circle to verify that the cosine and secant functions are even and that the sine, cosecant, tangent, and cotangent functions are odd.

57. Error Analysis Describe the error.

Your classmate uses a calculator to evaluate $\tan(\pi/2)$ and gets a result of 0.0274224385.

58. Verifying Expressions Are Not Equal Verify that

$$\sin(t_1 + t_2) \neq \sin t_1 + \sin t_2$$

by approximating sin 0.25, sin 0.75, and sin 1.

59. Using Technology With a graphing utility in *radian* and *parametric* modes, enter the equations

X1T = cos T and Y1T = sin T

and use the settings below.

Tmin = 0, Tmax = 6.3, Tstep = 0.1

Xmin = −1.5, Xmax = 1.5, Xscl = 1

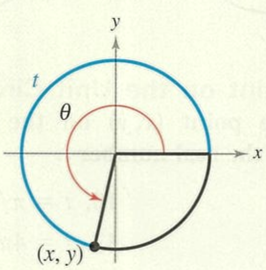
Ymin = −1, Ymax = 1, Yscl = 1

(a) Graph the entered equations and describe the graph.

(b) Use the *trace* feature to move the cursor around the graph. What do the t-values represent? What do the x- and y-values represent?

(c) What are the least and greatest values of x and y?

60. HOW DO YOU SEE IT? Use the figure below.



(a) Are all of the trigonometric functions of t defined? Explain.

(b) For those trigonometric functions that are defined, determine whether the sign of the trigonometric function is positive or negative. Explain.

61. Think About It Because $f(t) = \sin t$ is an odd function and $g(t) = \cos t$ is an even function, what can be said about the function $h(t) = f(t)g(t)$?

62. Think About It Because $f(t) = \sin t$ and $g(t) = \tan t$ are odd functions, what can be said about the function $h(t) = f(t)g(t)$?

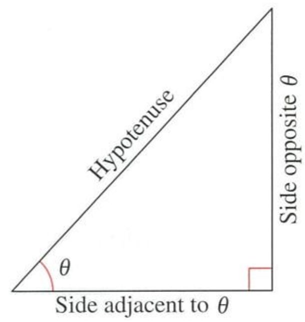
4.3 Right Triangle Trigonometry

Right triangle trigonometry has many real-life applications. For example, in Exercise 72 on page 287, you will use right triangle trigonometry to analyze the height of a helium-filled balloon.

- **Evaluate trigonometric functions of acute angles.**
- **Use fundamental trigonometric identities.**
- **Use trigonometric functions to model and solve real-life problems.**

The Six Trigonometric Functions

This section introduces the trigonometric functions from a *right triangle* perspective. Consider the right triangle shown below, in which one acute angle is labeled θ. Relative to the angle θ, the three sides of the triangle are the **hypotenuse,** the **opposite side** (the side opposite the angle θ), and the **adjacent side** (the side adjacent to the angle θ).



Using the lengths of these three sides, you can form six ratios that define the six trigonometric functions of the acute angle θ.

sine **cosecant** **cosine** **secant** **tangent** **cotangent**

In the definitions below,

$$0° < \theta < 90°$$

(θ lies in the first quadrant). For such angles, the value of each trigonometric function is *positive*.

Right Triangle Definitions of Trigonometric Functions

Let θ be an *acute* angle of a right triangle. The six trigonometric functions of the angle θ are defined below. (Note that the functions in the second row are the *reciprocals* of the corresponding functions in the first row.)

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

The abbreviations

opp, *adj,* and *hyp*

represent the lengths of the three sides of a right triangle.

opp = the length of the side *opposite* θ

adj = the length of the side *adjacent to* θ

hyp = the length of the *hypotenuse*

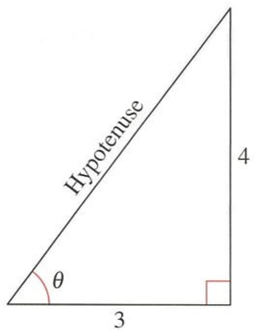


Figure 4.19

EXAMPLE 1 Evaluating Trigonometric Functions

See LarsonPrecalculus.com for an interactive version of this type of example.

Use the triangle in Figure 4.19 to find the values of the six trigonometric functions of θ.

Solution By the Pythagorean Theorem, $(\text{hyp})^2 = (\text{opp})^2 + (\text{adj})^2$, it follows that

$$\begin{aligned}\text{hyp} &= \sqrt{4^2 + 3^2}\\ &= \sqrt{25}\\ &= 5.\end{aligned}$$

So, the six trigonometric functions of θ are

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4}{5} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{5}{4}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{3}{5} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{5}{3}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{3} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{3}{4}.$$

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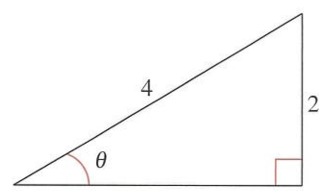
Use the triangle below to find the values of the six trigonometric functions of θ.



HISTORICAL NOTE

Georg Joachim Rheticus (1514–1576) was the leading Teutonic mathematical astronomer of the 16th century. He was the first to define the trigonometric functions as ratios of the sides of a right triangle.

In Example 1, you were given the lengths of two sides of the right triangle, but not the angle θ. Often, you will be asked to find the trigonometric functions of a *given* acute angle θ. To do this, construct a right triangle having θ as one of its angles.

EXAMPLE 2 Evaluating Trigonometric Functions of 45°

Find the values of sin 45°, cos 45°, and tan 45°.

Solution Construct a right triangle having 45° as one of its acute angles, as shown in Figure 4.20. Choose 1 as the length of the adjacent side. From geometry, you know that the other acute angle is also 45°. So, the triangle is isosceles and the length of the opposite side is also 1. By the Pythagorean Theorem, the length of the hypotenuse is $\sqrt{2}$.

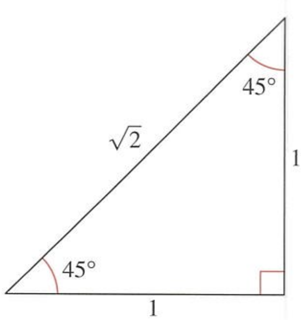


Figure 4.20

$$\sin 45^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45^\circ = \frac{\text{adj}}{\text{hyp}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45^\circ = \frac{\text{opp}}{\text{adj}} = \frac{1}{1} = 1$$

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Find the values of cot 45°, sec 45°, and csc 45°.

EXAMPLE 3 Evaluating Trigonometric Functions of 30° and 60°

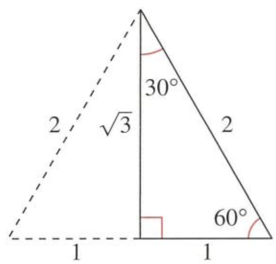

Figure 4.21

Use the equilateral triangle shown in Figure 4.21 to find the values of sin 60°, cos 60°, sin 30°, and cos 30°.

Solution For $\theta = 60°$, you have adj $= 1$, opp $= \sqrt{3}$, and hyp $= 2$. So,

$$\sin 60° = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{3}}{2} \quad \text{and} \quad \cos 60° = \frac{\text{adj}}{\text{hyp}} = \frac{1}{2}.$$

For $\theta = 30°$, adj $= \sqrt{3}$, opp $= 1$, and hyp $= 2$. So,

$$\sin 30° = \frac{\text{opp}}{\text{hyp}} = \frac{1}{2} \quad \text{and} \quad \cos 30° = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{3}}{2}.$$

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Use the equilateral triangle shown in Figure 4.21 to find the values of tan 60° and tan 30°.

REMARK The angles 30°, 45°, and 60° ($\pi/6$, $\pi/4$, and $\pi/3$ radians, respectively) occur frequently in trigonometry, so you should learn to construct the triangles shown in Figures 4.20 and 4.21.

Sines, Cosines, and Tangents of Special Angles

$$\sin 30° = \sin\frac{\pi}{6} = \frac{1}{2} \qquad \cos 30° = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2} \qquad \tan 30° = \tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$$

$$\sin 45° = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} \qquad \cos 45° = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} \qquad \tan 45° = \tan\frac{\pi}{4} = 1$$

$$\sin 60° = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \qquad \cos 60° = \cos\frac{\pi}{3} = \frac{1}{2} \qquad \tan 60° = \tan\frac{\pi}{3} = \sqrt{3}$$

Note that $\sin 30° = \frac{1}{2} = \cos 60°$. This occurs because 30° and 60° are complementary angles. In general, it can be shown from the right triangle definitions that *cofunctions of complementary angles are equal*. That is, if θ is an acute angle, then the relationships below are true.

$$\sin(90° - \theta) = \cos\theta \qquad \cos(90° - \theta) = \sin\theta \qquad \tan(90° - \theta) = \cot\theta$$

$$\cot(90° - \theta) = \tan\theta \qquad \sec(90° - \theta) = \csc\theta \qquad \csc(90° - \theta) = \sec\theta$$

To use a calculator to evaluate trigonometric functions of angles measured in degrees, remember to set the calculator to *degree* mode.

EXAMPLE 4 Using a Calculator

Use a calculator to evaluate sec 5° 40′ 12″.

Solution Begin by converting to decimal degree form. $\left[\text{Recall that } 1' = \frac{1}{60}(1°) \text{ and } 1'' = \frac{1}{3600}(1°).\right]$

$$5°\,40'\,12'' = 5° + \left(\frac{40}{60}\right)^{\circ} + \left(\frac{12}{3600}\right)^{\circ} = 5.67°$$

Then, use a calculator to evaluate sec 5.67°.

Function	Calculator Keystrokes	Display
sec 5° 40′ 12″ = sec 5.67°	(COS (5.67)) x^{-1} ENTER	1.0049166

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Use a calculator to evaluate csc 34° 30′ 36″.

Trigonometric Identities

Trigonometric identities are relationships between trigonometric functions.

Fundamental Trigonometric Identities

Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

REMARK Do not confuse, for example, $\sin^2\theta$ with $\sin\theta^2$. With $\sin^2\theta$, you are squaring $\sin\theta$. With $\sin\theta^2$, you are squaring θ and then finding the sine.

Note that $\sin^2\theta$ represents $(\sin\theta)^2$, $\cos^2\theta$ represents $(\cos\theta)^2$, and so on.

EXAMPLE 5 Applying Trigonometric Identities

Let θ be an acute angle such that $\sin\theta = 0.6$. Find the value of (a) $\cos\theta$ and (b) $\tan\theta$ using trigonometric identities.

Solution

a. To find the value of $\cos\theta$, use the Pythagorean identity

$$\sin^2\theta + \cos^2\theta = 1.$$

So, you have

$(0.6)^2 + \cos^2\theta = 1$	Substitute 0.6 for $\sin\theta$.
$\cos^2\theta = 1 - (0.6)^2$	Subtract $(0.6)^2$ from each side.
$\cos^2\theta = 0.64$	Simplify.
$\cos\theta = \sqrt{0.64}$	Extract positive square root.
$\cos\theta = 0.8.$	Simplify.

b. Now, knowing the sine and cosine of θ, you can find the tangent of θ.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{0.6}{0.8} = 0.75$$

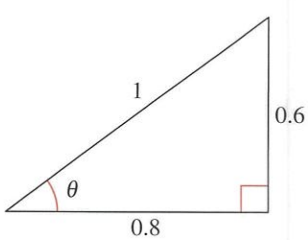

Figure 4.22

Use the definitions of $\cos\theta$ and $\tan\theta$ and the triangle shown in Figure 4.22 to check these results.

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Let θ be an acute angle such that $\cos\theta = 0.96$. Find the value of (a) $\sin\theta$ and (b) $\tan\theta$ using trigonometric identities.

EXAMPLE 6 Applying Trigonometric Identities

Let θ be an acute angle such that $\tan \theta = \frac{1}{3}$. Find the value of (a) $\cot \theta$ and (b) $\sec \theta$ using trigonometric identities.

Solution

a. $\cot \theta = \dfrac{1}{\tan \theta}$ — Reciprocal identity

$= \dfrac{1}{1/3}$ — Substitute $\frac{1}{3}$ for $\tan \theta$.

$= 3$ — Simplify.

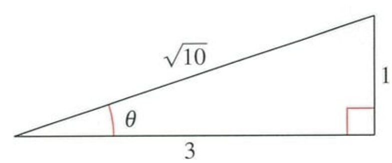
b. $\sec^2 \theta = 1 + \tan^2 \theta$ — Pythagorean identity

$\sec^2 \theta = 1 + \left(\dfrac{1}{3}\right)^2$ — Substitute $\frac{1}{3}$ for $\tan \theta$.

$\sec^2 \theta = \dfrac{10}{9}$ — Simplify.

$\sec \theta = \dfrac{\sqrt{10}}{3}$ — Extract positive square root and simplify.

Use the definitions of $\cot \theta$ and $\sec \theta$ and the triangle below to check these results.



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Let θ be an acute angle such that $\tan \theta = 2$. Find the value of (a) $\cot \theta$ and (b) $\sec \theta$ using trigonometric identities.

EXAMPLE 7 Using Trigonometric Identities

Use trigonometric identities to transform the left side of the equation into the right side $(0 < \theta < \pi/2)$.

a. $\sin \theta \csc \theta = 1$ **b.** $(\csc \theta + \cot \theta)(\csc \theta - \cot \theta) = 1$

Solution

a. $\sin \theta \csc \theta = \left(\dfrac{1}{\csc \theta}\right) \csc \theta = 1$ — Use a reciprocal identity and simplify.

b. $(\csc \theta + \cot \theta)(\csc \theta - \cot \theta)$

$= \csc^2 \theta - \csc \theta \cot \theta + \csc \theta \cot \theta - \cot^2 \theta$ — FOIL Method

$= \csc^2 \theta - \cot^2 \theta$ — Simplify.

$= 1$ — Pythagorean identity

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Use trigonometric identities to transform the left side of the equation into the right side $(0 < \theta < \pi/2)$.

a. $\tan \theta \csc \theta = \sec \theta$ **b.** $(\csc \theta + 1)(\csc \theta - 1) = \cot^2 \theta$

Applications Involving Right Triangles

Many applications of trigonometry involve **solving right triangles.** In this type of application, you are usually given one side of a right triangle and one of the acute angles and are asked to find one of the other sides, *or* you are given two sides and are asked to find one of the acute angles.

In Example 8, you are given the **angle of elevation,** which represents the angle from the horizontal upward to an object. In other applications you may be given the **angle of depression,** which represents the angle from the horizontal downward to an object. (See Figure 4.23.)

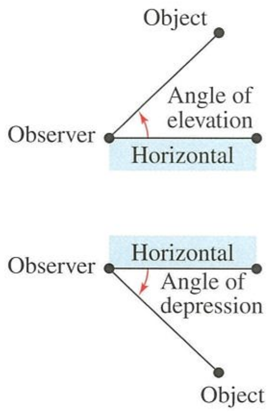

Figure 4.23

EXAMPLE 8 Solving a Right Triangle

A surveyor stands 115 feet from the base of the Washington Monument, as shown in Figure 4.24. The surveyor measures the angle of elevation to the top of the monument to be 78.3°. How tall is the Washington Monument?

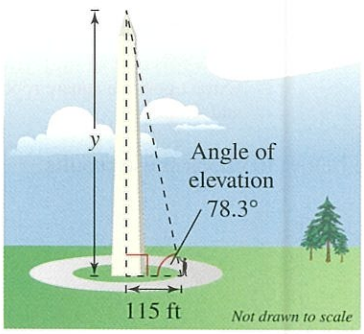

Figure 4.24

Solution From Figure 4.24,

$$\tan 78.3° = \frac{\text{opp}}{\text{adj}} = \frac{y}{115}$$

where y is the height of the monument. So, the height of the Washington Monument is

$$y = 115 \tan 78.3°$$
$$\approx 115(4.8288)$$
$$\approx 555 \text{ feet.}$$

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The angle of elevation to the top of a flagpole at a distance of 19 feet from its base is 64.6°. How tall is the flagpole?

EXAMPLE 9 Solving a Right Triangle

A lighthouse is 200 yards from a bike path along the edge of a lake. A walkway to the lighthouse is 400 yards long. (See Figure 4.25.) Find the acute angle θ between the bike path and the walkway.

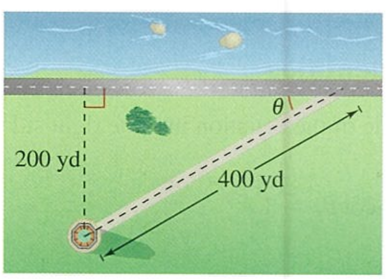

Figure 4.25

Solution From Figure 4.25, the sine of the angle θ is

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{200}{400} = \frac{1}{2}.$$

You should recognize that $\theta = 30°$.

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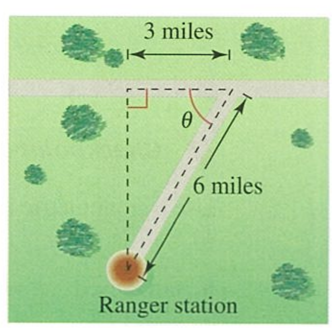
Find the acute angle θ between the two paths shown below.

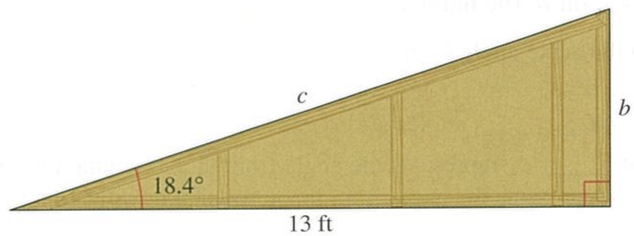


In Example 9, you were able to recognize that the special angle $\theta = 30°$ satisfies the equation $\sin \theta = \frac{1}{2}$. However, when θ is not a special angle, you can *estimate* its value. For example, to estimate the acute angle θ in the equation $\sin \theta = 0.6$, you could reason that $\sin 30° = \frac{1}{2} = 0.5000$ and $\sin 45° = 1/\sqrt{2} \approx 0.7071$, so θ lies somewhere between 30° and 45°. In a later section, you will study a method of determining a more precise value of θ.

EXAMPLE 10 Solving a Right Triangle

Skateboarders can go to a skatepark, which is a recreational environment built with many different types of ramps and rails.

Find the length c and the height b of the skateboard ramp below.



Solution From the figure,

$$\cos 18.4° = \frac{\text{adj}}{\text{hyp}} = \frac{13}{c}.$$

So, the length of the skateboard ramp is

$$c = \frac{13}{\cos 18.4°} \approx \frac{13}{0.9489} \approx 13.7 \text{ feet.}$$

Also from the figure,

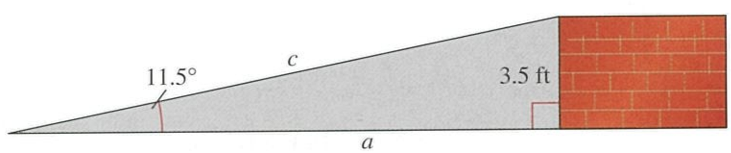
$$\tan 18.4° = \frac{\text{opp}}{\text{adj}} = \frac{b}{13}.$$

So, the height is

$$b = 13 \tan 18.4° \approx 13(0.3327) \approx 4.3 \text{ feet.}$$

✓ Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Find the length c and the horizontal length a of the loading ramp below.



Summarize (Section 4.3)

1. State the right triangle definitions of the six trigonometric functions *(page 277)*. For examples of evaluating trigonometric functions of acute angles, see Examples 1–4.
2. List the reciprocal, quotient, and Pythagorean identities *(page 280)*. For examples of using these identities, see Examples 5–7.
3. Describe real-life applications of trigonometric functions *(pages 282 and 283, Examples 8–10)*.

4.3 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary

1. Match each trigonometric function with its right triangle definition.

(a) sine (b) cosine (c) tangent (d) cosecant (e) secant (f) cotangent

(i) $\dfrac{\text{hypotenuse}}{\text{adjacent}}$ (ii) $\dfrac{\text{adjacent}}{\text{opposite}}$ (iii) $\dfrac{\text{hypotenuse}}{\text{opposite}}$ (iv) $\dfrac{\text{adjacent}}{\text{hypotenuse}}$ (v) $\dfrac{\text{opposite}}{\text{hypotenuse}}$ (vi) $\dfrac{\text{opposite}}{\text{adjacent}}$

In Exercises 2–4, fill in the blanks.

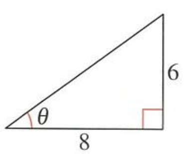
2. Relative to the acute angle θ, the three sides of a right triangle are the ________ side, the ________ side, and the ________.

3. Cofunctions of ________ angles are equal.

4. An angle of ________ represents the angle from the horizontal upward to an object, whereas an angle of ________ represents the angle from the horizontal downward to an object.

Skills and Applications

Evaluating Trigonometric Functions **In Exercises 5–10, find the exact values of the six trigonometric functions of the angle θ.**

5.


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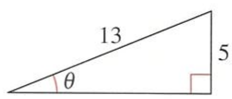
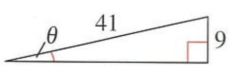


7.


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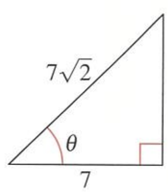

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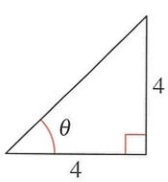
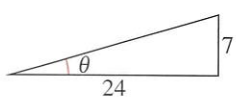
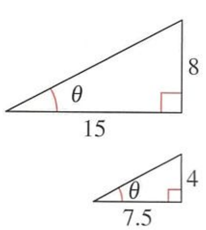
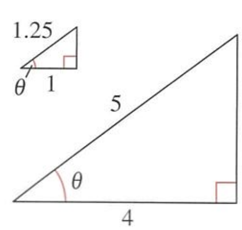
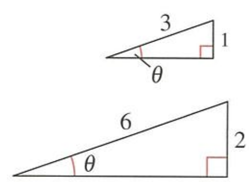


10.


Evaluating Trigonometric Functions **In Exercises 11–14, find the exact values of the six trigonometric functions of the angle θ for each of the two triangles. Explain why the function values are the same.**

11.


12.


13.


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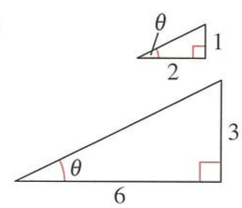


Evaluating Trigonometric Functions **In Exercises 15–22, sketch a right triangle corresponding to the trigonometric function of the acute angle θ. Then find the exact values of the other five trigonometric functions of θ.**

15. $\cos\theta = \frac{15}{17}$

16. $\sin\theta = \frac{3}{5}$

17. $\sec\theta = \frac{6}{5}$

18. $\tan\theta = \frac{4}{5}$

19. $\sin\theta = \frac{1}{5}$

20. $\sec\theta = \frac{17}{7}$

21. $\cot\theta = 3$

22. $\csc\theta = 9$

Evaluating Trigonometric Functions of 30°, 45°, and 60° **In Exercises 23–28, construct an appropriate triangle to find the missing values. ($0° \le \theta \le 90°$, $0 \le \theta \le \pi/2$)**

	Function	θ (deg)	θ (rad)	Function Value
23.	tan	30°		
24.	cos	45°		
25.	sin		$\frac{\pi}{4}$	
26.	tan		$\frac{\pi}{3}$	
27.	sec		$\frac{\pi}{4}$	
28.	csc		$\frac{\pi}{6}$	

Using a Calculator In Exercises 29–36, use a calculator to evaluate each function. Round your answers to four decimal places. (Be sure the calculator is in the correct mode.)

29. (a) $\sin 20°$ (b) $\cos 70°$

30. (a) $\tan 23.5°$ (b) $\cot 66.5°$

31. (a) $\sin 14.21°$ (b) $\csc 14.21°$

32. (a) $\cot 79.56°$ (b) $\sec 79.56°$

33. (a) $\cos 4° 50' 15''$ (b) $\sec 4° 50' 15''$

34. (a) $\sec 42° 12'$ (b) $\csc 48° 7'$

35. (a) $\cot 17° 15'$ (b) $\tan 17° 15'$

36. (a) $\sec 56° 8' 10''$ (b) $\cos 56° 8' 10''$

Applying Trigonometric Identities In Exercises 37–42, use the given function value(s) and the trigonometric identities to find the exact value of each indicated trigonometric function.

37. $\sin 60° = \frac{\sqrt{3}}{2}$, $\cos 60° = \frac{1}{2}$

(a) $\sin 30°$ (b) $\cos 30°$

(c) $\tan 60°$ (d) $\cot 60°$

38. $\sin 30° = \frac{1}{2}$, $\tan 30° = \frac{\sqrt{3}}{3}$

(a) $\csc 30°$ (b) $\cot 60°$

(c) $\cos 30°$ (d) $\cot 30°$

39. $\cos \theta = \frac{1}{3}$

(a) $\sin \theta$ (b) $\tan \theta$

(c) $\sec \theta$ (d) $\csc(90° - \theta)$

40. $\sec \theta = 5$

(a) $\cos \theta$ (b) $\cot \theta$

(c) $\cot(90° - \theta)$ (d) $\sin \theta$

41. $\cot \alpha = 3$

(a) $\tan \alpha$ (b) $\csc \alpha$

(c) $\cot(90° - \alpha)$ (d) $\sin \alpha$

42. $\cos \beta = \frac{\sqrt{7}}{4}$

(a) $\sec \beta$ (b) $\sin \beta$

(c) $\cot \beta$ (d) $\sin(90° - \beta)$

Using Trigonometric Identities In Exercises 43–52, use trigonometric identities to transform the left side of the equation into the right side $(0 < \theta < \pi/2)$.

43. $\tan \theta \cot \theta = 1$

44. $\cos \theta \sec \theta = 1$

45. $\tan \alpha \cos \alpha = \sin \alpha$

46. $\cot \alpha \sin \alpha = \cos \alpha$

47. $(1 + \sin \theta)(1 - \sin \theta) = \cos^2 \theta$

48. $(1 + \cos \theta)(1 - \cos \theta) = \sin^2 \theta$

49. $(\sec \theta + \tan \theta)(\sec \theta - \tan \theta) = 1$

50. $\sin^2 \theta - \cos^2 \theta = 2 \sin^2 \theta - 1$

51. $\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = \csc \theta \sec \theta$

52. $\frac{\tan \beta + \cot \beta}{\tan \beta} = \csc^2 \beta$

Finding Special Angles of a Triangle In Exercises 53–58, find each value of θ in degrees $(0° < \theta < 90°)$ and radians $(0 < \theta < \pi/2)$ without using a calculator.

53. (a) $\sin \theta = \frac{1}{2}$ (b) $\csc \theta = 2$

54. (a) $\cos \theta = \frac{\sqrt{2}}{2}$ (b) $\tan \theta = 1$

55. (a) $\sec \theta = 2$ (b) $\cot \theta = 1$

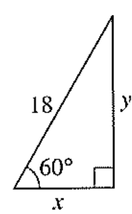
56. (a) $\tan \theta = \sqrt{3}$ (b) $\csc \theta = \sqrt{2}$

57. (a) $\csc \theta = \frac{2\sqrt{3}}{3}$ (b) $\sin \theta = \frac{\sqrt{2}}{2}$

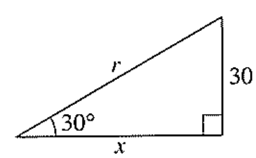
58. (a) $\cot \theta = \frac{\sqrt{3}}{3}$ (b) $\sec \theta = \sqrt{2}$

Finding Side Lengths of a Triangle In Exercises 59–62, find the exact values of the indicated variables.

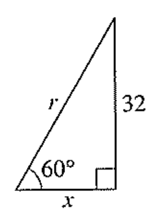
59. Find x and y.



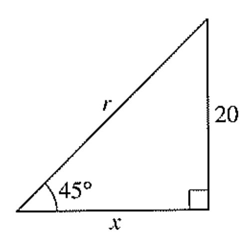
60. Find x and r.



61. Find x and r.



62. Find x and r.



63. **Empire State Building** You are standing 45 meters from the base of the Empire State Building. You estimate that the angle of elevation to the top of the 86th floor (the observatory) is 82°. The total height of the building is another 123 meters above the 86th floor. What is the approximate height of the building? One of your friends is on the 86th floor. What is the distance between you and your friend?

64. Height of a Tower A six-foot person walks from the base of a broadcasting tower directly toward the tip of the shadow cast by the tower. When the person is 132 feet from the tower and 3 feet from the tip of the shadow, the person's shadow starts to appear beyond the tower's shadow.

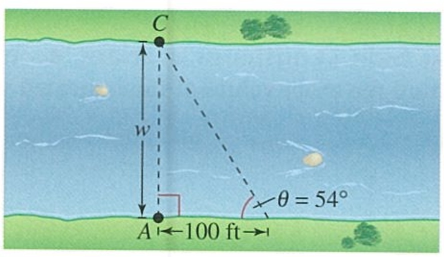
(a) Draw a right triangle that gives a visual representation of the problem. Label the known quantities of the triangle and use a variable to represent the height of the tower.

(b) Use a trigonometric function to write an equation involving the unknown quantity.

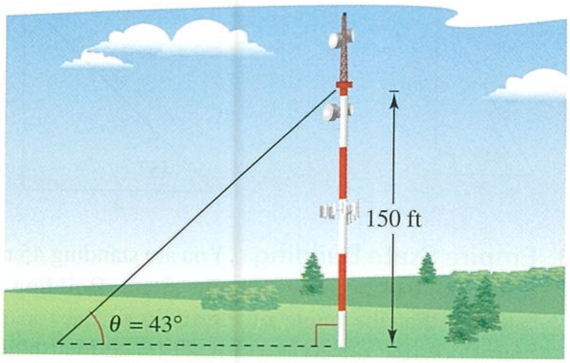
(c) What is the height of the tower?

65. Angle of Elevation You are skiing down a mountain with a vertical height of 1250 feet. The distance from the top of the mountain to the base is 2500 feet. What is the angle of elevation from the base to the top of the mountain?

66. Biology A biologist wants to know the width w of a river to properly set instruments for an experiment. From point A, the biologist walks downstream 100 feet and sights to point C (see figure). From this sighting, it is determined that $\theta = 54°$. How wide is the river?



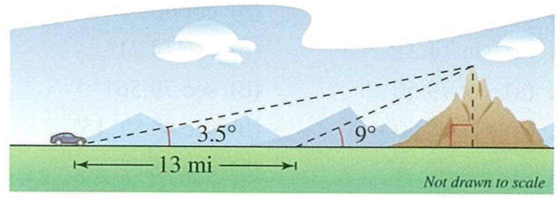
67. Guy Wire A guy wire runs from the ground to a cell tower. The wire is attached to the cell tower 150 feet above the ground. The angle formed between the wire and the ground is 43° (see figure).



(a) How long is the guy wire?

(b) How far from the base of the tower is the guy wire anchored to the ground?

68. Height of a Mountain In traveling across flat land, you see a mountain directly in front of you. Its angle of elevation (to the peak) is 3.5°. After you drive 13 miles closer to the mountain, the angle of elevation is 9° (see figure). Approximate the height of the mountain.



69. Machine Shop Calculations A steel plate has the form of one-fourth of a circle with a radius of 60 centimeters. Two two-centimeter holes are drilled in the plate, positioned as shown in the figure. Find the coordinates of the center of each hole.

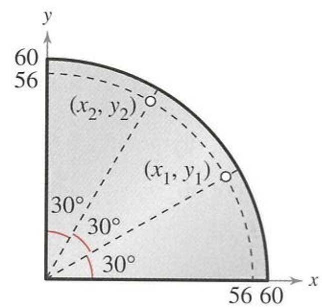


Figure for 69

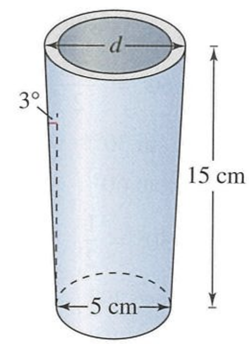


Figure for 70

70. Machine Shop Calculations A tapered shaft has a diameter of 5 centimeters at the small end and is 15 centimeters long (see figure). The taper is 3°. Find the diameter d of the large end of the shaft.

71. Geometry Use a compass to sketch a quarter of a circle of radius 10 centimeters. Using a protractor, construct an angle of 20° in standard position (see figure). Drop a perpendicular line from the point of intersection of the terminal side of the angle and the arc of the circle. By actual measurement, calculate the coordinates (x, y) of the point of intersection and use these measurements to approximate the six trigonometric functions of a 20° angle.