2.7 Nonlinear Inequalities

Nonlinear inequalities have many real-life applications. For example, in Exercises 67 and 68 on page 186, you will use a polynomial inequality to model the height of a projectile.

- **Solve polynomial inequalities.**
- **Solve rational inequalities.**
- **Use nonlinear inequalities to model and solve real-life problems.**

Polynomial Inequalities

To solve a polynomial inequality such as $x^2 - 2x - 3 < 0$, use the fact that a polynomial can change signs only at its *zeros* (the x-values that make the polynomial equal to zero). Between two consecutive zeros, a polynomial must be entirely positive or entirely negative. This means that when the real zeros of a polynomial are put in order, they divide the real number line into intervals in which the polynomial has no sign changes. These zeros are the **key numbers** of the inequality, and the resulting open intervals are the *test intervals* for the inequality. For example, the polynomial $x^2 - 2x - 3$ factors as

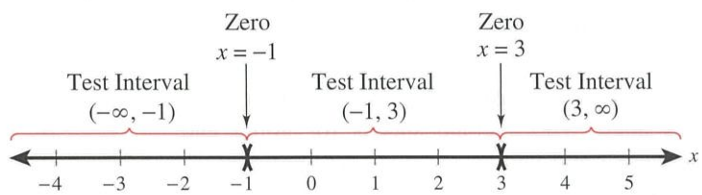
$$x^2 - 2x - 3 = (x + 1)(x - 3)$$

so it has two zeros,

$$x = -1 \quad \text{and} \quad x = 3.$$

These zeros divide the real number line into three test intervals:

$$(-\infty, -1), \quad (-1, 3), \quad \text{and} \quad (3, \infty). \qquad \text{(See figure below.)}$$



Three test intervals for $x^2 - 2x - 3$

To solve the inequality $x^2 - 2x - 3 < 0$, you need to test only one value from each of these test intervals. When a value from a test interval satisfies the original inequality, you can conclude that the interval is a solution of the inequality.

Use the same basic approach, generalized below, to find the solution set of any polynomial inequality.

REMARK The solution set of

$$x^2 - 2x - 3 < 0$$

discussed above, is the open interval $(-1, 3)$. Use Step 3 to verify this. By choosing the representative x-values $x = -2$, $x = 0$, and $x = 4$, you will find that the value of the polynomial is negative only in $(-1, 3)$.

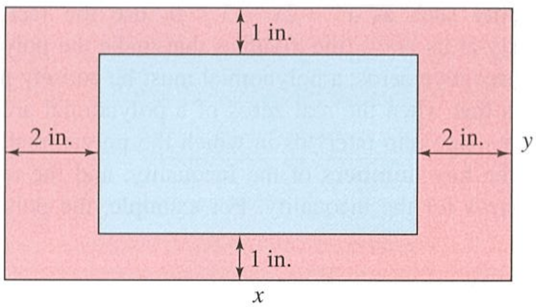
Test Intervals for a Polynomial Inequality

To determine the intervals on which the values of a polynomial are entirely negative or entirely positive, use the steps below.

1. Find all real zeros of the polynomial, and arrange the zeros in increasing order. These zeros are the key numbers of the inequality.
2. Use the key numbers of the inequality to determine the test intervals.
3. Choose one representative x-value in each test interval and evaluate the polynomial at that value. When the value of the polynomial is negative, the polynomial has negative values for every x-value in the interval. When the value of the polynomial is positive, the polynomial has positive values for every x-value in the interval.

71. Page Design A rectangular page contains 64 square inches of print. The margins at the top and bottom of the page are each 1 inch deep. The margins on each side are $1\frac{1}{2}$ inches wide. What should the dimensions of the page be to use the least amount of paper?

72. Page Design A page that is x inches wide and y inches high contains 30 square inches of print. The top and bottom margins are each 1 inch deep, and the margins on each side are 2 inches wide (see figure).



(a) Write a functions for the total area A of the page in terms of x.

(b) Determine the domain of the function based on the physical constraints of the problem.

(c) Use a graphing utility to graph the area function and approximate the dimensions of the page that use the least amount of paper.

73. Average Speed A driver's average speed is 50 miles per hour on a round trip between two cities 100 miles apart. The average speeds for going and returning were x and y miles per hour, respectively.

(a) Show that $y = (25x)/(x - 25)$.

(b) Determine the vertical and horizontal asymptotes of the graph of the function.

(c) Use a graphing utility to graph the function.

(d) Complete the table.

x	30	35	40	45	50	55	60
y							

(e) Are the results in the table what you expected? Explain.

(f) Is it possible to average 20 miles per hour in one direction and still average 50 miles per hour on the round trip? Explain.

74. Medicine The concentration C of a chemical in the bloodstream t hours after injection into muscle tissue is given by

$$C = \frac{3t^2 + t}{t^3 + 50}, \quad t > 0.$$

Use a graphing utility to graph the function. Determine the horizontal asymptote of the graph of the function and interpret its meaning in the context of the problem.

Exploration

True or False? **In Exercises 75–77, determine whether the statement is true or false. Justify your answer.**

75. The graph of a polynomial function can have infinitely many vertical asymptotes.

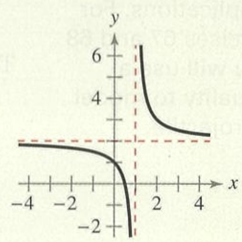
76. The graph of a rational function can never cross one of its asymptotes.

77. The graph of a rational function can have a vertical asymptote, a horizontal asymptote, and a slant asymptote.

78. HOW DO YOU SEE IT? The graph of a rational function

$$f(x) = \frac{N(x)}{D(x)}$$

is shown below. Determine which of the statements about the function is false. Justify your answer.



(a) $D(1) = 0$.

(b) The degree of $N(x)$ and $D(x)$ are equal.

(c) The ratio of the leading coefficients of $N(x)$ and $D(x)$ is 1.

79. Writing Is every rational function a polynomial function? Is every polynomial function a rational function? Explain.

Writing a Rational Function **In Exercises 80–82, write a rational function f whose graph has the specified characteristics. (There are many correct answers.)**

80. Vertical asymptote: None

Horizontal asymptote: $y = 2$

81. Vertical asymptotes: $x = -2, x = 1$

Horizontal asymptote: None

82. Vertical asymptote: $x = 2$

Slant asymptote: $y = x + 1$

Zero of the function: $x = -2$

Project: Department of Defense To work an extended application analyzing the total numbers of military personnel on active duty from 1984 through 2014, visit this text's website at *LarsonPrecalculus.com. (Source: U.S. Department of Defense)*

EXAMPLE 1 Solving a Polynomial Inequality

Solve $x^2 - x - 6 < 0$. Then graph the solution set.

ALGEBRA HELP To review the techniques for factoring polynomials, see Appendix A.5.

Solution Factoring the polynomial

$$x^2 - x - 6 = (x + 2)(x - 3)$$

shows that the key numbers are $x = -2$ and $x = 3$. So, the inequality's test intervals are

$$(-\infty, -2),\quad (-2, 3),\quad \text{and}\quad (3, \infty) \qquad \text{Test intervals}$$

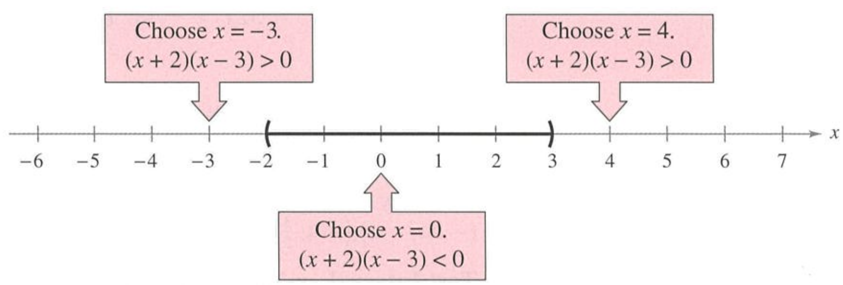
In each test interval, choose a representative x-value and evaluate the polynomial.

Test Interval	x-Value	Polynomial Value	Conclusion
$(-\infty, -2)$	$x = -3$	$(-3)^2 - (-3) - 6 = 6$	Positive
$(-2, 3)$	$x = 0$	$(0)^2 - (0) - 6 = -6$	Negative
$(3, \infty)$	$x = 4$	$(4)^2 - (4) - 6 = 6$	Positive

The inequality is satisfied for all x-values in $(-2, 3)$. This implies that the solution set of the inequality

$$x^2 - x - 6 < 0$$

is the interval $(-2, 3)$, as shown on the number line below. Note that the original inequality contains a "less than" symbol. This means that the solution set does not contain the endpoints of the test interval $(-2, 3)$.



✓ **Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Solve $x^2 - x - 20 < 0$. Then graph the solution set.

As with linear inequalities, you can check the reasonableness of a solution by substituting x-values into the original inequality. For instance, to check the solution found in Example 1, substitute several x-values from the interval $(-2, 3)$ into the inequality

$$x^2 - x - 6 < 0.$$

Regardless of which x-values you choose, the inequality should be satisfied.

You can also use a graph to check the result of Example 1. Sketch the graph of

$$y = x^2 - x - 6$$

as shown in Figure 2.31. Notice that the graph is below the x-axis on the interval $(-2, 3)$.

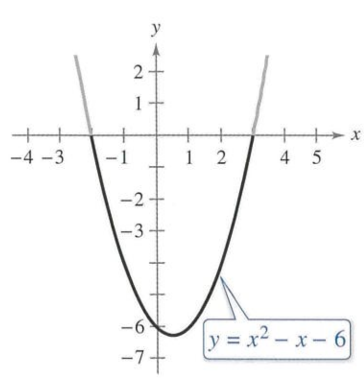


Figure 2.31

In Example 1, the polynomial inequality is in general form (with the polynomial on one side and zero on the other). Whenever this is not the case, you should begin by writing the inequality in general form.

EXAMPLE 2 Solving a Polynomial Inequality

See LarsonPrecalculus.com for an interactive version of this type of example.

Solve $4x^2 - 5x > 6$.

Algebraic Solution

$$4x^2 - 5x - 6 > 0 \quad \text{Write in general form.}$$
$$(x - 2)(4x + 3) > 0 \quad \text{Factor.}$$

Key numbers: $x = -\frac{3}{4}$, $x = 2$

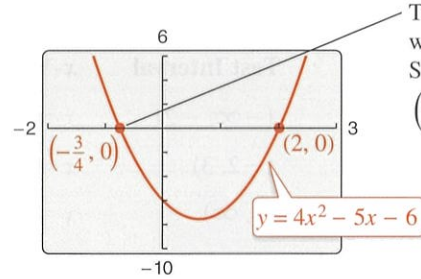
Test intervals: $\left(-\infty, -\frac{3}{4}\right)$, $\left(-\frac{3}{4}, 2\right)$, $(2, \infty)$

Test: Is $(x - 2)(4x + 3) > 0$?

Testing these intervals shows that the polynomial $4x^2 - 5x - 6$ is positive on the open intervals $\left(-\infty, -\frac{3}{4}\right)$ and $(2, \infty)$. So, the solution set of the inequality is $\left(-\infty, -\frac{3}{4}\right) \cup (2, \infty)$.

Graphical Solution

First write the polynomial inequality $4x^2 - 5x > 6$ as $4x^2 - 5x - 6 > 0$. Then use a graphing utility to graph $y = 4x^2 - 5x - 6$.



The graph is above the x-axis when $x < -\frac{3}{4}$ or when $x > 2$. So, the solution set is $\left(-\infty, -\frac{3}{4}\right) \cup (2, \infty)$.

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Solve $2x^2 + 3x < 5$ (a) algebraically and (b) graphically.

EXAMPLE 3 Solving a Polynomial Inequality

Solve $2x^3 - 3x^2 - 32x > -48$. Then graph the solution set.

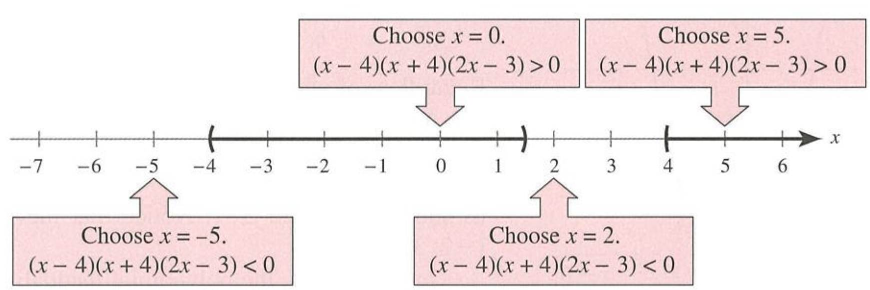
Solution

$$2x^3 - 3x^2 - 32x + 48 > 0 \quad \text{Write in general form.}$$
$$(x - 4)(x + 4)(2x - 3) > 0 \quad \text{Factor by grouping.}$$

The key numbers are $x = -4$, $x = \frac{3}{2}$, and $x = 4$, and the test intervals are $(-\infty, -4)$, $\left(-4, \frac{3}{2}\right)$, $\left(\frac{3}{2}, 4\right)$, and $(4, \infty)$.

Test Interval	x-Value	Polynomial Value	Conclusion
$(-\infty, -4)$	$x = -5$	$2(-5)^3 - 3(-5)^2 - 32(-5) + 48 = -117$	Negative
$\left(-4, \frac{3}{2}\right)$	$x = 0$	$2(0)^3 - 3(0)^2 - 32(0) + 48 = 48$	Positive
$\left(\frac{3}{2}, 4\right)$	$x = 2$	$2(2)^3 - 3(2)^2 - 32(2) + 48 = -12$	Negative
$(4, \infty)$	$x = 5$	$2(5)^3 - 3(5)^2 - 32(5) + 48 = 63$	Positive

The inequality is satisfied on the open intervals $\left(-4, \frac{3}{2}\right)$ and $(4, \infty)$. So, the solution set is $\left(-4, \frac{3}{2}\right) \cup (4, \infty)$, as shown on the number line below.



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Solve $3x^3 - x^2 - 12x > -4$. Then graph the solution set.

You may find it easier to determine the sign of a polynomial from its *factored* form. For instance, in Example 2, when you substitute the test value $x = 1$ into the factored form

$$(x - 2)(4x + 3)$$

the sign pattern of the factors is

$$(\,-\,)(\,+\,)$$

which yields a negative result. Use factored forms to determine the signs of the polynomials in other examples in this section.

When solving a polynomial inequality, be sure to account for the inequality symbol. For instance, in Example 2, note that the original inequality symbol is "greater than" and the solution consists of two open intervals. If the original inequality had been

$$4x^2 - 5x \geq 6$$

then the solution set would have been

$$\left(-\infty, -\tfrac{3}{4}\right] \cup [2, \infty).$$

Each of the polynomial inequalities in Examples 1, 2, and 3 has a solution set that consists of a single interval or the union of two intervals. When solving the exercises for this section, watch for unusual solution sets, as illustrated in Example 4.

EXAMPLE 4 Unusual Solution Sets

a. The solution set of

$$x^2 + 2x + 4 > 0$$

consists of the entire set of real numbers, $(-\infty, \infty)$. In other words, the value of the quadratic polynomial $x^2 + 2x + 4$ is positive for every real value of x.

b. The solution set of

$$x^2 + 2x + 1 \leq 0$$

consists of the single real number $\{-1\}$, because the inequality has only one key number, $x = -1$, and it is the only value that satisfies the inequality.

c. The solution set of

$$x^2 + 3x + 5 < 0$$

is empty. In other words, $x^2 + 3x + 5$ is not less than zero for any value of x.

d. The solution set of

$$x^2 - 4x + 4 > 0$$

consists of all real numbers except $x = 2$. This solution set can be written in interval notation as

$$(-\infty, 2) \cup (2, \infty).$$

✔ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

What is unusual about the solution set of each inequality?

a. $x^2 + 6x + 9 < 0$

b. $x^2 + 4x + 4 \leq 0$

c. $x^2 - 6x + 9 > 0$

d. $x^2 - 2x + 1 \geq 0$

Rational Inequalities

The concepts of key numbers and test intervals can be extended to rational inequalities. Use the fact that the value of a rational expression can change sign at its *zeros* (the x-values for which its numerator is zero) and at its *undefined values* (the x-values for which its denominator is zero). These two types of numbers make up the *key numbers* of a rational inequality. When solving a rational inequality, begin by writing the inequality in general form, that is, with zero on the right side of the inequality.

EXAMPLE 5 Solving a Rational Inequality

Solve $\dfrac{2x-7}{x-5} \le 3$. Then graph the solution set.

REMARK By writing 3 as $\frac{3}{1}$, you should be able to see that the least common denominator is $(x-5)(1) = x-5$. So, rewriting the general form as

$$\frac{2x-7}{x-5} - \frac{3(x-5)}{x-5} \le 0$$

and subtracting gives the result shown.

Solution

$$\frac{2x-7}{x-5} \le 3 \qquad \text{Write original inequality.}$$

$$\frac{2x-7}{x-5} - 3 \le 0 \qquad \text{Write in general form.}$$

$$\frac{2x-7-3x+15}{x-5} \le 0 \qquad \text{Find the LCD and subtract fractions.}$$

$$\frac{-x+8}{x-5} \le 0 \qquad \text{Simplify.}$$

Key numbers: $x = 5,\ x = 8$ Zeros and undefined values of rational expression

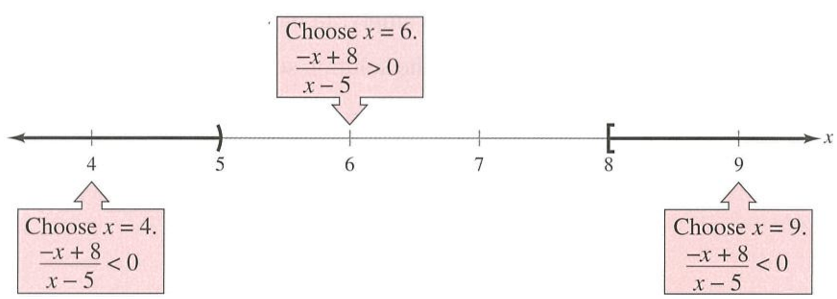
Test intervals: $(-\infty, 5), (5, 8), (8, \infty)$

Test: Is $\dfrac{-x+8}{x-5} \le 0$?

Testing these intervals, as shown in the figure below, the inequality is satisfied on the open intervals $(-\infty, 5)$ and $(8, \infty)$. Moreover,

$$\frac{-x+8}{x-5} = 0$$

when $x = 8$, so the solution set is $(-\infty, 5) \cup [8, \infty)$. (Be sure to use a bracket to signify that $x = 8$ is included in the solution set.)



Checkpoint Audio-video solution in English & Spanish at LarsonPrecalculus.com

Solve each inequality. Then graph the solution set.

a. $\dfrac{x-2}{x-3} \ge -3$

b. $\dfrac{4x-1}{x-6} > 3$

Applications

One common application of inequalities comes from business and involves profit, revenue, and cost. The formula that relates these three quantities is

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

$$P = R - C.$$

EXAMPLE 6 **Profit from a Product**

The marketing department of a calculator manufacturer determines that the demand for a new model of calculator is

$$p = 100 - 0.00001x, \quad 0 \le x \le 10{,}000{,}000 \qquad \text{Demand equation}$$

where p is the price per calculator (in dollars) and x represents the number of calculators sold. (According to this model, no one would be willing to pay \$100 for the calculator. At the other extreme, the company could not *give* away more than 10 million calculators.) The revenue for selling x calculators is

$$R = xp = x(100 - 0.00001x). \qquad \text{Revenue equation}$$

The total cost of producing x calculators is \$10 per calculator plus a one-time development cost of \$2,500,000. So, the total cost is

$$C = 10x + 2{,}500{,}000. \qquad \text{Cost equation}$$

What prices can the company charge per calculator to obtain a profit of at least \$190,000,000?

Solution

Verbal model: $\text{Profit} = \text{Revenue} - \text{Cost}$

Equation: $P = R - C$

$$P = 100x - 0.00001x^2 - (10x + 2{,}500{,}000)$$

$$P = -0.00001x^2 + 90x - 2{,}500{,}000$$

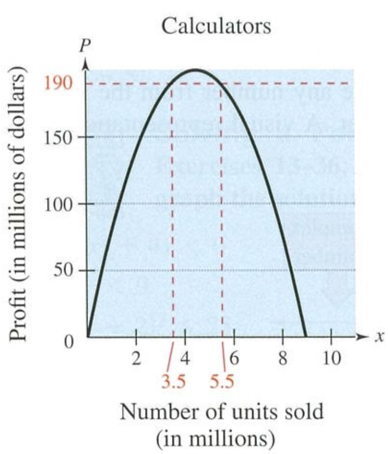


Figure 2.32

To answer the question, solve the inequality

$$P \ge 190{,}000{,}000$$

$$-0.00001x^2 + 90x - 2{,}500{,}000 \ge 190{,}000{,}000.$$

Write the inequality in general form, find the key numbers and the test intervals, and then test a value in each test interval to find that the solution is

$$3{,}500{,}000 \le x \le 5{,}500{,}000$$

as shown in Figure 2.32. Substituting the x-values in the original demand equation shows that prices of

$$\$45.00 \le p \le \$65.00$$

yield a profit of at least \$190,000,000.

✓ **Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

The revenue and cost equations for a product are

$$R = x(60 - 0.0001x) \quad \text{and} \quad C = 12x + 1{,}800{,}000$$

where R and C are measured in dollars and x represents the number of units sold. How many units must be sold to obtain a profit of at least \$3,600,000?

Solving an Inequality **In Exercises 61–66, solve the inequality. (Round your answers to two decimal places.)**

61. $0.3x^2 + 6.26 < 10.8$ **62.** $-1.3x^2 + 3.78 > 2.12$

63. $-0.5x^2 + 12.5x + 1.6 > 0$

64. $1.2x^2 + 4.8x + 3.1 < 5.3$

65. $\dfrac{1}{2.3x - 5.2} > 3.4$ **66.** $\dfrac{2}{3.1x - 3.7} > 5.8$

Height of a Projectile

In Exercises 67 and 68, use the position equation

$$s = -16t^2 + v_0 t + s_0$$

where s represents the height of an object (in feet), v_0 represents the initial velocity of the object (in feet per second), s_0 represents the initial height of the object (in feet), and t represents the time (in seconds).

67. A projectile is fired straight upward from ground level ($s_0 = 0$) with an initial velocity of 160 feet per second.

(a) At what instant will it be back at ground level?

(b) When will the height exceed 384 feet?

68. A projectile is fired straight upward from ground level ($s_0 = 0$) with an initial velocity of 128 feet per second.

(a) At what instant will it be back at ground level?

(b) When will the height be less than 128 feet?

69. Cost, Revenue, and Profit The revenue and cost equations for a product are $R = x(75 - 0.0005x)$ and $C = 30x + 250{,}000$, where R and C are measured in dollars and x represents the number of units sold. How many units must be sold to obtain a profit of at least \$750,000? What is the price per unit?

70. Cost, Revenue, and Profit The revenue and cost equations for a product are $R = x(50 - 0.0002x)$ and $C = 12x + 150{,}000$, where R and C are measured in dollars and x represents the number of units sold. How many units must be sold to obtain a profit of at least \$1,650,000? What is the price per unit?

Finding the Domain of an Expression **In Exercises 71–76, find the domain of the expression. Use a graphing utility to verify your result.**

71. $\sqrt{4 - x^2}$ **72.** $\sqrt{x^2 - 9}$

73. $\sqrt{x^2 - 9x + 20}$ **74.** $\sqrt{49 - x^2}$

75. $\sqrt{\dfrac{x}{x^2 - 2x - 35}}$ **76.** $\sqrt{\dfrac{x}{x^2 - 9}}$

77. School Enrollment The table shows the numbers N (in millions) of students enrolled in elementary and secondary schools in the United States from 2005 through 2014. *(Source: National Center for Education Statistics)*

DATA

Year	Number, N
2005	49.11
2006	49.32
2007	49.29
2008	49.27
2009	49.36
2010	49.48
2011	49.52
2012	49.77
2013	49.94
2014	49.99

Spreadsheet at LarsonPrecalculus.com

(a) Use a graphing utility to create a scatter plot of the data. Let t represent the year, with $t = 5$ corresponding to 2005.

(b) Use the *regression* feature of the graphing utility to find a *quartic* model for the data. (A quartic model has the form $at^4 + bt^3 + ct^2 + dt + e$, where a, b, c, d, and e are constant and t is variable.)

(c) Graph the model and the scatter plot in the same viewing window. How well does the model fit the data?

(d) According to the model, after 2014, when did the number of students enrolled in elementary and secondary schools fall below 48 million?

(e) Is the model valid for long-term predictions of student enrollment? Explain.

78. Safe Load The maximum safe load uniformly distributed over a one-foot section of a two-inch-wide wooden beam can be approximated by the model

$$\text{Load} = 168.5d^2 - 472.1$$

where d is the depth of the beam.

(a) Evaluate the model for $d = 4$, $d = 6$, $d = 8$, $d = 10$, and $d = 12$. Use the results to create a bar graph.

(b) Determine the minimum depth of the beam that will safely support a load of 2000 pounds.

79. Geometry A rectangular playing field with a perimeter of 100 meters is to have an area of at least 500 square meters. Within what bounds must the length of the rectangle lie?

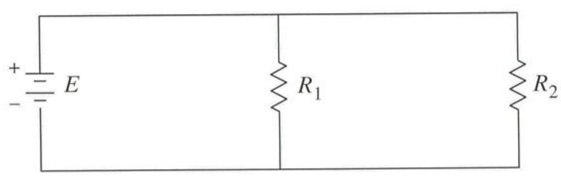
digidreamgrafix/Shutterstock.com

80. Geometry A rectangular parking lot with a perimeter of 440 feet is to have an area of at least 8000 square feet. Within what bounds must the length of the rectangle lie?

81. Resistors When two resistors of resistances R_1 and R_2 are connected in parallel (see figure), the total resistance R satisfies the equation

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

Find R_1 for a parallel circuit in which $R_2 = 2$ ohms and R must be at least 1 ohm.



82. Teachers' Salaries The table shows the mean salaries S (in thousands of dollars) of public school classroom teachers in the United States from 2002 through 2013.

DATA

Spreadsheet at LarsonPrecalculus.com

Year	Salary, S
2002	44.7
2003	45.7
2004	46.5
2005	47.5
2006	49.1
2007	51.1
2008	52.8
2009	54.3
2010	55.2
2011	56.1
2012	55.4
2013	56.4

A model that approximates these data is

$$S = \frac{40.32 + 3.53t}{1 + 0.039t}, \quad 2 \le t \le 13$$

where t represents the year, with $t = 2$ corresponding to 2002. *(Source: National Center for Education Statistics)*

(a) Use a graphing utility to create a scatter plot of the data. Then graph the model in the same viewing window.

(b) How well does the model fit the data? Explain.

(c) Use the model to predict when the salary for classroom teachers will exceed \$65,000.

(d) Is the model valid for long-term predictions of classroom teacher salaries? Explain.

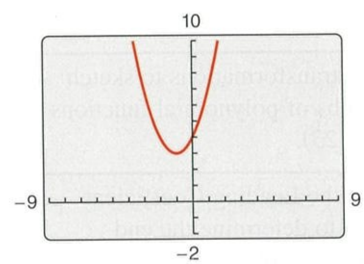
Exploration

True or False? **In Exercises 83 and 84, determine whether the statement is true or false. Justify your answer.**

83. The zeros of the polynomial $x^3 - 2x^2 - 11x + 12$ divide the real number line into three test intervals.

84. The solution set of the inequality $\frac{3}{2}x^2 + 3x + 6 \ge 0$ is the entire set of real numbers.

85. Graphical Reasoning Use a graphing utility to verify the results in Example 4. For instance, the graph of $y = x^2 + 2x + 4$ is shown below. Notice that the y-values are greater than 0 for all values of x, as stated in Example 4(a). Use the graphing utility to graph $y = x^2 + 2x + 1$, $y = x^2 + 3x + 5$, and $y = x^2 - 4x + 4$. Explain how you can use the graphs to verify the results of parts (b), (c), and (d) of Example 4.



86. HOW DO YOU SEE IT? Consider the polynomial

$$(x - a)(x - b)$$

and the real number line shown below.

(a) Identify the points on the line at which the polynomial is zero.

(b) For each of the three subintervals of the real number line, write the sign of each factor and the sign of the product.

(c) At what x-values does the polynomial change signs?

Conjecture **In Exercises 87–90, (a) find the interval(s) for b such that the equation has at least one real solution and (b) write a conjecture about the interval(s) based on the values of the coefficients.**

87. $x^2 + bx + 9 = 0$

88. $x^2 + bx - 9 = 0$

89. $3x^2 + bx + 10 = 0$

90. $2x^2 + bx + 5 = 0$

Chapter Summary

	What Did You Learn?	Explanation/Examples	Review Exercises
Section 2.1	Analyze graphs of quadratic functions *(p. 114)*.	Let a, b, and c be real numbers with $a \neq 0$. The function $f(x) = ax^2 + bx + c$ is a quadratic function. Its graph is a "U"-shaped curve called a parabola.	1, 2
	Write quadratic functions in standard form and use the results to sketch their graphs *(p. 117)*.	The quadratic function $f(x) = a(x - h)^2 + k$, $a \neq 0$, is in standard form. The graph of f is a parabola whose axis is the vertical line $x = h$ and whose vertex is (h, k). When $a > 0$, the parabola opens upward, and when $a < 0$, the parabola opens downward.	3–8
	Find minimum and maximum values of quadratic functions in real-life applications *(p. 119)*.	Consider $f(x) = ax^2 + bx + c$ with vertex $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$. When $a > 0$, f has a *minimum* at $x = -b/(2a)$. When $a < 0$, f has a *maximum* at $x = -b/(2a)$.	9, 10
Section 2.2	Use transformations to sketch graphs of polynomial functions *(p. 123)*.	The graph of a polynomial function is continuous (no breaks, holes, or gaps) and has only smooth, rounded turns.	11, 12
	Use the Leading Coefficient Test to determine the end behaviors of graphs of polynomial functions *(p. 125)*.	Consider the graph of $f(x) = a_nx^n + \cdot \cdot \cdot + a_1x + a_0$, $a_n \neq 0$. **When n is odd:** If $a_n > 0$, then the graph falls to the left and rises to the right. If $a_n < 0$, then the graph rises to the left and falls to the right. **When n is even:** If $a_n > 0$, then the graph rises to the left and to the right. If $a_n < 0$, then the graph falls to the left and to the right.	13–16
	Find real zeros of polynomial functions and use them as sketching aids *(p. 127)*.	When f is a polynomial function and a is a real number, the following are equivalent: (1) $x = a$ is a *zero* of f, (2) $x = a$ is a *solution* of the equation $f(x) = 0$, (3) $(x - a)$ is a *factor* of the polynomial $f(x)$, and (4) $(a, 0)$ is an *x-intercept* of the graph of f.	17–20
	Use the Intermediate Value Theorem to help locate real zeros of polynomial functions *(p. 130)*.	Let a and b be real numbers such that $a < b$. If f is a polynomial function such that $f(a) \neq f(b)$, then, in the interval $[a, b]$, f takes on every value between $f(a)$ and $f(b)$.	21, 22
Section 2.3	Use long division to divide polynomials by other polynomials *(p. 136)*.	Dividend: $x^2 + 3x + 5$; Divisor: $x + 1$; Quotient: $x + 2$; Remainder: 3 $\frac{x^2 + 3x + 5}{x + 1} = x + 2 + \frac{3}{x + 1}$	23, 24
	Use synthetic division to divide polynomials by binomials of the form $(x - k)$ *(p. 139)*.	Divisor: $x + 3$ Dividend: $x^4 - 10x^2 - 2x + 4$ -3 | 1 0 −10 −2 4 −3 9 3 −3 1 −3 −1 1 (1) ← Remainder: 1 Quotient: $x^3 - 3x^2 - x + 1$	25, 26
	Use the Remainder Theorem and the Factor Theorem *(p. 140)*.	**The Remainder Theorem:** If a polynomial $f(x)$ is divided by $x - k$, then the remainder is $r = f(k)$. **The Factor Theorem:** A polynomial $f(x)$ has a factor $(x - k)$ if and only if $f(k) = 0$.	27, 28

	What Did You Learn?	Explanation/Examples	Review Exercises
Section 2.4	Use the imaginary unit i to write complex number *(p. 145)*.	When a and b are real numbers, $a + bi$ is a complex number. Two complex numbers $a + bi$ and $c + di$, written in standard form, are equal to each other if and only if $a = c$ and $b = d$.	29, 30
	Add, subtract, and multiply complex number *(p. 146)*.	**Sum:** $(a + bi) + (c + di) = (a + c) + (b + d)i$ **Difference:** $(a + bi) - (c + di) = (a - c) + (b - d)i$	31–34
	Use complex conjugates to write the quotient of two complex numbers in standard form *(p. 148)*.	To write $(a + bi)/(c + di)$ in standard form, where c and d are not both zero, multiply the numerator and denominator by the complex conjugate of the denominator, $c - di$.	35–38
	Find complex solutions of quadratic equations *(p. 149)*.	When a is a positive real number, the principal square root of $-a$ is defined as $\sqrt{-a} = \sqrt{a}i$.	39, 40
Section 2.5	Use the Fundamental Theorem of Algebra to determine the numbers of zeros of polynomial functions *(p. 152)*.	**The Fundamental Theorem of Algebra** If $f(x)$ is a polynomial of degree n, where $n > 0$, then f has at least one zero in the complex number system.	41, 42
	Find rational zeros of polynomial functions *(p. 153)*, and find complex zeros using conjugate pairs *(p. 156)*.	The Rational Zero Test relates the possible rational zeros of a polynomial to the leading coefficient and constant term. **Complex Zeros:** Let f be a polynomial function that has real coefficients. If $a + bi$, where $b \neq 0$, is a zero of the function, then the complex conjugate $a - bi$ is also a zero of the function.	43, 44
	Find zeros of polynomial by factoring *(p. 157)*, use Descartes's Rule of Signs and the Upper and Lower Bound Rules *(p. 159)*, and find zeros of polynomials in real-life applications *(p. 161)*.	Every polynomial of degree $n > 0$ with real coefficients can be written as the product of linear and quadratic factors with real coefficients, where the quadratic factors have no real zeros.	45–48
Section 2.6	Find domains *(p. 166)*, and vertical and horizontal asymptotes *(p. 167)*, of graphs of rational functions.	The domain of a rational function of x includes all real numbers except x-values that make the denominator zero. The line $x = a$ is a vertical asymptote of the graph of f when $f(x) \to \infty$ or $f(x) \to -\infty$ as $x \to a$, either from the right or from the left. The line $y = b$ is a horizontal asymptote of the graph of f when $f(x) \to b$ as $x \to \infty$ or $x \to -\infty$.	49, 50
	Sketch the graphs of rational functions *(p. 169)*, including functions with slant asymptotes *(p. 172)*.	Consider a rational function whose denominator is of degree 1 or greater. If the degree of the numerator is exactly *one more* than the degree of the denominator, then the graph of the function has a slant asymptote.	51–58
	Use rational functions to model and solve real-life problems *(p. 173)*.	A rational function can help you model the cost of removing a given percent of the smokestack pollutants at a utility company that burns coals. (See Example 8.)	59, 60
Section 2.7	Solve polynomial *(p. 178)*, and rational *(p. 182)* inequalities.	Use the concepts of key numbers and text intervals to solve both polynomial and rational inequalities.	61–64
	Use nonlinear inequalities to model and solve real-life problems *(p. 183)*.	A common application of nonlinear inequalities involves profit P, revenue R, and cost C. (See Example 6.)	65

Review Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

2.1 Sketching Graphs of Quadratic Functions In Exercises 1 and 2, sketch the graph of each quadratic function and compare it with the graph of $y = x^2$.

1. (a) $g(x) = -2x^2$
   (b) $h(x) = x^2 + 2$

2. (a) $h(x) = (x - 3)^2$
   (b) $k(x) = \frac{1}{2}x - 1$

Using Standard Form to Graph a Parabola In Exercises 3–8, write the quadratic function in standard form and sketch its graph. Identify the vertex, axis of symmetry, and x-intercept(s).

3. $g(x) = x^2 - 2x$
4. $f(x) = x^2 + 8x + 10$
5. $h(x) = 3 + 4x - x^2$
6. $f(t) = -2t^2 + 4t + 1$
7. $h(x) = 4x^2 + 4x + 13$
8. $f(x) = \frac{1}{3}(x^2 + 5x - 4)$

9. **Geometry** The perimeter of a rectangle is 1000 meters.
   (a) Write the width y as a function of the length x. Use the result to write the area A as a function of x.
   (b) Of all possible rectangles with perimeters of 1000 meters, find the dimensions of the one with the maximum area.

10. **Minimum Cost** A soft-drink manufacturer has a daily production cost of $C = 70{,}000 - 120x + 0.055x^2$, where C is the total cost (in dollars) and x is the number of units produced. How many units should they produce each day to yield a minimum cost?

2.2 Sketching a Transformation of a Monomial Function In Exercises 11 and 12, sketch the graphs of $y = x^n$ and the transformation.

11. $y = x^4$, $f(x) = 6 - x^4$
12. $y = x^5$, $f(x) = \frac{1}{2}x^5 + 3$

Applying the Leading Coefficient Test In Exercises 13–16, describe the left-hand and right-hand behavior of the graph of the polynomial function.

13. $f(x) = -2x^2 - 5x + 12$
14. $f(x) = 4x - \frac{1}{2}x^3$
15. $g(x) = -3x^3 - 8x^4 + x^5$
16. $h(x) = 5 + 9x^6 - 6x^5$

Sketching the Graph of a Polynomial Function In Exercises 17–20, sketch the graph of the function by (a) applying the Leading Coefficient Test, (b) finding the real zeros of the polynomial, (c) plotting sufficient solution points, and (d) drawing a continuous curve through the points.

17. $g(x) = 2x^3 + 4x^2$
18. $h(x) = 3x^2 - x^4$
19. $f(x) = -x^3 + x^2 - 2$
20. $f(x) = x(x^3 + x^2 - 5x + 3)$

Using the Intermediate Value Theorem In Exercises 21 and 22, (a) use the Intermediate Value Theorem and the *table* feature of a graphing utility to find intervals one unit in length in which the polynomial function is guaranteed to have a zero. (b) Adjust the table to approximate the zeros of the function to the nearest thousandth. Use the *zero* or *root* feature of the graphing utility to verify your results.

21. $f(x) = 3x^3 - x^2 + 3$
22. $f(x) = x^4 - 5x - 1$

2.3 Long Division of Polynomials In Exercises 23 and 24, use long division to divide.

23. $\dfrac{30x^2 - 3x + 8}{5x - 3}$
24. $\dfrac{5x^3 - 21x^2 - 25x - 4}{x^2 - 5x - 1}$

Using Synthetic Division In Exercises 25 and 26, use synthetic division to divide.

25. $\dfrac{2x^3 - 25x^2 + 66x + 48}{x - 8}$
26. $\dfrac{x^4 - 2x^2 + 9x}{x + 3}$

Factoring a Polynomial In Exercises 27 and 28, (a) verify the given factor(s) of $f(x)$, (b) find the remaining factors of $f(x)$, (c) use your results to write the complete factorization of $f(x)$, (d) list all real zeros of f, and (e) confirm your results by using a graphing utility to graph the function.

Function	Factor(s)
27. $f(x) = 2x^3 + 11x^2 - 21x - 90$	$(x + 6)$
28. $f(x) = x^4 - 4x^3 - 7x^2 + 22x + 24$	$(x + 2), (x - 3)$

2.4 Writing a Complex Number in Standard Form In Exercises 29 and 30, write the complex number in standard form.

29. $4 + \sqrt{-9}$
30. $-5i + i^2$

Performing Operations with Complex Numbers In Exercises 31–34, perform the operation and write the result in standard form.

31. $(6 - 4i) + (-9 + i)$
32. $(7 - 2i) - (3 - 8i)$
33. $-3i(-2 + 5i)$
34. $(4 + i)(3 - 10i)$

Quotient of Complex Numbers in Standard Form In Exercises 35 and 36, write the quotient in standard form.

35. $\dfrac{4}{1 - 2i}$
36. $\dfrac{3 + 2i}{5 + i}$

Performing Operations with Complex Numbers **In Exercises 37 and 38, perform the operation and write the result in standard form.**

37. $\dfrac{4}{2-3i}+\dfrac{2}{1+i}$

38. $\dfrac{1}{2+i}-\dfrac{5}{1+4i}$

Complex Solutions of a Quadratic Equation **In Exercises 39 and 40, use the Quadratic Formula to solve the quadratic equation.**

39. $x^2-2x+10=0$

40. $6x^2+3x+27=0$

2.5 Zeros of Polynomial Functions **In Exercises 41 and 42, determine the number of zeros of the polynomial function.**

41. $g(x)=x^2-2x-8$

42. $h(t)=t^2-t^5$

Using the Rational Zero Test **In Exercises 43 and 44, find the rational zeros of the function.**

43. $f(x)=4x^3-27x^2+11x+42$

44. $f(x)=x^4+x^3-11x^2+x-12$

Finding the Zeros of a Polynomial Function **In Exercises 45 and 46, write the polynomial as the product of linear factors and list all the zeros of the function.**

45. $g(x)=x^3-7x^2+36$

46. $f(x)=x^4+8x^3+8x^2-72x-153$

47. **Using Descartes's Rule of Signs** Use Descartes's Rule of Signs to determine the possible numbers of positive and negative real zeros of $h(x)=-2x^5+4x^3-2x^2+5$.

48. **Verifying Upper and Lower Bounds** Use synthetic division to verify the upper and lower bounds of the real zeros of $f(x)=4x^3-3x^2+4x-3$.
    (a) Upper: $x=1$ (b) Lower: $x=-\frac{1}{4}$

2.6 Finding Domain and Asymptotes **In Exercises 49 and 50, find the domain and the vertical and horizontal asymptotes of the graph of the rational function.**

49. $f(x)=\dfrac{3x}{x+10}$

50. $f(x)=\dfrac{8}{x^2-10x+24}$

Sketching the Graph of a Rational Function **In Exercises 51–58, (a) state the domain of the function, (b) identify all intercepts, (c) find any asymptotes, and (d) plot additional solution points as needed to sketch the graph of the rational function.**

51. $f(x)=\dfrac{4}{x}$

52. $h(x)=\dfrac{x-4}{x-7}$

53. $f(x)=\dfrac{x}{x^2-16}$

54. $f(x)=\dfrac{-8x}{x^2+4}$

55. $f(x)=\dfrac{6x^2-11x+3}{3x^2-x}$

56. $f(x)=\dfrac{6x^2-7x+2}{4x^2-1}$

57. $f(x)=\dfrac{2x^3}{x^2+1}$

58. $f(x)=\dfrac{2x^2+2}{x+1}$

59. **Seizure of Illegal Drugs** The cost C (in millions of dollars) for the federal government to seize $p\%$ of an illegal drug as it enters the country is given by

$$C=\frac{528p}{100-p},\quad 0\le p\le 100.$$

(a) Use a graphing utility to graph the cost function.
(b) Find the costs of seizing 25%, 50%, and 75% of the drug.
(c) According to the model, it is possible to seize 100% of the drug? Explain.

60. **Page Design** A page that is x inches wide and y inches high contains 30 square inches of print. The top and bottom margins are each 2 inches deep, and the margins on each side are 2 inches wide.
(a) Write a function for the total area A of the page in terms of x.
(b) Determine the domain of the function based of the physical constraints of the problem.
(c) Use a graphing utility to graph the area function and approximate the dimensions of the page that use the least amount of paper.

2.7 Solving an Inequality **In Exercises 61–64, solve the inequality. Then graph the solution set.**

61. $12x^2+5x<2$

62. $x^3-16x\ge 0$

63. $\dfrac{2}{x+1}\ge\dfrac{3}{x-1}$

64. $\dfrac{x^2-9x+20}{x}<0$

65. **Biology** A biologist introduces 200 ladybugs into a crop field. The population P of the ladybugs can be approximated by the model

$$P=\frac{1000(1+3t)}{5+t}$$

where t is the time in days. Find the time required for the population to increase to at least 2000 ladybugs.

Exploration

True or False? **In Exercises 66 and 67, determine whether the statement is true or false. Justify your answer.**

66. A fourth-degree polynomial with real coefficients can have -5, $-8i$, $4i$, and 5 as its zeros.

67. The domain of a rational function can never be the set of all real numbers.

68. **Writing** Describe what is meant by an asymptote of a graph.

Chapter Test

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

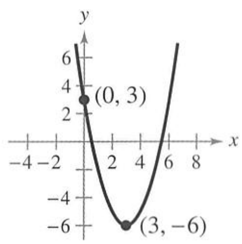


Figure for 2

Take this test as you would take a test in class. When you are finished, check your work against the answers given in the back of the book.

1. Sketch the graph of each quadratic function and compare it with the graph of $y = x^2$.
   (a) $g(x) = -x^2 + 4$ (b) $g(x) = \left(x - \frac{3}{2}\right)^2$
2. Write the standard form of the equation of the parabola shown at the left.
3. The path of a ball is modeled by the function $f(x) = -\frac{1}{20}x^2 + 3x + 5$, where $f(x)$ is the height (in feet) of the ball and x is the horizontal distance (in feet) from where the ball was thrown.
   (a) What is the maximum height of the ball?
   (b) Which number determines the height at which the ball was thrown? Does changing this value change the coordinates of the maximum height of the ball? Explain.
4. Describe the left-hand and right-hand behavior of the graph of the function $h(t) = -\frac{3}{4}t^5 + 2t^2$. Then sketch its graph.
5. Divide using long division.
   $$\frac{3x^3 + 4x - 1}{x^2 + 1}$$
6. Divide using synthetic division.
   $$\frac{2x^4 - 3x^2 + 4x - 1}{x + 2}$$
7. Use synthetic division to show that $x = \frac{5}{2}$ is a zero of the function
   $f(x) = 2x^3 - 5x^2 - 6x + 15.$
   Use the result to factor the polynomial function completely and list all the zeros of the function.
8. Perform each operation and write the result in standard form.
   (a) $\sqrt{-16} - 2(7 + 2i)$
   (b) $(5 - i)(3 + 4i)$
9. Write the quotient in standard form: $\dfrac{8}{1 + 2i}$.

In Exercises 10 and 11, find a polynomial function with real coefficients that has the given zeros. (There are many correct answers.)

10. $0, 2, 3i$
11. $1, 1, 2 + \sqrt{3}i$

In Exercises 12 and 13, find all the zeros of the function.

12. $f(x) = 3x^3 + 14x^2 - 7x - 10$
13. $f(x) = x^4 - 9x^2 - 22x - 24$

In Exercises 14–16, identify any intercepts and asymptotes of the graph of the function. Then sketch the graph of the function.

14. $h(x) = \dfrac{3}{x^2} - 1$
15. $f(x) = \dfrac{2x^2 - 5x - 12}{x^2 - 16}$
16. $g(x) = \dfrac{x^2 + 2}{x - 1}$

In Exercises 17 and 18, solve the inequality. The graph the solution set.

17. $2x^2 + 5x > 12$
18. $\dfrac{2}{x} \le \dfrac{1}{x + 6}$

Proofs in Mathematics

These two pages contain proofs of four important theorems about polynomial functions. The first two theorems are from Section 2.3, and the second two theorems are from Section 2.5.

> **The Remainder Theorem** *(p. 140)*
>
> If a polynomial $f(x)$ is divided by $x - k$, then the remainder is
>
> $r = f(k)$.

Proof

Using the Division Algorithm with the divisor $(x - k)$, you have

$$f(x) = (x - k)q(x) + r(x).$$

Either $r(x) = 0$ or the degree of $r(x)$ is less than the degree of $x - k$, so you know that $r(x)$ must be a constant. That is, $r(x) = r$. Now, by evaluating $f(x)$ at $x = k$, you have

$$\begin{aligned} f(k) &= (k - k)q(k) + r \\ &= (0)q(k) + r \\ &= r. \end{aligned}$$

To be successful in algebra, it is important that you understand the connection among *factors* of a polynomial, *zeros* of a polynomial function, and *solutions* or *roots* of a polynomial equation. The Factor Theorem is the basis for this connection.

> **The Factor Theorem** *(p. 141)*
>
> A polynomial $f(x)$ has a factor $(x - k)$ if and only if $f(k) = 0$.

Proof

Using the Division Algorithm with the factor $(x - k)$, you have

$$f(x) = (x - k)q(x) + r(x).$$

By the Remainder Theorem, $r(x) = r = f(k)$, and you have

$$f(x) = (x - k)q(x) + f(k)$$

where $q(x)$ is a polynomial of lesser degree than $f(x)$. If $f(k) = 0$, then

$$f(x) = (x - k)q(x)$$

and you see that $(x - k)$ is a factor of $f(x)$. Conversely, if $(x - k)$ is a factor of $f(x)$, then division of $f(x)$ by $(x - k)$ yields a remainder of 0. So, by the Remainder Theorem, you have $f(k) = 0$.

THE FUNDAMENTAL THEOREM OF ALGEBRA

The Fundamental Theorem of Algebra, which is closely related to the Linear Factorization Theorem, has a long and interesting history. In the early work with polynomial equations, the Fundamental Theorem of Algebra was thought to have been false, because imaginary solutions were not considered. In fact, in the very early work by mathematicians such as Abu al-Khwarizmi (c. 800 A.D.), negative solutions were also not considered.

Once imaginary numbers were considered, several mathematicians attempted to give a general proof of the Fundamental Theorem of Algebra. These included Jean Le Rond d'Alembert (1746), Leonhard Euler (1749), Joseph-Louis Lagrange (1772), and Pierre Simon Laplace (1795). The mathematician usually credited with the first complete and correct proof of the Fundamental Theorem of Algebra is Carl Friedrich Gauss, who published the proof in 1816.

Linear Factorization Theorem *(p. 152)*

If $f(x)$ is a polynomial of degree n, where $n > 0$, then $f(x)$ has precisely n linear factors

$$f(x) = a_n(x - c_1)(x - c_2) \cdot \cdot \cdot (x - c_n)$$

where $c_1, c_2, \ldots, c_n$ are complex numbers.

Proof

Using the Fundamental Theorem of Algebra, you know that f must have at least one zero, c_1. Consequently, $(x - c_1)$ is a factor of $f(x)$, and you have

$$f(x) = (x - c_1)f_1(x).$$

If the degree of $f_1(x)$ is greater than zero, then you again apply the Fundamental Theorem of Algebra to conclude that f_1 must have a zero c_2, which implies that

$$f(x) = (x - c_1)(x - c_2)f_2(x).$$

It is clear that the degree of $f_1(x)$ is $n - 1$, that the degree of $f_2(x)$ is $n - 2$, and that you can repeatedly apply the Fundamental Theorem of Algebra n times until you obtain

$$f(x) = a_n(x - c_1)(x - c_2) \cdot \cdot \cdot (x - c_n)$$

where a_n is the leading coefficient of the polynomial $f(x)$. ■

Factors of a Polynomial *(p. 157)*

Every polynomial of degree $n > 0$ with real coefficients can be written as the product of linear and quadratic factors with real coefficients, where the quadratic factors have no real zeros.

Proof

To begin, use the Linear Factorization Theorem to conclude that $f(x)$ can be *completely* factored in the form

$$f(x) = d(x - c_1)(x - c_2)(x - c_3) \cdot \cdot \cdot (x - c_n).$$

If each c_i is real, then there is nothing more to prove. If any c_i is imaginary ($c_i = a + bi, b \neq 0$), then you know that the conjugate $c_j = a - bi$ is also a zero, because the coefficients of $f(x)$ are real. By multiplying the corresponding factors, you obtain

$$\begin{aligned}(x - c_i)(x - c_j) &= [x - (a + bi)][x - (a - bi)] \\ &= [(x - a) - bi][(x - a) + bi] \\ &= (x - a)^2 + b^2 \\ &= x^2 - 2ax + (a^2 + b^2)\end{aligned}$$

where each coefficient is real. ■

P.S. Problem Solving

1. Verifying the Remainder Theorem Show that if $f(x) = ax^3 + bx^2 + cx + d$, then $f(k) = r$, where $r = ak^3 + bk^2 + ck + d$, using long division. In other words, verify the Remainder Theorem for a third-degree polynomial function.

2. Babylonian Mathematics In 2000 B.C., the Babylonians solved polynomial equations by referring to tables of values. One such table gave the values of $y^3 + y^2$. To be able to use this table, the Babylonians sometimes used the method below to manipulate the equation.

$$ax^3 + bx^2 = c \qquad \text{Original equation}$$

$$\frac{a^3x^3}{b^3} + \frac{a^2x^2}{b^2} = \frac{a^2c}{b^3} \qquad \text{Multiply each side by } \frac{a^2}{b^3}.$$

$$\left(\frac{ax}{b}\right)^3 + \left(\frac{ax}{b}\right)^2 = \frac{a^2c}{b^3} \qquad \text{Rewrite.}$$

Then they would find $(a^2c)/b^3$ in the $y^3 + y^2$ column of the table. They knew that the corresponding y-value was equal to $(ax)/b$, so they could conclude that $x = (by)/a$.

(a) Calculate $y^3 + y^2$ for $y = 1, 2, 3, \ldots, 10$. Record the values in a table.

(b) Use the table from part (a) and the method above to solve each equation.

(i) $x^3 + x^2 = 252$

(ii) $x^3 + 2x^2 = 288$

(iii) $3x^3 + x^2 = 90$

(iv) $2x^3 + 5x^2 = 2500$

(v) $7x^3 + 6x^2 = 1728$

(vi) $10x^3 + 3x^2 = 297$

(c) Using the methods from this chapter, verify your solution of each equation.

3. Finding Dimensions At a glassware factory, molten cobalt glass is poured into molds to make paperweights. Each mold is a rectangular prism whose height is 3 inches greater than the length of each side of the square base. A machine pours 20 cubic inches of liquid glass into each mold. What are the dimensions of the mold?

4. True or False? Determine whether the statement is true or false. If false, provide one or more reasons why the statement is false and correct the statement. Let $f(x) = ax^3 + bx^2 + cx + d$, $a \neq 0$, and let $f(2) = -1$. Then

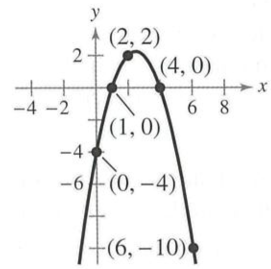
$$\frac{f(x)}{x+1} = q(x) + \frac{2}{x+1}$$

where $q(x)$ is a second-degree polynomial.

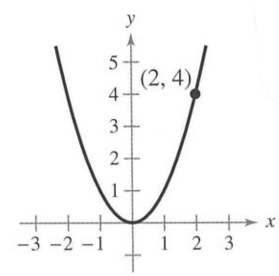
5. Finding the Equation of a Parabola The parabola shown in the figure has an equation of the form $y = ax^2 + bx + c$. Find the equation of this parabola using each method.

(a) Find the equation analytically.

(b) Use the regression feature of a graphing utility to find the equation.



6. Finding the Slope of a Tangent Line One of the fundamental themes of calculus is to find the slope of the tangent line to a curve at a point. To see how this can be done, consider the point (2, 4) on the graph of the quadratic function $f(x) = x^2$, as shown in the figure.



(a) Find the slope m_1 of the line joining (2, 4) and (3, 9). Is the slope of the tangent line at (2, 4) greater than or less than the slope of the line through (2, 4) and (3, 9)?

(b) Find the slope m_2 of the line joining (2, 4) and (1, 1). Is the slope of the tangent line at (2, 4) greater than or less than the slope of the line through (2, 4) and (1, 1)?

(c) Find the slope m_3 of the line joining (2, 4) and (2.1, 4.41). Is the slope of the tangent line at (2, 4) greater than or less than the slope of the line through (2, 4) and (2.1, 4.41)?

(d) Find the slope m_h of the line joining (2, 4) and $(2 + h, f(2 + h))$ in terms of the nonzero number h.

(e) Evaluate the slope formula from part (d) for $h = -1$, 1, and 0.1. Compare these values with those in parts (a)–(c).

(f) What can you conclude the slope $m_{\tan}$ of the tangent line at (2, 4) to be? Explain.

7. Writing Cubic Functions For each part, write a cubic function of the form $f(x) = (x - k)q(x) + r$ whose graph has the specified characteristics. (There are many correct answers.)

(a) Passes through the point $(2, 5)$ and rises to the right

(b) Passes through the point $(-3, 1)$ and falls to the right

8. Multiplicative Inverse of a Complex Number The multiplicative inverse of a complex number z is a complex number z_m such that $z \cdot z_m = 1$. Find the multiplicative inverse of each complex number.

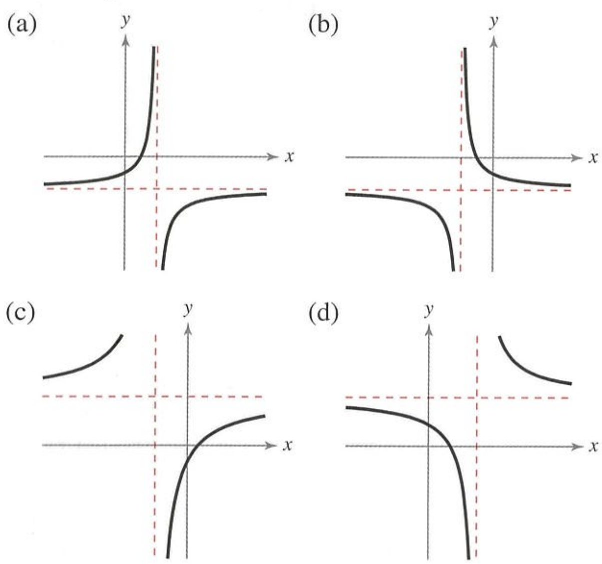
(a) $z = 1 + i$ (b) $z = 3 - i$ (c) $z = -2 + 8i$

9. Proof Prove that the product of a complex number $a + bi$ and its complex conjugate is a real number.

10. Matching Match the graph of the rational function

$$f(x) = \frac{ax + b}{cx + d}$$

with the given conditions.



(i)	(ii)	(iii)	(iv)
$a > 0$	$a > 0$	$a < 0$	$a > 0$
$b < 0$	$b > 0$	$b > 0$	$b < 0$
$c > 0$	$c < 0$	$c > 0$	$c > 0$
$d < 0$	$d < 0$	$d < 0$	$d > 0$

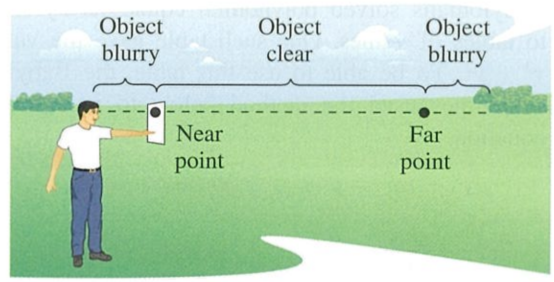
11. Effects of Values on a Graph Consider the function

$$f(x) = \frac{ax}{(x - b)^2}.$$

(a) Determine the effect on the graph of f when $b \neq 0$ and a is varied. Consider cases in which a is positive and a is negative.

(b) Determine the effect on the graph of f when $a \neq 0$ and b is varied.

12. Distinct Vision The endpoints of the interval over which distinct vision is possible are called the *near point* and *far point* of the eye (see figure). With increasing age, these points normally change. The table shows the approximate near points y (in inches) for various ages x (in years).



Age, x	Near Point, y
16	3.0
32	4.7
44	9.8
50	19.7
60	39.4

(a) Use the *regression* feature of a graphing utility to find a quadratic model for the data. Use the graphing utility to plot the data and graph the model in the same viewing window.

(b) Find a rational model for the data. Take the reciprocals of the near points to generate the points $(x, 1/y)$. Use the *regression* feature of the graphing utility to find a linear model for the data. The resulting line has the form

$$\frac{1}{y} = ax + b.$$

Solve for y. Use the graphing utility to plot the data and graph the model in the same viewing window.

(c) Use the *table* feature of the graphing utility to construct a table showing the predicted near point based on each model for each of the ages in the original table. How well do the models fit the original data?

(d) Use both models to estimate the near point for a person who is 25 years old. Which model is a better fit?

(e) Do you think either model can be used to predict the near point for a person who is 70 years old? Explain.

13. Zeros of a Cubic Function Can a cubic function with real coefficients have two real zeros and one complex zero? Explain.