1.5 Analyzing Graphs of Functions

Graphs of functions can help you visualize relationships between variables in real life. For example, in Exercise 90 on page 59, you will use the graph of a function to visually represent the temperature in a city over a 24-hour period.

- **Use the Vertical Line Test for functions.**
- **Find the zeros of functions.**
- **Determine intervals on which functions are increasing or decreasing.**
- **Determine relative minimum and relative maximum values of functions.**
- **Determine the average rate of change of a function.**
- **Identify even and odd functions.**

The Graph of a Function

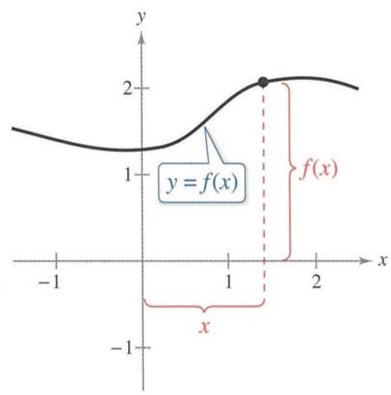
In Section 1.4, you studied functions from an algebraic point of view. In this section, you will study functions from a graphical perspective.

The **graph of a function** f is the collection of ordered pairs $(x, f(x))$ such that x is in the domain of f. As you study this section, remember that

$x =$ the directed distance from the y-axis

$y = f(x) =$ the directed distance from the x-axis

as shown in the figure at the right.



EXAMPLE 1 Finding the Domain and Range of a Function

Use the graph of the function f, shown in Figure 1.32, to find (a) the domain of f, (b) the function values $f(-1)$ and $f(2)$, and (c) the range of f.

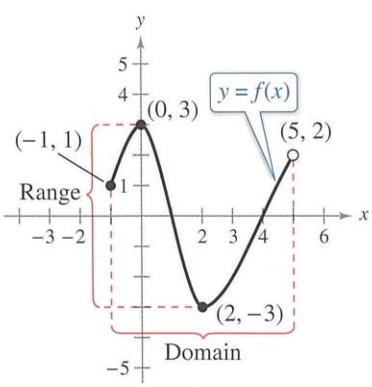


Figure 1.32

Solution

a. The closed dot at $(-1, 1)$ indicates that $x = -1$ is in the domain of f, whereas the open dot at $(5, 2)$ indicates that $x = 5$ is not in the domain. So, the domain of f is all x in the interval $[-1, 5)$.

b. One point on the graph of f is $(-1, 1)$, so $f(-1) = 1$. Another point on the graph of f is $(2, -3)$, so $f(2) = -3$.

c. The graph does not extend below $f(2) = -3$ or above $f(0) = 3$, so the range of f is the interval $[-3, 3]$.

REMARK The use of dots (open or closed) at the extreme left and right points of a graph indicates that the graph does not extend beyond these points. If such dots are not on the graph, then assume that the graph extends beyond these points.

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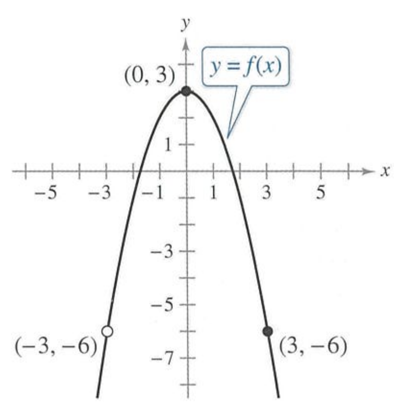
Use the graph of the function f to find (a) the domain of f, (b) the function values $f(0)$ and $f(3)$, and (c) the range of f.



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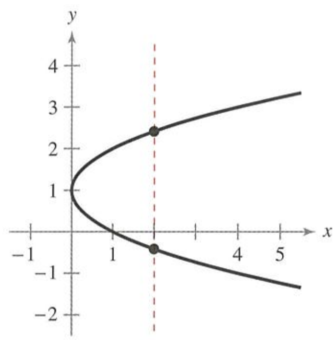
By the definition of a function, at most one y-value corresponds to a given x-value. So, no two points on the graph of a function have the same x-coordinate, or lie on the same vertical line. It follows, then, that a vertical line can intersect the graph of a function at most once. This observation provides a convenient visual test called the **Vertical Line Test** for functions.

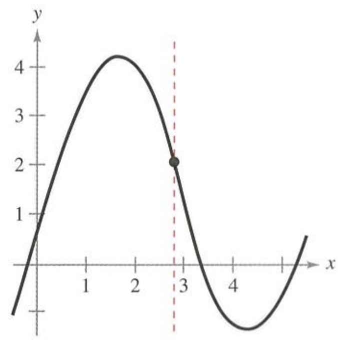
> **Vertical Line Test for Functions**
>
> A set of points in a coordinate plane is the graph of y as a function of x if and only if no *vertical* line intersects the graph at more than one point.

EXAMPLE 2 Vertical Line Test for Functions

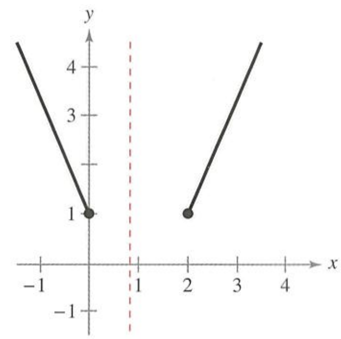
Use the Vertical Line Test to determine whether each graph represents y as a function of x.



(a)



(b)



(c)

Solution

a. This *is not* a graph of y as a function of x, because there are vertical lines that intersect the graph twice. That is, for a particular input x, there is more than one output y.

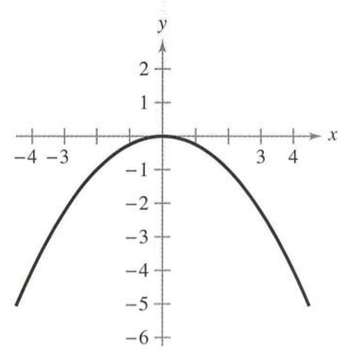
b. This *is* a graph of y as a function of x, because every vertical line intersects the graph at most once. That is, for a particular input x, there is at most one output y.

c. This *is* a graph of y as a function of x, because every vertical line intersects the graph at most once. That is, for a particular input x, there is at most one output y. (Note that when a vertical line does not intersect the graph, it simply means that the function is undefined for that particular value of x.)

▷ TECHNOLOGY Most graphing utilities graph functions of x more easily than other types of equations. For example, the graph shown in (a) above represents the equation $x - (y - 1)^2 = 0$. To duplicate this graph using a graphing utility, you must first solve the equation for y to obtain $y = 1 \pm \sqrt{x}$, and then graph the two equations $y_1 = 1 + \sqrt{x}$ and $y_2 = 1 - \sqrt{x}$ in the same viewing window.

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Use the Vertical Line Test to determine whether the graph represents y as a function of x.



ALGEBRA HELP The solution to Example 3 involves solving equations. To review the techniques for solving equations, see Appendix A.5.

Zeros of a Function

If the graph of a function of x has an x-intercept at $(a, 0)$, then a is a **zero** of the function.

> **Zeros of a Function**
>
> The **zeros of a function** $y = f(x)$ are the x-values for which $f(x) = 0$.

EXAMPLE 3 Finding the Zeros of Functions

Find the zeros of each function algebraically.

a. $f(x) = 3x^2 + x - 10$

b. $g(x) = \sqrt{10 - x^2}$

c. $h(t) = \dfrac{2t - 3}{t + 5}$

Solution To find the zeros of a function, set the function equal to zero and solve for the independent variable.

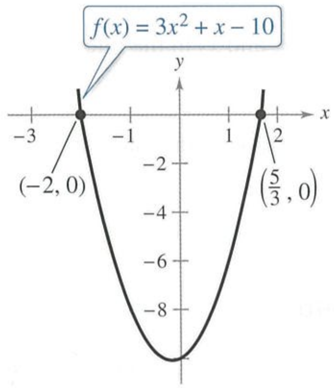
a. $3x^2 + x - 10 = 0$ — Set $f(x)$ equal to 0.

$(3x - 5)(x + 2) = 0$ — Factor.

$3x - 5 = 0 \Rightarrow x = \frac{5}{3}$ — Set 1st factor equal to 0 and solve.

$x + 2 = 0 \Rightarrow x = -2$ — Set 2nd factor equal to 0 and solve.

The zeros of f are $x = \frac{5}{3}$ and $x = -2$. In Figure 1.33, note that the graph of f has $\left(\frac{5}{3}, 0\right)$ and $(-2, 0)$ as its x-intercepts.



Zeros of f: $x = -2$, $x = \frac{5}{3}$
Figure 1.33

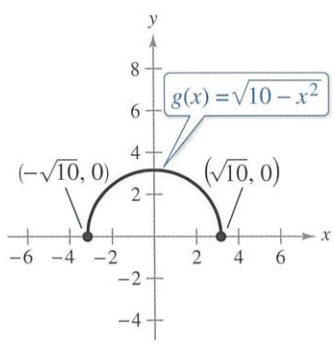
b. $\sqrt{10 - x^2} = 0$ — Set $g(x)$ equal to 0.

$10 - x^2 = 0$ — Square each side.

$10 = x^2$ — Add x^2 to each side.

$\pm\sqrt{10} = x$ — Extract square roots.

The zeros of g are $x = -\sqrt{10}$ and $x = \sqrt{10}$. In Figure 1.34, note that the graph of g has $\left(-\sqrt{10}, 0\right)$ and $\left(\sqrt{10}, 0\right)$ as its x-intercepts.



Zeros of g: $x = \pm\sqrt{10}$
Figure 1.34

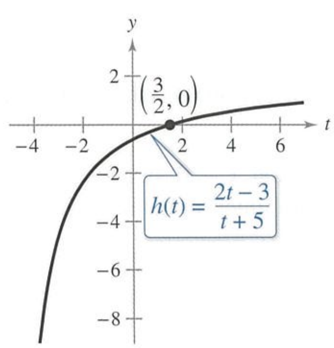
c. $\dfrac{2t - 3}{t + 5} = 0$ — Set $h(t)$ equal to 0.

$2t - 3 = 0$ — Multiply each side by $t + 5$.

$2t = 3$ — Add 3 to each side.

$t = \dfrac{3}{2}$ — Divide each side by 2.

The zero of h is $t = \frac{3}{2}$. In Figure 1.35, note that the graph of h has $\left(\frac{3}{2}, 0\right)$ as its t-intercept.



Zero of h: $t = \frac{3}{2}$
Figure 1.35

✓ Checkpoint Audio-video solution in English & Spanish at LarsonPrecalculus.com

Find the zeros of each function.

a. $f(x) = 2x^2 + 13x - 24$ **b.** $g(t) = \sqrt{t - 25}$ **c.** $h(x) = \dfrac{x^2 - 2}{x - 1}$

Increasing and Decreasing Functions

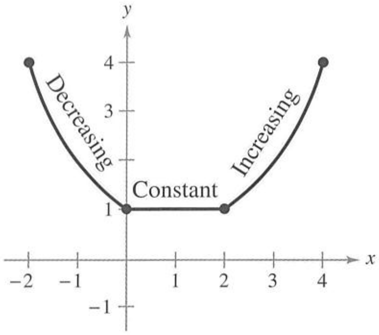


Figure 1.36

The more you know about the graph of a function, the more you know about the function itself. Consider the graph shown in Figure 1.36. As you move from *left to right,* this graph falls from $x = -2$ to $x = 0$, is constant from $x = 0$ to $x = 2$, and rises from $x = 2$ to $x = 4$.

Increasing, Decreasing, and Constant Functions

A function f is **increasing** on an interval when, for any x_1 and x_2 in the interval,

$$x_1 < x_2 \quad \text{implies} \quad f(x_1) < f(x_2).$$

A function f is **decreasing** on an interval when, for any x_1 and x_2 in the interval,

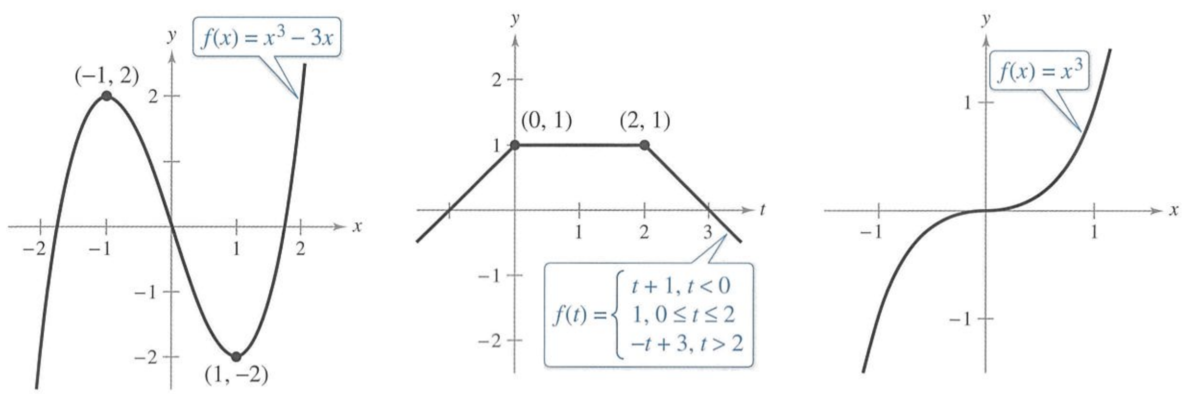
$$x_1 < x_2 \quad \text{implies} \quad f(x_1) > f(x_2).$$

A function f is **constant** on an interval when, for any x_1 and x_2 in the interval,

$$f(x_1) = f(x_2).$$

EXAMPLE 4 Describing Function Behavior

Determine the open intervals on which each function is increasing, decreasing, or constant.



(a) (b) (c)

Solution

a. This function is increasing on the interval $(-\infty, -1)$, decreasing on the interval $(-1, 1)$, and increasing on the interval $(1, \infty)$.

b. This function is increasing on the interval $(-\infty, 0)$, constant on the interval $(0, 2)$, and decreasing on the interval $(2, \infty)$.

c. This function may appear to be constant on an interval near $x = 0$, but for all real values of x_1 and x_2, if $x_1 < x_2$, then $(x_1)^3 < (x_2)^3$. So, the function is increasing on the interval $(-\infty, \infty)$.

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Graph the function

$$f(x) = x^3 + 3x^2 - 1.$$

Then determine the open intervals on which the function is increasing, decreasing, or constant.

Relative Minimum and Relative Maximum Values

The points at which a function changes its increasing, decreasing, or constant behavior are helpful in determining the **relative minimum** or **relative maximum** values of the function.

REMARK A relative minimum or relative maximum is also referred to as a local minimum or local maximum.

Definitions of Relative Minimum and Relative Maximum

A function value $f(a)$ is a **relative minimum** of f when there exists an interval (x_1, x_2) that contains a such that

$$x_1 < x < x_2 \quad \text{implies} \quad f(a) \le f(x).$$

A function value $f(a)$ is a **relative maximum** of f when there exists an interval (x_1, x_2) that contains a such that

$$x_1 < x < x_2 \quad \text{implies} \quad f(a) \ge f(x).$$

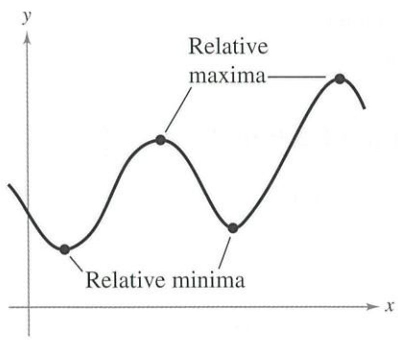


Figure 1.37

Figure 1.37 shows several different examples of relative minima and relative maxima. In Section 2.1, you will study a technique for finding the *exact point* at which a second-degree polynomial function has a relative minimum or relative maximum. For the time being, however, you can use a graphing utility to find reasonable approximations of these points.

EXAMPLE 5 Approximating a Relative Minimum

Use a graphing utility to approximate the relative minimum of the function

$$f(x) = 3x^2 - 4x - 2.$$

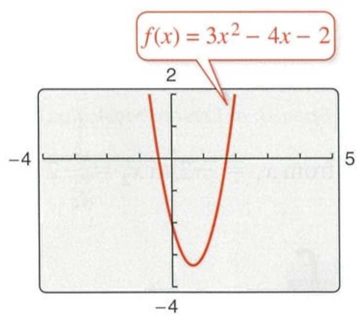


Figure 1.38

Solution The graph of f is shown in Figure 1.38. By using the *zoom* and *trace* features or the *minimum* feature of a graphing utility, you can approximate that the relative minimum of the function occurs at the point

$$(0.67, -3.33).$$

So, the relative minimum is approximately -3.33. Later, in Section 2.1, you will learn how to determine that the exact point at which the relative minimum occurs is $\left(\frac{2}{3}, -\frac{10}{3}\right)$ and the exact relative minimum is $-\frac{10}{3}$.

✓ **Checkpoint** Audio-video solution in English & Spanish at LarsonPrecalculus.com

Use a graphing utility to approximate the relative maximum of the function

$$f(x) = -4x^2 - 7x + 3.$$

You can also use the *table* feature of a graphing utility to numerically approximate the relative minimum of the function in Example 5. Using a table that begins at 0.6 and increments the value of x by 0.01, you can approximate that the minimum of

$$f(x) = 3x^2 - 4x - 2$$

occurs at the point $(0.67, -3.33)$.

TECHNOLOGY When you use a graphing utility to approximate the x- and y-values of the point where a relative minimum or relative maximum occurs, the *zoom* feature will often produce graphs that are nearly flat. To overcome this problem, manually change the vertical setting of the viewing window. The graph will stretch vertically when the values of Ymin and Ymax are closer together.

Average Rate of Change

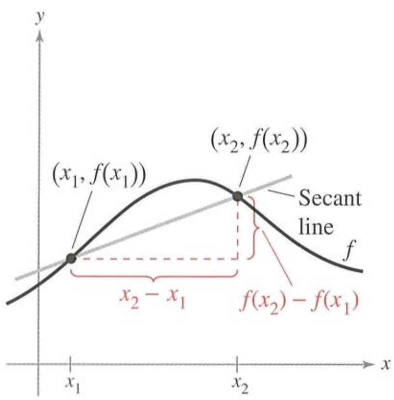


Figure 1.39

In Section 1.3, you learned that the slope of a line can be interpreted as a *rate of change*. For a nonlinear graph, the **average rate of change** between any two points $(x_1, f(x_1))$ and $(x_2, f(x_2))$ is the slope of the line through the two points (see Figure 1.39). The line through the two points is called a **secant line,** and the slope of this line is denoted as m_{sec}.

$$\text{Average rate of change of } f \text{ from } x_1 \text{ to } x_2 = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x} = m_{\text{sec}}$$

EXAMPLE 6 Average Rate of Change of a Function

Find the average rates of change of $f(x) = x^3 - 3x$ (a) from $x_1 = -2$ to $x_2 = -1$ and (b) from $x_1 = 0$ to $x_2 = 1$ (see Figure 1.40).

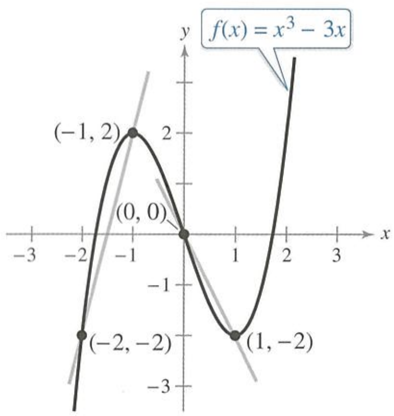


Figure 1.40

Solution

a. The average rate of change of f from $x_1 = -2$ to $x_2 = -1$ is

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(-1) - f(-2)}{-1 - (-2)} = \frac{2 - (-2)}{1} = 4.$$

Secant line has positive slope.

b. The average rate of change of f from $x_1 = 0$ to $x_2 = 1$ is

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(1) - f(0)}{1 - 0} = \frac{-2 - 0}{1} = -2.$$

Secant line has negative slope.

✔ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Find the average rates of change of $f(x) = x^2 + 2x$ (a) from $x_1 = -3$ to $x_2 = -2$ and (b) from $x_1 = -2$ to $x_2 = 0$.

EXAMPLE 7 Finding Average Speed

The distance s (in feet) a moving car is from a stoplight is given by the function

$$s(t) = 20t^{3/2}$$

where t is the time (in seconds). Find the average speed of the car (a) from $t_1 = 0$ to $t_2 = 4$ seconds and (b) from $t_1 = 4$ to $t_2 = 9$ seconds.

Average speed is an average rate of change.

Solution

a. The average speed of the car from $t_1 = 0$ to $t_2 = 4$ seconds is

$$\frac{s(t_2) - s(t_1)}{t_2 - t_1} = \frac{s(4) - s(0)}{4 - 0} = \frac{160 - 0}{4} = 40 \text{ feet per second.}$$

b. The average speed of the car from $t_1 = 4$ to $t_2 = 9$ seconds is

$$\frac{s(t_2) - s(t_1)}{t_2 - t_1} = \frac{s(9) - s(4)}{9 - 4} = \frac{540 - 160}{5} = 76 \text{ feet per second.}$$

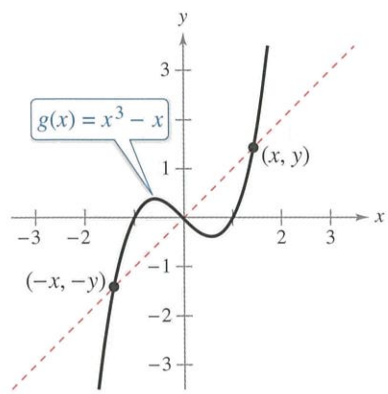
✔ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

In Example 7, find the average speed of the car (a) from $t_1 = 0$ to $t_2 = 1$ second and (b) from $t_1 = 1$ second to $t_2 = 4$ seconds.

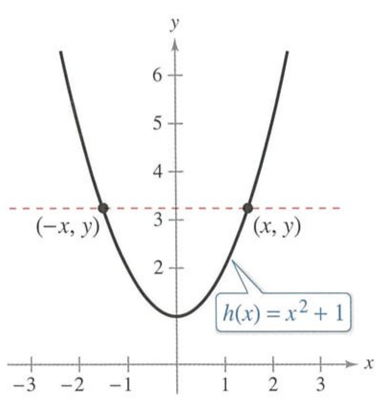
Even and Odd Functions

In Section 1.2, you studied different types of symmetry of a graph. In the terminology of functions, a function is said to be **even** when its graph is symmetric with respect to the y-axis and **odd** when its graph is symmetric with respect to the origin. The symmetry tests in Section 1.2 yield the tests for even and odd functions below.

> **Tests for Even and Odd Functions**
>
> A function $y = f(x)$ is **even** when, for each x in the domain of f, $f(-x) = f(x)$.
>
> A function $y = f(x)$ is **odd** when, for each x in the domain of f, $f(-x) = -f(x)$.



(a) Symmetric to origin: Odd Function



(b) Symmetric to y-axis: Even Function

Figure 1.41

EXAMPLE 8 Even and Odd Functions

See LarsonPrecalculus.com for an interactive version of this type of example.

a. The function $g(x) = x^3 - x$ is odd because $g(-x) = -g(x)$, as follows.

$$g(-x) = (-x)^3 - (-x) \quad \text{Substitute } -x \text{ for } x.$$
$$= -x^3 + x \quad \text{Simplify.}$$
$$= -(x^3 - x) \quad \text{Distributive Property}$$
$$= -g(x) \quad \text{Test for odd function}$$

b. The function $h(x) = x^2 + 1$ is even because $h(-x) = h(x)$, as follows.

$$h(-x) = (-x)^2 + 1 = x^2 + 1 = h(x) \quad \text{Test for even function}$$

Figure 1.41 shows the graphs and symmetry of these two functions.

✓ **Checkpoint** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Determine whether each function is even, odd, or neither. Then describe the symmetry.

a. $f(x) = 5 - 3x$ **b.** $g(x) = x^4 - x^2 - 1$ **c.** $h(x) = 2x^3 + 3x$

Summarize (Section 1.5)

1. State the Vertical Line Test for functions *(page 50)*. For an example of using the Vertical Line Test, see Example 2.
2. Explain how to find the zeros of a function *(page 51)*. For an example of finding the zeros of functions, see Example 3.
3. Explain how to determine intervals on which functions are increasing or decreasing *(page 52)*. For an example of describing function behavior, see Example 4.
4. Explain how to determine relative minimum and relative maximum values of functions *(page 53)*. For an example of approximating a relative minimum, see Example 5.
5. Explain how to determine the average rate of change of a function *(page 54)*. For examples of determining average rates of change, see Examples 6 and 7.
6. State the definitions of an even function and an odd function *(page 55)*. For an example of identifying even and odd functions, see Example 8.

1.5 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

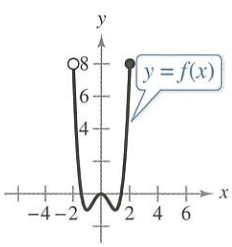
Vocabulary: Fill in the blanks.

1. The ________ ________ ________ is used to determine whether a graph represents y as a function of x.
2. The ________ of a function $y = f(x)$ are the values of x for which $f(x) = 0$.
3. A function f is ________ on an interval when, for any x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) > f(x_2)$.
4. A function value $f(a)$ is a relative ________ of f when there exists an interval (x_1, x_2) containing a such that $x_1 < x < x_2$ implies $f(a) \geq f(x)$.
5. The ________ ________ ________ ________ between any two points $(x_1, f(x_1))$ and $(x_2, f(x_2))$ is the slope of the line through the two points, and this line is called the ________ line.
6. A function f is ________ when, for each x in the domain of f, $f(-x) = -f(x)$.

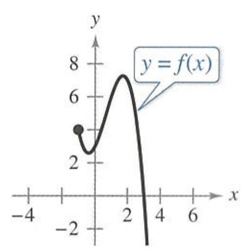
Skills and Applications

Domain, Range, and Values of a Function In Exercises 7–10, use the graph of the function to find the domain and range of f and each function value.

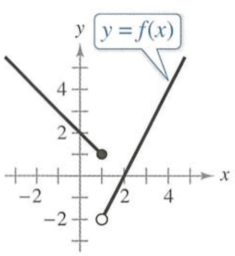
7. (a) $f(-1)$ (b) $f(0)$ (c) $f(1)$ (d) $f(2)$



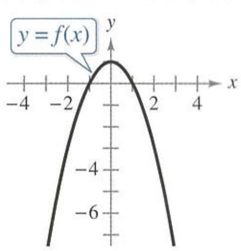
8. (a) $f(-1)$ (b) $f(0)$ (c) $f(1)$ (d) $f(3)$



9. (a) $f(2)$ (b) $f(1)$ (c) $f(3)$ (d) $f(-1)$

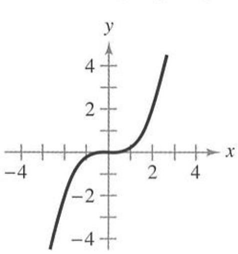


10. (a) $f(-2)$ (b) $f(1)$ (c) $f(0)$ (d) $f(2)$



Vertical Line Test for Functions In Exercises 11–14, use the Vertical Line Test to determine whether the graph represents y as a function of x. To print an enlarged copy of the graph, go to *MathGraphs.com.*

11.



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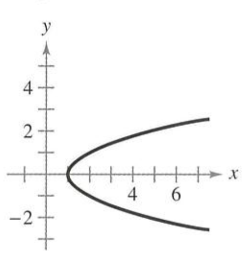
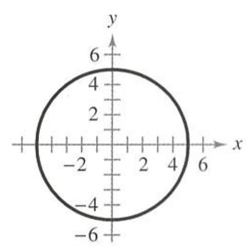
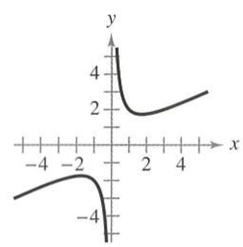


13.



14.



Finding the Zeros of a Function In Exercises 15–26, find the zeros of the function algebraically.

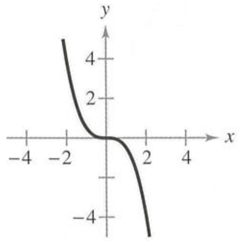
15. $f(x) = 3x + 18$
16. $f(x) = 15 - 2x$
17. $f(x) = 2x^2 - 7x - 30$
18. $f(x) = 3x^2 + 22x - 16$
19. $f(x) = \dfrac{x + 3}{2x^2 - 6}$
20. $f(x) = \dfrac{x^2 - 9x + 14}{4x}$
21. $f(x) = \frac{1}{3}x^3 - 2x$
22. $f(x) = -25x^4 + 9x^2$
23. $f(x) = x^3 - 4x^2 - 9x + 36$
24. $f(x) = 4x^3 - 24x^2 - x + 6$
25. $f(x) = \sqrt{2x} - 1$
26. $f(x) = \sqrt{3x + 2}$

Graphing and Finding Zeros In Exercises 27–32, (a) use a graphing utility to graph the function and find the zeros of the function and (b) verify your results from part (a) algebraically.

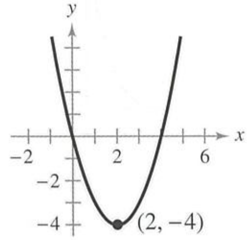
27. $f(x) = x^2 - 6x$
28. $f(x) = 2x^2 - 13x - 7$
29. $f(x) = \sqrt{2x + 11}$
30. $f(x) = \sqrt{3x - 14} - 8$
31. $f(x) = \dfrac{3x - 1}{x - 6}$
32. $f(x) = \dfrac{2x^2 - 9}{3 - x}$

Describing Function Behavior **In Exercises 33–40, determine the open intervals on which the function is increasing, decreasing, or constant.**

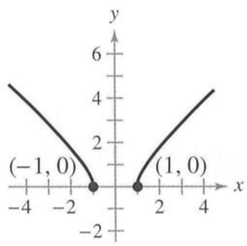
33. $f(x) = -\frac{1}{2}x^3$



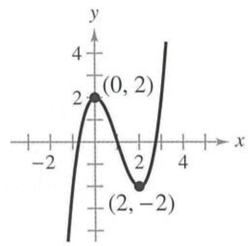
34. $f(x) = x^2 - 4x$



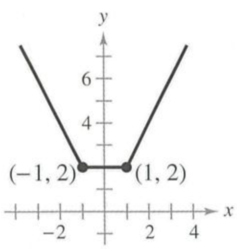
35. $f(x) = \sqrt{x^2 - 1}$



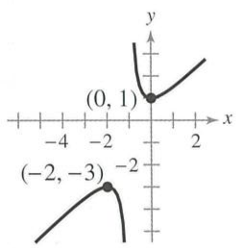
36. $f(x) = x^3 - 3x^2 + 2$



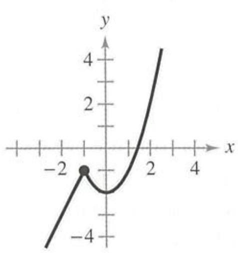
37. $f(x) = |x + 1| + |x - 1|$



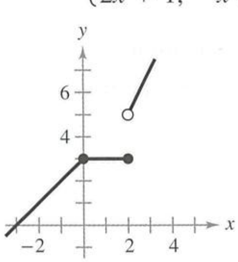
38. $f(x) = \dfrac{x^2 + x + 1}{x + 1}$



39. $f(x) = \begin{cases} 2x + 1, & x \le -1 \\ x^2 - 2, & x > -1 \end{cases}$



40. $f(x) = \begin{cases} x + 3, & x \le 0 \\ 3, & 0 < x \le 2 \\ 2x + 1, & x > 2 \end{cases}$



Describing Function Behavior **In Exercises 41–48, use a graphing utility to graph the function and visually determine the open intervals on which the function is increasing, decreasing, or constant. Use a table of values to verify your results.**

41. $f(x) = 3$
42. $g(x) = x$
43. $g(x) = \frac{1}{2}x^2 - 3$
44. $f(x) = 3x^4 - 6x^2$
45. $f(x) = \sqrt{1 - x}$
46. $f(x) = x\sqrt{x + 3}$
47. $f(x) = x^{3/2}$
48. $f(x) = x^{2/3}$

Approximating Relative Minima or Maxima **In Exercises 49–54, use a graphing utility to approximate (to two decimal places) any relative minima or maxima of the function.**

49. $f(x) = x(x + 3)$
50. $f(x) = -x^2 + 3x - 2$
51. $h(x) = x^3 - 6x^2 + 15$
52. $f(x) = x^3 - 3x^2 - x + 1$
53. $h(x) = (x - 1)\sqrt{x}$
54. $g(x) = x\sqrt{4 - x}$

Graphical Reasoning **In Exercises 55–60, graph the function and determine the interval(s) for which $f(x) \ge 0$.**

55. $f(x) = 4 - x$
56. $f(x) = 4x + 2$
57. $f(x) = 9 - x^2$
58. $f(x) = x^2 - 4x$
59. $f(x) = \sqrt{x - 1}$
60. $f(x) = |x + 5|$

Average Rate of Change of a Function **In Exercises 61–64, find the average rate of change of the function from x_1 to x_2.**

	Function	x-Values
61.	$f(x) = -2x + 15$	$x_1 = 0, x_2 = 3$
62.	$f(x) = x^2 - 2x + 8$	$x_1 = 1, x_2 = 5$
63.	$f(x) = x^3 - 3x^2 - x$	$x_1 = -1, x_2 = 2$
64.	$f(x) = -x^3 + 6x^2 + x$	$x_1 = 1, x_2 = 6$

65. **Research and Development** The amounts (in billions of dollars) the U.S. federal government spent on research and development for defense from 2010 through 2014 can be approximated by the model

$$y = 0.5079t^2 - 8.168t + 95.08$$

where t represents the year, with $t = 0$ corresponding to 2010. *(Source: American Association for the Advancement of Science)*

(a) Use a graphing utility to graph the model.

(b) Find the average rate of change of the model from 2010 to 2014. Interpret your answer in the context of the problem.

66. Finding Average Speed Use the information in Example 7 to find the average speed of the car from $t_1 = 0$ to $t_2 = 9$ seconds. Explain why the result is less than the value obtained in part (b) of Example 7.

Physics **In Exercises 67–70, (a) use the position equation $s = -16t^2 + v_0t + s_0$ to write a function that represents the situation, (b) use a graphing utility to graph the function, (c) find the average rate of change of the function from t_1 to t_2, (d) describe the slope of the secant line through t_1 and t_2, (e) find the equation of the secant line through t_1 and t_2, and (f) graph the secant line in the same viewing window as your position function.**

67. An object is thrown upward from a height of 6 feet at a velocity of 64 feet per second.

$t_1 = 0, t_2 = 3$

68. An object is thrown upward from a height of 6.5 feet at a velocity of 72 feet per second.

$t_1 = 0, t_2 = 4$

69. An object is thrown upward from ground level at a velocity of 120 feet per second.

$t_1 = 3, t_2 = 5$

70. An object is dropped from a height of 80 feet.

$t_1 = 1, t_2 = 2$

Even, Odd, or Neither? **In Exercises 71–76, determine whether the function is even, odd, or neither. Then describe the symmetry.**

71. $f(x) = x^6 - 2x^2 + 3$ **72.** $g(x) = x^3 - 5x$

73. $h(x) = x\sqrt{x+5}$ **74.** $f(x) = x\sqrt{1-x^2}$

75. $f(s) = 4s^{3/2}$ **76.** $g(s) = 4s^{2/3}$

Even, Odd, or Neither? **In Exercises 77–82, sketch a graph of the function and determine whether it is even, odd, or neither. Verify your answer algebraically.**

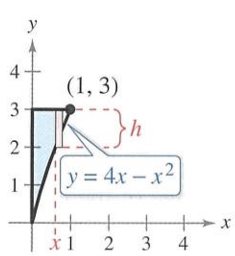
77. $f(x) = -9$ **78.** $f(x) = 5 - 3x$

79. $f(x) = -|x - 5|$ **80.** $h(x) = x^2 - 4$

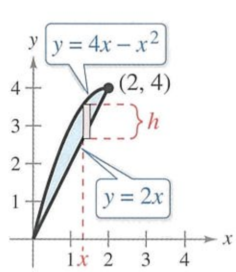
81. $f(x) = \sqrt[3]{4x}$ **82.** $f(x) = \sqrt[3]{x-4}$

Height of a Rectangle **In Exercises 83 and 84, write the height h of the rectangle as a function of x.**

83.

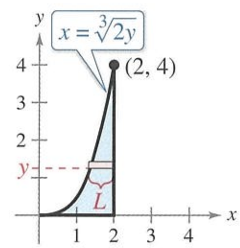


84.

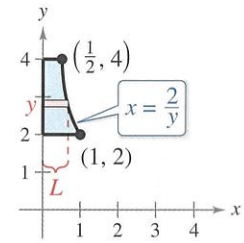


Length of a Rectangle **In Exercises 85 and 86, write the length L of the rectangle as a function of y.**

85.



86.

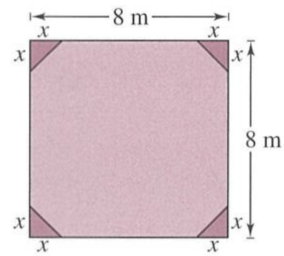


87. Error Analysis Describe the error.

The function $f(x) = 2x^3 - 5$ is odd because $f(-x) = -f(x)$, as follows.

$$\begin{aligned} f(-x) &= 2(-x)^3 - 5 \\ &= -2x^3 - 5 \\ &= -(2x^3 - 5) \\ &= -f(x) \end{aligned}$$

88. Geometry Corners of equal size are cut from a square with sides of length 8 meters (see figure).



(a) Write the area A of the resulting figure as a function of x. Determine the domain of the function.

(b) Use a graphing utility to graph the area function over its domain. Use the graph to find the range of the function.

(c) Identify the figure that results when x is the maximum value in the domain of the function. What would be the length of each side of the figure?

89. Coordinate Axis Scale Each function described below models the specified data for the years 2006 through 2016, with $t = 6$ corresponding to 2006. Estimate a reasonable scale for the vertical axis (e.g., hundreds, thousands, millions, etc.) of the graph and justify your answer. (There are many correct answers.)

(a) $f(t)$ represents the average salary of college professors.

(b) $f(t)$ represents the U.S. population.

(c) $f(t)$ represents the percent of the civilian workforce that is unemployed.

(d) $f(t)$ represents the number of games a college football team wins.

90. Temperature The table shows the temperatures y (in degrees Fahrenheit) in a city over a 24-hour period. Let x represent the time of day, where $x = 0$ corresponds to 6 A.M.

DATA

Time, x	Temperature, y
0	34
2	50
4	60
6	64
8	63
10	59
12	53
14	46
16	40
18	36
20	34
22	37
24	45

Spreadsheet at LarsonPrecalculus.com

These data can be approximated by the model

$$y = 0.026x^3 - 1.03x^2 + 10.2x + 34, \quad 0 \le x \le 24.$$

(a) Use a graphing utility to create a scatter plot of the data. Then graph the model in the same viewing window.

(b) How well does the model fit the data?

(c) Use the graph to approximate the times when the temperature was increasing and decreasing.

(d) Use the graph to approximate the maximum and minimum temperatures during this 24-hour period.

(e) Could this model predict the temperatures in the city during the next 24-hour period? Why or why not?

Exploration

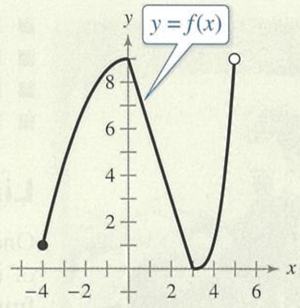
True or False? **In Exercises 91–93, determine whether the statement is true or false. Justify your answer.**

91. A function with a square root cannot have a domain that is the set of real numbers.

92. It is possible for an odd function to have the interval $[0, \infty)$ as its domain.

93. It is impossible for an even function to be increasing on its entire domain.

94. HOW DO YOU SEE IT? Use the graph of the function to answer parts (a)–(e).



(a) Find the domain and range of f.

(b) Find the zero(s) of f.

(c) Determine the open intervals on which f is increasing, decreasing, or constant.

(d) Approximate any relative minimum or relative maximum values of f.

(e) Is f even, odd, or neither?

Think About It **In Exercises 95 and 96, find the coordinates of a second point on the graph of a function f when the given point is on the graph and the function is (a) even and (b) odd.**

95. $\left(-\frac{5}{3}, -7\right)$ **96.** $(2a, 2c)$

97. Writing Use a graphing utility to graph each function. Write a paragraph describing any similarities and differences you observe among the graphs.

(a) $y = x$ (b) $y = x^2$ (c) $y = x^3$

(d) $y = x^4$ (e) $y = x^5$ (f) $y = x^6$

98. Graphical Reasoning Graph each of the functions with a graphing utility. Determine whether each function is even, odd, or neither.

$f(x) = x^2 - x^4$ $\quad g(x) = 2x^3 + 1$

$h(x) = x^5 - 2x^3 + x$ $\quad j(x) = 2 - x^6 - x^8$

$k(x) = x^5 - 2x^4 + x - 2$ $\quad p(x) = x^9 + 3x^5 - x^3 + x$

What do you notice about the equations of functions that are odd? What do you notice about the equations of functions that are even? Can you describe a way to identify a function as odd or even by inspecting the equation? Can you describe a way to identify a function as neither odd nor even by inspecting the equation?

99. Even, Odd, or Neither? Determine whether g is even, odd, or neither when f is an even function. Explain.

(a) $g(x) = -f(x)$ (b) $g(x) = f(-x)$

(c) $g(x) = f(x) - 2$ (d) $g(x) = f(x - 2)$

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1.6 A Library of Parent Functions

Piecewise-defined functions model many real-life situations. For example, in Exercise 47 on page 66, you will write a piecewise-defined function to model the depth of snow during a snowstorm.

- **Identify and graph linear and squaring functions.**
- **Identify and graph cubic, square root, and reciprocal functions.**
- **Identify and graph step and other piecewise-defined functions.**
- **Recognize graphs of parent functions.**

Linear and Squaring Functions

One of the goals of this text is to enable you to recognize the basic shapes of the graphs of different types of functions. For example, you know that the graph of the **linear function** $f(x) = ax + b$ is a line with slope $m = a$ and y-intercept at $(0, b)$. The graph of a linear function has the characteristics below.

- The domain of the function is the set of all real numbers.
- When $m \neq 0$, the range of the function is the set of all real numbers.
- The graph has an x-intercept at $(-b/m, 0)$ and a y-intercept at $(0, b)$.
- The graph is increasing when $m > 0$, decreasing when $m < 0$, and constant when $m = 0$.

EXAMPLE 1 **Writing a Linear Function**

Write the linear function f for which $f(1) = 3$ and $f(4) = 0$.

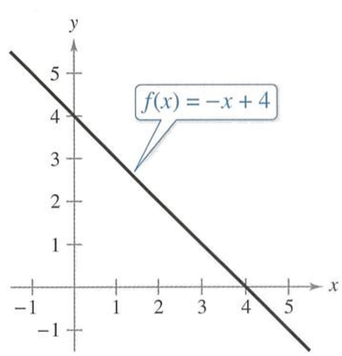
Solution To find the equation of the line that passes through $(x_1, y_1) = (1, 3)$ and $(x_2, y_2) = (4, 0)$, first find the slope of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 3}{4 - 1} = \frac{-3}{3} = -1$$

Next, use the point-slope form of the equation of a line.

$$y - y_1 = m(x - x_1) \quad \text{Point-slope form}$$
$$y - 3 = -1(x - 1) \quad \text{Substitute for } x_1, y_1, \text{ and } m.$$
$$y = -x + 4 \quad \text{Simplify.}$$
$$f(x) = -x + 4 \quad \text{Function notation}$$

The figure below shows the graph of this function.



✔ **Checkpoint** Audio-video solution in English & Spanish at LarsonPrecalculus.com

Write the linear function f for which $f(-2) = 6$ and $f(4) = -9$.