CHAPTER 8
Matrices and Determinants

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Section 8.1 Matrices and Systems of Equations

1. square

2. main diagonal

3. augmented

4. coefficient

5. row-equivalent

6. reduced row-echelon form

7. Because the matrix has one row and two columns, its dimension is 1×2.

8. Because the matrix has one row and four columns, its dimension is 1×4.

9. Because the matrix has three rows and one column, its dimension is 3×1.

10. Because the matrix has three rows and four columns, its dimension is 3×4.

11. Because the matrix has two rows and two columns, its dimension is 2×2.

12. Because the matrix has two rows and three columns, its dimension is 2×3.

13. Because the matrix has three rows and three columns, its dimension is 3×3.

14. Because the matrix has three rows and two columns, its dimension is 3×2.

15. $\begin{cases} 2x - y = 7 \\ x + y = 2 \end{cases}$

$$\begin{bmatrix} 2 & -1 & \vdots & 7 \\ 1 & 1 & \vdots & 2 \end{bmatrix}$$

16. $\begin{cases} 5x + 2y = 13 \\ -3x + 4y = -24 \end{cases}$

$$\begin{bmatrix} 5 & 2 & \vdots & 13 \\ -3 & 4 & \vdots & -24 \end{bmatrix}$$

17. $\begin{cases} x - y + 2z = 2 \\ 4x - 3y + z = -1 \\ 2x + y = 0 \end{cases}$

$$\begin{bmatrix} 1 & -1 & 2 & \vdots & 2 \\ 4 & -3 & 1 & \vdots & -1 \\ 2 & 1 & 0 & \vdots & 0 \end{bmatrix}$$

18. $\begin{cases} -2x - 4y + z = 13 \\ 6x - 7z = 22 \\ 3x - y + z = 9 \end{cases}$

$$\begin{bmatrix} -2 & -4 & 1 & \vdots & 13 \\ 6 & 0 & -7 & \vdots & 22 \\ 3 & -1 & 1 & \vdots & 9 \end{bmatrix}$$

19. $\begin{cases} 3x - 5y + 2z = 12 \\ 12x - 7z = 10 \end{cases}$

$$\begin{bmatrix} 3 & -5 & 2 & \vdots & 12 \\ 12 & 0 & -7 & \vdots & 10 \end{bmatrix}$$

20. $\begin{cases} 9x - y - 3z = 21 \\ -15y + 13z = -8 \end{cases}$

$$\begin{bmatrix} 9 & -1 & -3 & \vdots & 21 \\ 0 & -15 & 13 & \vdots & -8 \end{bmatrix}$$

21. $\begin{bmatrix} 1 & 1 & \vdots & 3 \\ 5 & -3 & \vdots & -1 \end{bmatrix}$

$$\begin{cases} x + y = 3 \\ 5x - 3y = -1 \end{cases}$$

22. $\begin{bmatrix} 5 & 2 & \vdots & 9 \\ 3 & -8 & \vdots & 0 \end{bmatrix}$

$$\begin{cases} 5x + 2y = 9 \\ 3x - 8y = 0 \end{cases}$$

23. $\begin{bmatrix} 2 & 0 & 5 & \vdots & -12 \\ 0 & 1 & -2 & \vdots & 7 \\ 6 & 3 & 0 & \vdots & 2 \end{bmatrix}$

$$\begin{cases} 2x + 5z = -12 \\ y - 2z = 7 \\ 6x + 3y = 2 \end{cases}$$

24. $\begin{bmatrix} 4 & -5 & -1 & \vdots & 18 \\ -11 & 0 & 6 & \vdots & 25 \\ 3 & 8 & 0 & \vdots & -29 \end{bmatrix}$

$$\begin{cases} 4x - 5y - z = 18 \\ -11x + 6z = 25 \\ 3x + 8y = -29 \end{cases}$$

25. $\begin{bmatrix} 9 & 12 & 3 & 0 & \vdots & 0 \\ -2 & 18 & 5 & 2 & \vdots & 10 \\ 1 & 7 & -8 & 0 & \vdots & -4 \\ 3 & 0 & 2 & 0 & \vdots & -10 \end{bmatrix}$

$$\begin{cases} 9x + 12y + 3z = 0 \\ -2x + 18y + 5z + 2w = 10 \\ x + 7y - 8z = -4 \\ 3x + 2z = -10 \end{cases}$$

26. $\begin{bmatrix} 6 & 2 & -1 & -5 & \vdots & -25 \\ -1 & 0 & 7 & 3 & \vdots & 7 \\ 4 & -1 & -10 & 6 & \vdots & 23 \\ 0 & 8 & 1 & -11 & \vdots & -21 \end{bmatrix}$

$$\begin{cases} 6x + 2y - z - 5w = -25 \\ -x + 7z + 3w = 7 \\ 4x - y - 10z + 6w = 23 \\ 8y + z - 11w = -21 \end{cases}$$

27. $\begin{bmatrix} -2 & 5 & 1 \\ 3 & -1 & -8 \end{bmatrix} \to \begin{bmatrix} 13 & 0 & -39 \\ 3 & -1 & -8 \end{bmatrix}$

Add 5 times Row 2 to Row 1.

28. $\begin{bmatrix} 3 & -1 & -4 \\ -4 & 3 & 7 \end{bmatrix} \to \begin{bmatrix} 3 & -1 & -4 \\ 5 & 0 & -5 \end{bmatrix}$

Add 3 times Row 1 to Row 2.

29. $\begin{bmatrix} 0 & -1 & -5 & 5 \\ -1 & 3 & -7 & 6 \\ 4 & -5 & 1 & 3 \end{bmatrix} \to \begin{bmatrix} -1 & 3 & -7 & 6 \\ 0 & -1 & -5 & 5 \\ 0 & 7 & -27 & 27 \end{bmatrix}$

Interchange Row 1 and Row 2. Then add 4 times the new Row 1 to Row 3.

30. $\begin{bmatrix} -1 & -2 & 3 & -2 \\ 2 & -5 & 1 & -7 \\ 5 & 4 & -7 & 6 \end{bmatrix} \to \begin{bmatrix} -1 & -2 & 3 & -2 \\ 0 & -9 & 7 & -11 \\ 0 & -6 & 8 & -4 \end{bmatrix}$

Add 2 times Row 1 to Row 2.

Add 5 times Row 1 to Row 3.

31. $\begin{bmatrix} 3 & 6 & 8 \\ 4 & -3 & 6 \end{bmatrix}$

$\frac{1}{3}R_1 \to \begin{bmatrix} 1 & \boxed{2} & \frac{8}{3} \\ 4 & -3 & 6 \end{bmatrix}$

32. $\begin{bmatrix} 1 & 4 & 3 \\ 2 & 10 & 5 \end{bmatrix}$

$-2R_1 + R_2 \to \begin{bmatrix} 1 & 4 & 3 \\ 0 & \boxed{2} & -1 \end{bmatrix}$

33. $\begin{bmatrix} 1 & 1 & 1 \\ 5 & -2 & 4 \end{bmatrix}$

$-5R_1 + R_2 \to \begin{bmatrix} 1 & 1 & 1 \\ 0 & \boxed{-7} & -1 \end{bmatrix}$

34. $\begin{bmatrix} -3 & 3 & 12 \\ 18 & -8 & 4 \end{bmatrix}$

$-\frac{1}{3}R_1 \to \begin{bmatrix} 1 & -1 & \boxed{-4} \\ 18 & -8 & 4 \end{bmatrix}$

35. $\begin{bmatrix} 1 & 5 & 4 & -1 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 1 & -7 \end{bmatrix}$

$-5R_2 + R_1 \to \begin{bmatrix} 1 & 0 & \boxed{14} & \boxed{-11} \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 1 & -7 \end{bmatrix}$

36. $\begin{bmatrix} 1 & 0 & 6 & 1 \\ 0 & -1 & 0 & 7 \\ 0 & 0 & -1 & 3 \end{bmatrix}$

$\begin{matrix} \\ -R_2 \to \\ -R_3 \to \end{matrix} \begin{bmatrix} 1 & 0 & 6 & 1 \\ 0 & 1 & 0 & \boxed{-7} \\ 0 & 0 & 1 & \boxed{-3} \end{bmatrix}$

37. $\begin{bmatrix} 1 & 1 & 4 & -1 \\ 3 & 8 & 10 & 3 \\ -2 & 1 & 12 & 6 \end{bmatrix}$

$\begin{matrix} \\ -3R_1 + R_2 \to \\ 2R_1 + R_3 \to \end{matrix} \begin{bmatrix} 1 & 1 & 4 & -1 \\ 0 & 5 & \boxed{-2} & \boxed{6} \\ 0 & 3 & \boxed{20} & \boxed{4} \end{bmatrix}$

$\frac{1}{5}R_2 \to \begin{bmatrix} 1 & 1 & 4 & -1 \\ 0 & 1 & -\frac{2}{5} & \frac{6}{5} \\ 0 & 3 & \boxed{20} & \boxed{4} \end{bmatrix}$

38. $\begin{bmatrix} 2 & 4 & 8 & 3 \\ 1 & -1 & -3 & 2 \\ 2 & 6 & 4 & 9 \end{bmatrix}$

$\frac{1}{2}R_1 \rightarrow \begin{bmatrix} 1 & \boxed{2} & \boxed{4} & \boxed{\frac{3}{2}} \\ 1 & -1 & -3 & 2 \\ 2 & 6 & 4 & 9 \end{bmatrix}$

$\begin{matrix} \\ -R_1 + R_2 \rightarrow \\ -2R_1 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 2 & 4 & \frac{3}{2} \\ 0 & \boxed{-3} & -7 & \frac{1}{2} \\ 0 & 2 & \boxed{-4} & \boxed{6} \end{bmatrix}$

39. (a) $\left[\begin{array}{cc:c} -3 & 4 & 22 \\ 6 & -4 & -28 \end{array}\right]$

(i) $R_1 + R_2 \rightarrow \left[\begin{array}{cc:c} 3 & 0 & -6 \\ 6 & -4 & -28 \end{array}\right]$

(ii) $-2R_1 + R_2 \rightarrow \left[\begin{array}{cc:c} 3 & 0 & -6 \\ 0 & -4 & -16 \end{array}\right]$

(iii) $-\frac{1}{4}R_2 \rightarrow \left[\begin{array}{cc:c} 3 & 0 & -6 \\ 0 & 1 & 4 \end{array}\right]$

(iv) $\frac{1}{3}R_1 \rightarrow \left[\begin{array}{cc:c} 1 & 0 & -2 \\ 0 & 1 & 4 \end{array}\right]$

The solution is $x = -2$ and $y = 4$.

(b) $\begin{cases} -3x + 4y = 22 \\ 6x - 4y = -28 \end{cases}$

$3x = -6$

$x = -2$

Back-substitute $x = -2$ into $-3x + 4y = 22$.

$-3(-2) + 4y = 22$

$4y = 16$

$y = 4$

The solution is $x = -2$ and $y = 4$.

(c) Answers vary. *Sample answer:* In this case, solving the system of linear equations using the elimination method was more efficient.

40. (a) $\left[\begin{array}{ccc:c} 7 & 13 & 1 & -4 \\ -3 & -5 & -1 & -4 \\ 3 & 6 & 1 & -2 \end{array}\right]$

(i) $R_2 + R_1 \rightarrow \left[\begin{array}{ccc:c} 4 & 8 & 0 & -8 \\ -3 & -5 & -1 & -4 \\ 3 & 6 & 1 & -2 \end{array}\right]$

(ii) $\frac{1}{4}R_1 \rightarrow \left[\begin{array}{ccc:c} 1 & 2 & 0 & -2 \\ -3 & -5 & -1 & -4 \\ 3 & 6 & 1 & -2 \end{array}\right]$

(iii) $R_3 + R_2 \rightarrow \left[\begin{array}{ccc:c} 1 & 2 & 0 & -2 \\ 0 & 1 & 0 & -6 \\ 3 & 6 & 1 & -2 \end{array}\right]$

(iv) $-3R_1 + R_3 \rightarrow \left[\begin{array}{ccc:c} 1 & 2 & 0 & -2 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 4 \end{array}\right]$

(v) $-2R_2 + R_1 \rightarrow \left[\begin{array}{ccc:c} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 4 \end{array}\right]$

The solution is $x = 10$, $y = -6$, and $z = 4$.

(b) $\begin{cases} 7x + 13y + z = -4 \\ -3x - 5y - z = -4 \\ 3x + 6y + z = -2 \end{cases}$

Add Equations 2 and 3.

$\begin{cases} 7x + 13y + z = -4 \\ y = -6 \\ 3x + 6y + z = -2 \end{cases}$

Add Equations 1 and −1 times Equation 3.

$\begin{cases} 7x + 13y + z = -4 \\ y = -6 \\ 4x + 7y = -2 \end{cases}$

Back-substitute $y = -6$ into Equations 1 and 3.

$y = -6$

$4x + 7(-6) = -2$

$4x = 40$

$x = 10$

$7(10) + 13(-6) + z = -4$

$z = 4$

The solution is $x = 10$, $y = -6$, and $z = 4$.

(c) Answers vary. *Sample answer:* In this case, writing the row-reduced echelon form of the augmented matrix was more efficient.

41. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

This matrix is in reduced row-echelon form.

42. $\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

This matrix is not in row-echelon form.

43. $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

This matrix is not in row-echelon form.

44. $\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

This matrix is in row-echelon form.

45. $\begin{bmatrix} 1 & 1 & 0 & 5 \\ -2 & -1 & 2 & -10 \\ 3 & 6 & 7 & 14 \end{bmatrix}$

$$\begin{matrix} \\ 2R_1 + R_2 \to \\ -3R_1 + R_3 \to \end{matrix} \begin{bmatrix} 1 & 1 & 0 & 5 \\ 0 & 1 & 2 & 0 \\ 0 & 3 & 7 & -1 \end{bmatrix}$$

$$\begin{matrix} \\ \\ -3R_2 + R_3 \to \end{matrix} \begin{bmatrix} 1 & 1 & 0 & 5 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

46. $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & 7 & -5 & 14 \\ -2 & -1 & -3 & 8 \end{bmatrix}$

$$\begin{matrix} \\ -3R_1 + R_2 \to \\ 2R_1 + R_3 \to \end{matrix} \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & -2 & 5 \\ 0 & 3 & -5 & 14 \end{bmatrix}$$

$$\begin{matrix} \\ \\ -3R_2 + R_3 \to \end{matrix} \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & -2 & 5 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

47. $\begin{bmatrix} 1 & -1 & -1 & 1 \\ 5 & -4 & 1 & 8 \\ -6 & 8 & 18 & 0 \end{bmatrix}$

$$\begin{matrix} \\ -5R_1 + R_2 \to \\ 6R_1 + R_3 \to \end{matrix} \begin{bmatrix} 1 & -1 & -1 & 1 \\ 0 & 1 & 6 & 3 \\ 0 & 2 & 12 & 6 \end{bmatrix}$$

$$\begin{matrix} \\ \\ -2R_2 + R_3 \to \end{matrix} \begin{bmatrix} 1 & -1 & -1 & 1 \\ 0 & 1 & 6 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

48. $\begin{bmatrix} 1 & -3 & 0 & -7 \\ -3 & 10 & 1 & 23 \\ 4 & -10 & 2 & -24 \end{bmatrix}$

$$\begin{matrix} \\ 3R_1 + R_2 \to \\ -4R_1 + R_3 \to \end{matrix} \begin{bmatrix} 1 & -3 & 0 & -7 \\ 0 & 1 & 1 & 2 \\ 0 & 2 & 2 & 4 \end{bmatrix}$$

$$\begin{matrix} \\ \\ -2R_2 + R_3 \to \end{matrix} \begin{bmatrix} 1 & -3 & 0 & -7 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

49. Use the reduced row-echelon form feature of a graphing utility.

$$\begin{bmatrix} 3 & 3 & 3 \\ -1 & 0 & -4 \\ 2 & 4 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

50. Use the reduced row-echelon form feature of a graphing utility.

$$\begin{bmatrix} 1 & 3 & 2 \\ 5 & 15 & 9 \\ 2 & 6 & 10 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

51. Use the reduced row-echelon form feature of a graphing utility.

$$\begin{bmatrix} 1 & 2 & 3 & -5 \\ 1 & 2 & 4 & -9 \\ -2 & -4 & -4 & 3 \\ 4 & 8 & 11 & -14 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

52. Use the reduced row-echelon form feature of a graphing utility.

$$\begin{bmatrix} -2 & 3 & -1 & -2 \\ 4 & -2 & 5 & 8 \\ 1 & 5 & -2 & 0 \\ 3 & 8 & -10 & -30 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

53. Use the reduced row-echelon form feature of a graphing utility.

$$\begin{bmatrix} -3 & 5 & 1 & 12 \\ 1 & -1 & 1 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 3 & 16 \\ 0 & 1 & 2 & 12 \end{bmatrix}$$

54. Use the reduced row-echelon form feature of a graphing utility.

$$\begin{bmatrix} 5 & 1 & 2 & 4 \\ -1 & 5 & 10 & -32 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 2 & -6 \end{bmatrix}$$

55. $\begin{cases} x - 2y = 4 \\ y = -1 \end{cases}$

$x - 2(-1) = 4$

$x = 2$

Solution: $(2, -1)$

56. $\begin{cases} x + 5y = 0 \\ y = 6 \end{cases}$

$x + 5(6) = 0$

$x = -30$

Solution: $(-30, 6)$

57. $\begin{cases} x - y + 2z = 4 \\ y - z = 2 \\ z = -2 \end{cases}$

$y - (-2) = 2$

$y = 0$

$x - 0 + 2(-2) = 4$

$x = 8$

Solution: $(8, 0, -2)$

58. $\begin{cases} x + 2y - 2z = -1 \\ y + z = 9 \\ z = -3 \end{cases}$

$y + (-3) = 9$

$y = 12$

$x + 2(12) - 2(-3) = -1$

$x = -31$

Solution: $(-31, 12, -3)$

59. $\begin{cases} x + 2y = 7 \\ -x + y = 8 \end{cases}$

$$\begin{bmatrix} 1 & 2 & \vdots & 7 \\ -1 & 1 & \vdots & 8 \end{bmatrix}$$

$$R_1 + R_2 \rightarrow \begin{bmatrix} 1 & 2 & \vdots & 7 \\ 0 & 3 & \vdots & 15 \end{bmatrix}$$

$$\tfrac{1}{3}R_2 \rightarrow \begin{bmatrix} 1 & 2 & \vdots & 7 \\ 0 & 1 & \vdots & 5 \end{bmatrix}$$

$\begin{cases} x + 2y = 7 \\ y = 5 \end{cases}$

$x + 2(5) = 7 \Rightarrow x = -3$

Solution: $(-3, 5)$

60. $\begin{cases} 2x + 6y = 16 \\ 2x + 3y = 7 \end{cases}$

$$\begin{bmatrix} 2 & 6 & \vdots & 16 \\ 2 & 3 & \vdots & 7 \end{bmatrix}$$

$$-R_1 + R_2 \rightarrow \begin{bmatrix} 2 & 6 & \vdots & 16 \\ 0 & -3 & \vdots & -9 \end{bmatrix}$$

$$\begin{matrix} \tfrac{1}{2}R_1 \rightarrow \\ -\tfrac{1}{3}R_2 \rightarrow \end{matrix} \begin{bmatrix} 1 & 3 & \vdots & 8 \\ 0 & 1 & \vdots & 3 \end{bmatrix}$$

$\begin{cases} x + 3y = 8 \\ y = 3 \end{cases}$

$y = 3$

$x + 3(3) = 8 \Rightarrow x = -1$

Solution: $(-1, 3)$

61. $\begin{cases} 3x - 2y = -27 \\ x + 3y = 13 \end{cases}$

$$\begin{bmatrix} 3 & -2 & \vdots & -27 \\ 1 & 3 & \vdots & 13 \end{bmatrix}$$

$$\begin{matrix} R_1 \\ R_2 \end{matrix} \begin{bmatrix} 1 & 3 & \vdots & 13 \\ 3 & -2 & \vdots & -27 \end{bmatrix}$$

$$-3R_1 + R_2 \rightarrow \begin{bmatrix} 1 & 3 & \vdots & 13 \\ 0 & -11 & \vdots & -66 \end{bmatrix}$$

$$-\tfrac{1}{11}R_2 \rightarrow \begin{bmatrix} 1 & 3 & \vdots & 13 \\ 0 & 1 & \vdots & 6 \end{bmatrix}$$

$\begin{cases} x + 3y = 13 \\ y = 6 \end{cases}$

$y = 6$

$x + 3(6) = 13 \Rightarrow x = -5$

Solution: $(-5, 6)$

62. $\begin{cases} -x + y = 4 \\ 2x - 4y = -34 \end{cases}$

$$\begin{bmatrix} -1 & 1 & \vdots & 4 \\ 2 & -4 & \vdots & -34 \end{bmatrix}$$

$$\begin{matrix} (-1)R_1 \to \\ \left(\frac{1}{2}\right)R_2 \to \end{matrix} \begin{bmatrix} 1 & -1 & \vdots & -4 \\ 1 & -2 & \vdots & -17 \end{bmatrix}$$

$$-R_1 + R_2 \to \begin{bmatrix} 1 & -1 & \vdots & -4 \\ 0 & -1 & \vdots & -13 \end{bmatrix}$$

$$(-1)R_2 \to \begin{bmatrix} 1 & -1 & \vdots & -4 \\ 0 & 1 & \vdots & 13 \end{bmatrix}$$

$$\begin{cases} x - y = -4 \\ y = 13 \end{cases}$$

$y = 13$

$x - 13 = -4 \Rightarrow x = 9$

Solution: $(9, 13)$

63. $\begin{cases} x + 2y - 3z = -28 \\ 4y + 2z = 0 \\ -x + y - z = -5 \end{cases}$

$$\begin{bmatrix} 1 & 2 & -3 & \vdots & -28 \\ 0 & 4 & 2 & \vdots & 0 \\ -1 & 1 & -1 & \vdots & -5 \end{bmatrix}$$

$$\begin{matrix} \\ \frac{1}{4}R_2 \to \\ R_1 + R_3 \to \end{matrix} \begin{bmatrix} 1 & 2 & -3 & \vdots & -28 \\ 0 & 1 & \frac{1}{2} & \vdots & 0 \\ 0 & 3 & -4 & \vdots & -33 \end{bmatrix}$$

$$-3R_2 + R_3 \to \begin{bmatrix} 1 & 2 & -3 & \vdots & -28 \\ 0 & 1 & \frac{1}{2} & \vdots & 0 \\ 0 & 0 & -\frac{11}{2} & \vdots & -33 \end{bmatrix}$$

$$-\frac{2}{11}R_3 \to \begin{bmatrix} 1 & 2 & -3 & \vdots & -28 \\ 0 & 1 & \frac{1}{2} & \vdots & 0 \\ 0 & 0 & 1 & \vdots & 6 \end{bmatrix}$$

$$\begin{cases} x + 2y - 3z = -28 \\ y + \frac{1}{2}z = 0 \\ z = 6 \end{cases}$$

$z = 6$

$y + \frac{1}{2}(6) = 0 \Rightarrow y = -3$

$x + 2(-3) - 3(6) = -28 \Rightarrow x = -4$

Solution: $(-4, -3, 6)$

64. $\begin{cases} 3x - 2y + z = 15 \\ -x + y + 2z = -10 \\ x - y - 4z = 14 \end{cases}$

$$\begin{bmatrix} 3 & -2 & 1 & \vdots & 15 \\ -1 & 1 & 2 & \vdots & -10 \\ 1 & -1 & -4 & \vdots & 14 \end{bmatrix}$$

$$\begin{matrix} R_3 \\ \\ R_1 \end{matrix} \begin{bmatrix} 1 & -1 & -4 & \vdots & 14 \\ -1 & 1 & 2 & \vdots & -10 \\ 3 & -2 & 1 & \vdots & 15 \end{bmatrix}$$

$$\begin{matrix} \\ R_1 + R_2 \to \\ -3R_1 + R_3 \to \end{matrix} \begin{bmatrix} 1 & -1 & -4 & \vdots & 14 \\ 0 & 0 & -2 & \vdots & 4 \\ 0 & 1 & 13 & \vdots & -27 \end{bmatrix}$$

$$\begin{matrix} \\ R_3 \\ R_2 \end{matrix} \begin{bmatrix} 1 & -1 & -4 & \vdots & 14 \\ 0 & 1 & 13 & \vdots & -27 \\ 0 & 0 & -2 & \vdots & 4 \end{bmatrix}$$

$$\begin{matrix} \\ \\ -\frac{1}{2}R_3 \to \end{matrix} \begin{bmatrix} 1 & -1 & -4 & \vdots & 14 \\ 0 & 1 & 13 & \vdots & -27 \\ 0 & 0 & 1 & \vdots & -2 \end{bmatrix}$$

$$\begin{cases} x - y - 4z = 14 \\ y + 13z = -27 \\ z = -2 \end{cases}$$

$z = -2$

$y + 13(-2) = -27 \Rightarrow y = -1$

$x - (-1) - 4(-2) = 14 \Rightarrow x = 5$

Solution: $(5, -1, -2)$

65. $\begin{cases} -3x + 2y = -22 \\ 3x + 4y = 4 \\ 4x - 8y = 32 \end{cases}$

$$\begin{bmatrix} -3 & 2 & \vdots & -22 \\ 3 & 4 & \vdots & 4 \\ 4 & -8 & \vdots & 32 \end{bmatrix}$$

$$R_1 + R_3 \to \begin{bmatrix} 1 & -6 & \vdots & 10 \\ 3 & 4 & \vdots & 4 \\ 4 & -8 & \vdots & 32 \end{bmatrix}$$

$$\begin{matrix} \\ -3R_1 + R_2 \to \\ -4R_1 + R_3 \to \end{matrix} \begin{bmatrix} 1 & -6 & \vdots & 10 \\ 0 & 22 & \vdots & -26 \\ 0 & 16 & \vdots & -8 \end{bmatrix}$$

$$\begin{matrix} \\ \frac{1}{22}R_2 \to \\ \frac{1}{16}R_3 \to \end{matrix} \begin{bmatrix} 1 & -1 & \vdots & 22 \\ 0 & 1 & \vdots & -\frac{13}{10} \\ 0 & 1 & \vdots & -\frac{1}{2} \end{bmatrix}$$

$$\begin{matrix} \\ \\ -R_2 + R_3 \to \end{matrix} \begin{bmatrix} 1 & -1 & \vdots & 22 \\ 0 & 1 & \vdots & -\frac{13}{10} \\ 0 & 0 & \vdots & \frac{9}{5} \end{bmatrix}$$

The system is inconsistent and there is no solution.

66. $\begin{cases} x + 2y = 0 \\ x + y = 6 \\ 3x - 2y = 8 \end{cases}$

$$\begin{bmatrix} 1 & 2 & \vdots & 0 \\ 1 & 1 & \vdots & 6 \\ 3 & -2 & \vdots & 8 \end{bmatrix}$$

$$\begin{matrix} \\ -R_1 + R_2 \rightarrow \\ -3R_1 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 2 & \vdots & 0 \\ 0 & -1 & \vdots & 6 \\ 0 & -8 & \vdots & 8 \end{bmatrix}$$

$$\begin{matrix} \\ \\ -8R_2 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 2 & \vdots & 0 \\ 0 & -1 & \vdots & 6 \\ 0 & 0 & \vdots & -40 \end{bmatrix}$$

The system is inconsistent and there is no solution.

67. Use the reduced row-echelon form feature of a graphing utility.

$\begin{cases} 3x + 2y - z + w = 0 \\ x - y + 4z + 2w = 25 \\ -2x + y + 2z - w = 2 \\ x + y + z + w = 6 \end{cases}$

$$\begin{bmatrix} 3 & 2 & -1 & 1 & \vdots & 0 \\ 1 & -1 & 4 & 2 & \vdots & 25 \\ -2 & 1 & 2 & -1 & \vdots & 2 \\ 1 & 1 & 1 & 1 & \vdots & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & \vdots & 3 \\ 0 & 1 & 0 & 0 & \vdots & -2 \\ 0 & 0 & 1 & 0 & \vdots & 5 \\ 0 & 0 & 0 & 1 & \vdots & 0 \end{bmatrix}$$

$x = 3$

$y = -2$

$z = 5$

$w = 0$

Solution: $(3, -2, 5, 0)$

68. Use the reduced row-echelon form feature of a graphing utility.

$\begin{cases} x - 4y + 3z - 2w = 9 \\ 3x - 2y + z - 4w = -13 \\ -4x + 3y - 2z + w = -4 \\ -2x + y - 4z + 3w = -10 \end{cases}$

$$\begin{bmatrix} 1 & -4 & 3 & -2 & \vdots & 9 \\ 3 & -2 & 1 & -4 & \vdots & -13 \\ -4 & 3 & -2 & 1 & \vdots & -4 \\ -2 & 1 & -4 & 3 & \vdots & -10 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & \vdots & -1 \\ 0 & 1 & 0 & 0 & \vdots & 0 \\ 0 & 0 & 1 & 0 & \vdots & 6 \\ 0 & 0 & 0 & 1 & \vdots & 4 \end{bmatrix}$$

$x = -1$

$y = 0$

$z = 6$

$w = 4$

Solution: $(-1, 0, 6, 4)$

69. $\begin{bmatrix} 1 & 0 & \vdots & 3 \\ 0 & 1 & \vdots & -4 \end{bmatrix}$

$x = 3$

$y = -4$

Solution: $(3, -4)$

70. $\begin{bmatrix} 1 & 0 & 0 & \vdots & 5 \\ 0 & 1 & 0 & \vdots & -3 \\ 0 & 0 & 1 & \vdots & 0 \end{bmatrix}$

$x = 5$

$y = -3$

$z = 0$

Solution: $(5, -3, 0)$

71. $\begin{cases} -2x + 6y = -22 \\ x + 2y = -9 \end{cases}$

$$\begin{bmatrix} -2 & 6 & \vdots & -22 \\ 1 & 2 & \vdots & -9 \end{bmatrix}$$

$$\begin{matrix} R_1 \\ R_2 \end{matrix} \begin{bmatrix} 1 & 2 & \vdots & -9 \\ -2 & 6 & \vdots & -22 \end{bmatrix}$$

$$\begin{matrix} \\ 2R_1 + R_2 \rightarrow \end{matrix} \begin{bmatrix} 1 & 2 & \vdots & -9 \\ 0 & 10 & \vdots & -40 \end{bmatrix}$$

$$\begin{matrix} \\ \frac{1}{10}R_2 \rightarrow \end{matrix} \begin{bmatrix} 1 & 2 & \vdots & -9 \\ 0 & 1 & \vdots & -4 \end{bmatrix}$$

$\begin{cases} x + 2y = -9 \\ y = -4 \end{cases}$

$y = -4$

$x + 2(-4) = -9 \Rightarrow x = -1$

Solution: $(-1, -4)$

72. $\begin{cases} 5x - 5y = -5 \\ -2x - 3y = 7 \end{cases}$

$$\begin{bmatrix} 5 & -5 & \vdots & -5 \\ -2 & -3 & \vdots & 7 \end{bmatrix}$$

$$\tfrac{1}{5}R_1 \rightarrow \begin{bmatrix} 1 & -1 & \vdots & -1 \\ -2 & -3 & \vdots & 7 \end{bmatrix}$$

$$2R_1 + R_2 \rightarrow \begin{bmatrix} 1 & -1 & \vdots & -1 \\ 0 & -5 & \vdots & 5 \end{bmatrix}$$

$$-\tfrac{1}{5}R_2 \rightarrow \begin{bmatrix} 1 & -1 & \vdots & -1 \\ 0 & 1 & \vdots & -1 \end{bmatrix}$$

$\begin{cases} x - y = -1 \\ y = -1 \end{cases}$

$y = -1$

$x - (-1) = -1 \Rightarrow x = -2$

Solution: $(-2, -1)$

73. $\begin{cases} x + 2y + z = 8 \\ 3x + 7y + 6z = 26 \end{cases}$

$$\begin{bmatrix} 1 & 2 & 1 & \vdots & 8 \\ 3 & 7 & 6 & \vdots & 26 \end{bmatrix}$$

$$-3R_1 + R_2 \rightarrow \begin{bmatrix} 1 & 2 & 1 & \vdots & 8 \\ 0 & 1 & 3 & \vdots & 2 \end{bmatrix}$$

$\begin{cases} x + 2y + z = 8 \\ y + 3z = 2 \end{cases}$

Let $z = a$.

$y + 3a = 2 \Rightarrow y = -3a + 2$

$x + 2(-3a + 2) + a = 8 \Rightarrow x = 5a + 4$

Solution: $(5a + 4, -3a + 2, a)$ where a is a real number

74. $\begin{cases} x + y + 4z = 5 \\ 2x + y - z = 9 \end{cases}$

$$\begin{bmatrix} 1 & 1 & 4 & \vdots & 5 \\ 2 & 1 & -1 & \vdots & 9 \end{bmatrix}$$

$$-2R_1 + R_2 \rightarrow \begin{bmatrix} 1 & 1 & 4 & \vdots & 5 \\ 0 & -1 & -9 & \vdots & -1 \end{bmatrix}$$

$\begin{cases} x + y + 4z = 5 \\ -y - 9z = -1 \end{cases}$

Let $z = a$.

$-y - 9a = -1 \Rightarrow y = -9a + 1$

$x + (-9a + 1) + 4a = 5 \Rightarrow x = 5a + 4$

Solution: $(5a + 4, -9a + 1, a)$ where a is a real number

75. $\begin{cases} x - 3z = -2 \\ 3x + y - 2z = 5 \\ 2x + 2y + z = 4 \end{cases}$

$$\begin{bmatrix} 1 & 0 & -3 & \vdots & -2 \\ 3 & 1 & -2 & \vdots & 5 \\ 2 & 2 & 1 & \vdots & 4 \end{bmatrix}$$

$$\begin{matrix} -3R_1 + R_2 \rightarrow \\ -2R_1 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 0 & -3 & \vdots & -2 \\ 0 & 1 & 7 & \vdots & 11 \\ 0 & 2 & 7 & \vdots & 8 \end{bmatrix}$$

$$-2R_2 + R_3 \rightarrow \begin{bmatrix} 1 & 0 & -3 & \vdots & -2 \\ 0 & 1 & 7 & \vdots & 11 \\ 0 & 0 & -7 & \vdots & -14 \end{bmatrix}$$

$$-\tfrac{1}{7}R_3 \rightarrow \begin{bmatrix} 1 & 0 & -3 & \vdots & -2 \\ 0 & 1 & 7 & \vdots & 11 \\ 0 & 0 & 1 & \vdots & 2 \end{bmatrix}$$

$\begin{cases} x - 3z = -2 \\ y + 7z = 11 \\ z = 2 \end{cases}$

$z = 2$

$y + 7(2) = 11 \Rightarrow y = -3$

$x - 3(2) = -2 \Rightarrow x = 4$

Solution: $(4, -3, 2)$

76. $\begin{cases} 2x - y + 3z = 24 \\ 2y - z = 14 \\ 7x - 5y = 6 \end{cases}$

$$\begin{bmatrix} 2 & -1 & 3 & \vdots & 24 \\ 0 & 2 & -1 & \vdots & 14 \\ 7 & -5 & 0 & \vdots & 6 \end{bmatrix}$$

$$R_3 + (-3)R_1 \rightarrow \begin{bmatrix} 1 & -2 & -9 & \vdots & -66 \\ 0 & 2 & -1 & \vdots & 14 \\ 7 & -5 & 0 & \vdots & 6 \end{bmatrix}$$

$$-7R_1 + R_3 \rightarrow \begin{bmatrix} 1 & -2 & -9 & \vdots & -66 \\ 0 & 2 & -1 & \vdots & 14 \\ 0 & 9 & 63 & \vdots & 468 \end{bmatrix}$$

$$4R_2 \rightarrow \begin{bmatrix} 1 & -2 & -9 & \vdots & -66 \\ 0 & 8 & -4 & \vdots & 56 \\ 0 & 9 & 63 & \vdots & 468 \end{bmatrix}$$

$$-R_3 + R_2 \rightarrow \begin{bmatrix} 1 & -2 & -9 & \vdots & -66 \\ 0 & -1 & -67 & \vdots & -412 \\ 0 & 9 & 63 & \vdots & 468 \end{bmatrix}$$

$$9R_2 + R_3 \rightarrow \begin{bmatrix} 1 & -2 & -9 & \vdots & -66 \\ 0 & -1 & -67 & \vdots & -412 \\ 0 & 0 & -540 & \vdots & -3240 \end{bmatrix}$$

$$\begin{matrix} \\ -R_2 \rightarrow \\ -\frac{1}{540}R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & -2 & -9 & \vdots & -66 \\ 0 & 1 & 67 & \vdots & 412 \\ 0 & 0 & 1 & \vdots & 6 \end{bmatrix}$$

$\begin{cases} x - 2y - 9z = -66 \\ y + 67z = 412 \\ z = 6 \end{cases}$

$z = 6$

$y + 67(6) = 412 \Rightarrow y = 10$

$x - 2(10) - 9(6) = -66 \Rightarrow x = 8$

Solution: $(8, 10, 6)$

77. $\begin{cases} -x + y - z = -14 \\ 2x - y + z = 21 \\ 3x + 2y + z = 19 \end{cases}$

$$\begin{bmatrix} -1 & 1 & -1 & \vdots & -14 \\ 2 & -1 & 1 & \vdots & 21 \\ 3 & 2 & 1 & \vdots & 19 \end{bmatrix}$$

$$-R_1 \rightarrow \begin{bmatrix} 1 & -1 & 1 & \vdots & 14 \\ 2 & -1 & 1 & \vdots & 21 \\ 3 & 2 & 1 & \vdots & 19 \end{bmatrix}$$

$$\begin{matrix} \\ -2R_1 + R_2 \rightarrow \\ -3R_1 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & -1 & 1 & \vdots & 14 \\ 0 & 1 & -1 & \vdots & -7 \\ 0 & 5 & -2 & \vdots & -23 \end{bmatrix}$$

$$-5R_2 + R_3 \rightarrow \begin{bmatrix} 1 & -1 & 1 & \vdots & 14 \\ 0 & 1 & -1 & \vdots & -7 \\ 0 & 0 & 3 & \vdots & 12 \end{bmatrix}$$

$$\frac{1}{3}R_3 \rightarrow \begin{bmatrix} 1 & -1 & 1 & \vdots & 14 \\ 0 & 1 & -1 & \vdots & -7 \\ 0 & 0 & 1 & \vdots & 4 \end{bmatrix}$$

$\begin{cases} x - y + z = 14 \\ y - z = -7 \\ z = 4 \end{cases}$

$z = 4$

$y - 4 = -7 \Rightarrow y = -3$

$x - (-3) + 4 = 14 \Rightarrow x = 7$

Solution: $(7, -3, 4)$

78. $\begin{cases} 2x + 2y - z = 2 \\ x - 3y + z = -28 \\ -x + y = 14 \end{cases}$

$$\begin{bmatrix} 2 & 2 & -1 & \vdots & 2 \\ 1 & -3 & 1 & \vdots & -28 \\ -1 & 1 & 0 & \vdots & 14 \end{bmatrix}$$

$$\begin{matrix} R_2 \\ R_1 \\ \\ \end{matrix}\begin{bmatrix} 1 & -3 & 1 & \vdots & -28 \\ 2 & 2 & -1 & \vdots & 2 \\ -1 & 1 & 0 & \vdots & 14 \end{bmatrix}$$

$$\begin{matrix} \\ R_3 \\ R_2 \end{matrix}\begin{bmatrix} 1 & -3 & 1 & \vdots & -28 \\ -1 & 1 & 0 & \vdots & 14 \\ 2 & 2 & -1 & \vdots & 2 \end{bmatrix}$$

$$\begin{matrix} \\ R_1 + R_2 \rightarrow \\ -2R_1 + R_3 \rightarrow \end{matrix}\begin{bmatrix} 1 & -3 & 1 & \vdots & -28 \\ 0 & -2 & 1 & \vdots & -14 \\ 0 & 8 & -3 & \vdots & 58 \end{bmatrix}$$

$$\begin{matrix} \\ \\ 4R_2 + R_3 \rightarrow \end{matrix}\begin{bmatrix} 1 & -3 & 1 & \vdots & -28 \\ 0 & -2 & 1 & \vdots & -14 \\ 0 & 0 & 1 & \vdots & 2 \end{bmatrix}$$

$$\begin{matrix} \\ -\frac{1}{2}R_2 \rightarrow \\ \\ \end{matrix}\begin{bmatrix} 1 & -3 & 1 & \vdots & -28 \\ 0 & 1 & -\frac{1}{2} & \vdots & 7 \\ 0 & 0 & 1 & \vdots & 2 \end{bmatrix}$$

$\begin{cases} x - 3y + z = -28 \\ y - \frac{1}{2}z = 7 \\ z = 2 \end{cases}$

$z = 2$

$y - \frac{1}{2}(2) = 7 \Rightarrow y = 8$

$x - 3(8) + 2 = -28 \Rightarrow x = -6$

Solution: $(-6, 8, 2)$

79. Use the reduced row-echelon form feature of a graphic utility.

$\begin{cases} 3x + 3y + 12z = 6 \\ x + y + 4z = 2 \\ 2x + 5y + 20z = 10 \\ -x + 2y + 8z = 4 \end{cases}$ $\begin{bmatrix} 3 & 3 & 12 & \vdots & 6 \\ 1 & 1 & 4 & \vdots & 2 \\ 2 & 5 & 20 & \vdots & 10 \\ -1 & 2 & 8 & \vdots & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & \vdots & 0 \\ 0 & 1 & 4 & \vdots & 2 \\ 0 & 0 & 0 & \vdots & 0 \\ 0 & 0 & 0 & \vdots & 0 \end{bmatrix} \Rightarrow \begin{cases} x = 0 \\ y + 4z = 2 \end{cases}$

Let $z = a$.

$y = 2 - 4a$

$x = 0$

Solution: $(0, 2 - 4a, a)$ where a is any real number

80. Use the reduced row-echelon form feature of a graphing utility.

$$\begin{cases} 2x + 10y + 2z = 6 \\ x + 5y + 2z = 6 \\ x + 5y + z = 3 \\ -3x - 15y - 3z = -9 \end{cases}$$

$$\begin{bmatrix} 2 & 10 & 2 & \vdots & 6 \\ 1 & 5 & 2 & \vdots & 6 \\ 1 & 5 & 1 & \vdots & 3 \\ -3 & -15 & -3 & \vdots & -9 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 5 & 0 & \vdots & 0 \\ 0 & 0 & 1 & \vdots & 3 \\ 0 & 0 & 0 & \vdots & 0 \\ 0 & 0 & 0 & \vdots & 0 \end{bmatrix}$$

$$\begin{cases} z = 3 \\ x + 5y = 0 \end{cases}$$

$z = 3$

$y = a$

$x + 5a = 0 \Rightarrow x = -5a$

Solution: $(-5a, a, 3)$ where a is a real number

81. Use the reduced row-echelon form feature of a graphing utility.

$$\begin{cases} 2x + y - z + 2w = -6 \\ 3x + 4y + w = 1 \\ x + 5y + 2z + 6w = -3 \\ 5x + 2y - z - w = 3 \end{cases}$$

$$\begin{bmatrix} 2 & 1 & -1 & 2 & \vdots & -6 \\ 3 & 4 & 0 & 1 & \vdots & 1 \\ 1 & 5 & 2 & 6 & \vdots & -3 \\ 5 & 2 & -1 & -1 & \vdots & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & \vdots & 1 \\ 0 & 1 & 0 & 0 & \vdots & 0 \\ 0 & 0 & 1 & 0 & \vdots & 4 \\ 0 & 0 & 0 & 1 & \vdots & -2 \end{bmatrix}$$

$x = 1$

$y = 0$

$z = 4$

$w = -2$

Solution: $(1, 0, 4, -2)$

82. Use the reduced row-echelon form feature of a graphing utility.

$$\begin{cases} x + 2y + 2z + 4w = 11 \\ 3x + 6y + 5z + 12w = 30 \\ x + 3y - 3z + 2w = -5 \\ 6x - y - z + w = -9 \end{cases}$$

$$\begin{bmatrix} 1 & 2 & 2 & 4 & \vdots & 11 \\ 3 & 6 & 5 & 12 & \vdots & 30 \\ 1 & 3 & -3 & 2 & \vdots & -5 \\ 6 & -1 & -1 & 1 & \vdots & -9 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & \vdots & -1 \\ 0 & 1 & 0 & 0 & \vdots & 1 \\ 0 & 0 & 1 & 0 & \vdots & 3 \\ 0 & 0 & 0 & 1 & \vdots & 1 \end{bmatrix}$$

$x = -1$

$y = 1$

$z = 3$

$w = 1$

Solution: $(-1, 1, 3, 1)$

83. Use the reduced row-echelon form feature of a graphing utility.

$$\begin{cases} x + y + z + w = 0 \\ 2x + 3y + z - 2w = 0 \\ 3x + 5y + z = 0 \end{cases}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & \vdots & 0 \\ 2 & 3 & 1 & -2 & \vdots & 0 \\ 3 & 5 & 1 & 0 & \vdots & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 2 & 0 & \vdots & 0 \\ 0 & 1 & -1 & 0 & \vdots & 0 \\ 0 & 0 & 0 & 1 & \vdots & 0 \end{bmatrix}$$

$$\begin{cases} x + 2z = 0 \\ y - z = 0 \\ w = 0 \end{cases}$$

Let $z = a$. Then $x = -2a$ and $y = a$.

Solution: $(-2a, a, a, 0)$ where a is a real number

84. $$\begin{cases} x + 2y + z + 3w = 0 \\ x - y + w = 0 \\ y - z + 2w = 0 \end{cases}$$

$$\begin{bmatrix} 1 & 2 & 1 & 3 & \vdots & 0 \\ 1 & -1 & 0 & 1 & \vdots & 0 \\ 0 & 1 & -1 & 2 & \vdots & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 2 & \vdots & 0 \\ 0 & 1 & 0 & 1 & \vdots & 0 \\ 0 & 0 & 1 & -1 & \vdots & 0 \end{bmatrix}$$

$$\begin{cases} x + 2w = 0 \\ y + w = 0 \\ z - w = 0 \end{cases}$$

Let $w = a$. Then $z = a$, $y = -a$, and $x = -2a$.

Solution: $(-2a, -a, a, a)$ where a is a real number

85. The dimension of the matrix is 4×1.

86. The matrix is in row-echelon form, not reduced row-echelon form.

87. $\begin{cases} a + b + c = 1 \\ 4a + 2b + c = -1 \\ 9a + 3b + c = -5 \end{cases}$

$\begin{bmatrix} 1 & 1 & 1 & \vdots & 1 \\ 4 & 2 & 1 & \vdots & -1 \\ 9 & 3 & 1 & \vdots & -5 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & \vdots & -1 \\ 0 & 1 & 0 & \vdots & 1 \\ 0 & 0 & 1 & \vdots & 1 \end{bmatrix}$

$a = 1$

$b = 1$

$c = 1$

So, $f(x) = -x^2 + x + 1$.

88. $\begin{cases} a + b + c = 2 \\ 4a + 2b + c = 9 \\ 9a + 3b + c = 20 \end{cases}$

$\begin{bmatrix} 1 & 1 & 1 & \vdots & 2 \\ 4 & 2 & 1 & \vdots & 9 \\ 9 & 3 & 1 & \vdots & 20 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & \vdots & 2 \\ 0 & 1 & 0 & \vdots & 1 \\ 0 & 0 & 1 & \vdots & -1 \end{bmatrix}$

$a = 2$

$b = 1$

$c = -1$

So, $f(x) = 2x^2 + x - 1$.

89. $\begin{cases} 4a - 2b + c = -15 \\ a - b + c = 7 \\ a + b + c = -3 \end{cases}$

$\begin{bmatrix} 4 & -2 & 1 & \vdots & -15 \\ 4 & -1 & 1 & \vdots & 7 \\ 1 & 1 & 1 & \vdots & -3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & \vdots & -9 \\ 0 & 1 & 0 & \vdots & -5 \\ 0 & 0 & 1 & \vdots & 11 \end{bmatrix}$

$a = -9$

$b = -5$

$c = 11$

So, $f(x) = -9x^2 - 5x + 11$.

90. $\begin{cases} 4a - 2b + c = -3 \\ a + b + c = -3 \\ 4a + 2b + c = -11 \end{cases}$

$\begin{bmatrix} 4 & -2 & 1 & \vdots & -3 \\ 1 & 1 & 1 & \vdots & -3 \\ 4 & 2 & 1 & \vdots & -11 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & \vdots & -2 \\ 0 & 1 & 0 & \vdots & -2 \\ 0 & 0 & 1 & \vdots & 1 \end{bmatrix}$

$a = -2$

$b = -2$

$c = 1$

So, $f(x) = -2x^2 - 2x + 1$.

91. $\begin{cases} a + b + c = 8 \\ 4a + 2b + c = 13 \\ 9a + 3b + c = 20 \end{cases}$

$\begin{bmatrix} 1 & 1 & 1 & \vdots & 8 \\ 4 & 2 & 1 & \vdots & 13 \\ 9 & 3 & 1 & \vdots & 20 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & \vdots & 1 \\ 0 & 1 & 0 & \vdots & 2 \\ 0 & 0 & 1 & \vdots & 5 \end{bmatrix}$

$a = 1$

$b = 2$

$c = 5$

So, $f(x) = x^2 + 2x + 5$.

92. $\begin{cases} a + b + c = 9 \\ 4a + 2b + c = 8 \\ 9a + 3b + c = 5 \end{cases}$

$\begin{bmatrix} 1 & 1 & 1 & \vdots & 9 \\ 4 & 2 & 1 & \vdots & 8 \\ 9 & 3 & 1 & \vdots & 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & \vdots & -1 \\ 0 & 1 & 0 & \vdots & 2 \\ 0 & 0 & 1 & \vdots & 8 \end{bmatrix}$

$a = 1$

$b = 2$

$c = 8$

So, $f(x) = -x^2 + 2x + 8$.

93. $\begin{cases} 12b + 66a = 831 \\ 66b + 506a = 5643 \end{cases}$

$\begin{bmatrix} 12 & 66 & \vdots & 831 \\ 66 & 506 & \vdots & 5643 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & \vdots & 28 \\ 0 & 1 & \vdots & 7.5 \end{bmatrix}$

$y = 7.5t + 28$

In 2020, the number of new cases of the waterborne disease will be $y = 7.5(15) + 28 = 140.5 \approx 141$ cases.

Because the data values increase in a linear pattern, this prediction is reasonable.

94. x = amount at 7%

y = amount at 8.5%

z = amount at 9.5%

$$\begin{cases} x + y + z = 2{,}000{,}000 \\ 0.07x + 0.085y + 0.095z = 169{,}750 \\ y - 4z = 0 \end{cases}$$

$$\begin{bmatrix} 1 & 1 & 1 & \vdots & 2{,}000{,}000 \\ 0.07 & 0.085 & 0.095 & \vdots & 169{,}750 \\ 0 & 1 & -4 & \vdots & 0 \end{bmatrix}$$

$$-0.07R_1 + R_2 \begin{bmatrix} 1 & 1 & 1 & \vdots & 2{,}000{,}000 \\ 0 & 0.015 & 0.025 & \vdots & 29{,}750 \\ 0 & 1 & -4 & \vdots & 0 \end{bmatrix}$$

$$1000R_2 \rightarrow \begin{bmatrix} 1 & 1 & 1 & \vdots & 2{,}000{,}000 \\ 0 & 15 & 25 & \vdots & 27{,}750{,}000 \\ 0 & 1 & -4 & \vdots & 0 \end{bmatrix}$$

$$R_2 + (-15)R_3 \rightarrow \begin{bmatrix} 1 & 1 & 1 & \vdots & 2{,}000{,}000 \\ 0 & 15 & 25 & \vdots & 29{,}750{,}000 \\ 0 & 0 & 85 & \vdots & 29{,}750{,}000 \end{bmatrix}$$

$$\begin{matrix} \\ \frac{1}{15}R_2 \rightarrow \\ \frac{1}{85}R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 1 & 1 & \vdots & 2{,}000{,}000 \\ 0 & 1 & \frac{5}{3} & \vdots & \frac{5{,}950{,}000}{3} \\ 0 & 0 & -1 & \vdots & 350{,}000 \end{bmatrix}$$

The matrix is now in row-echelon form, and the corresponding system is shown.

$$\begin{cases} x + y + z = 2{,}000{,}000 \\ y + \frac{5}{3}z = \frac{5{,}950{,}000}{3} \\ z = 350{,}000 \end{cases}$$

$$y + \frac{5}{3}(350{,}000) = \frac{5{,}950{,}000}{3}$$
$$y = 1{,}400{,}000$$
$$x + 1{,}400{,}000 + 350{,}000 = 2{,}000{,}000$$
$$x = 250{,}000$$

The natural history museum borrowed \$250,000 at 7%, \$1,400,000 at 8.5%, and \$350,000 at 9.5%.

95. x = amount at 8%

y = amount at 9%

z = amount at 12%

$$\begin{cases} x + y + z = 2{,}000{,}000 \\ 0.08x + 0.09y + 0.12z = 186{,}000 \\ x - 2z = 0 \end{cases}$$

$$\begin{bmatrix} 1 & 1 & 1 & \vdots & 2{,}000{,}000 \\ 0.08 & 0.09 & 0.12 & \vdots & 186{,}000 \\ 1 & 0 & -2 & \vdots & 0 \end{bmatrix}$$

$$\begin{matrix} \\ -0.08R_1 + R_2 \\ -R_1 + R_3 \end{matrix} \begin{bmatrix} 1 & 1 & 1 & \vdots & 2{,}000{,}000 \\ 0 & 0.01 & 0.04 & \vdots & 26{,}000 \\ 0 & -1 & -3 & \vdots & -2{,}000{,}000 \end{bmatrix}$$

$$100R_2 \rightarrow \begin{bmatrix} 1 & 1 & 1 & \vdots & 2{,}000{,}000 \\ 0 & 1 & 4 & \vdots & 2{,}600{,}000 \\ 0 & -1 & -3 & \vdots & -2{,}000{,}000 \end{bmatrix}$$

$$\begin{matrix} \\ \\ R_2 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 1 & 1 & \vdots & 2{,}000{,}000 \\ 0 & 1 & 4 & \vdots & 2{,}600{,}000 \\ 0 & 0 & 1 & \vdots & 600{,}000 \end{bmatrix}$$

The matrix is now in row-echelon form, and the corresponding system is shown.

$$\begin{cases} x + y + z = 2{,}000{,}000 \\ y + 4z = 2{,}600{,}000 \\ z = 600{,}000 \end{cases}$$

Using back-substitution, you can determine the solution.

$$y + 4(600{,}000) = 2{,}600{,}000$$
$$y = 200{,}000$$
$$x + 200{,}000 + 600{,}000 = 2{,}000{,}000$$
$$x = 1{,}200{,}000$$

So, the zoo borrowed 1,200,000 at 8%, \$200,000 at 9%, and \$600,000 at 12%.

96. (a) $(0, 5.0), (15, 9.6), (30, 12.4)$

$y = ax^2 + bx + c$

$$\begin{cases} c = 5 \\ 225a + 15b + c = 9.6 \Rightarrow 225a + 15b = 4.6 \\ 900a + 30b + c = 12.4 \Rightarrow 900a + 30b = 7.4 \end{cases}$$

$$\begin{bmatrix} 225 & 15 & \vdots & 4.6 \\ 900 & 30 & \vdots & 7.4 \end{bmatrix}$$

$$-4R_1 + R_2 \rightarrow \begin{bmatrix} 225 & 15 & \vdots & 4.6 \\ 0 & -30 & \vdots & -11 \end{bmatrix}$$

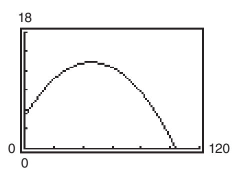
$$\begin{matrix} \frac{1}{225}R_1 \rightarrow \\ \left(-\frac{1}{30}\right)R_2 \rightarrow \end{matrix} \begin{bmatrix} 1 & \frac{1}{15} & \vdots & \frac{23}{1125} \\ 0 & 1 & \vdots & \frac{11}{30} \end{bmatrix}$$

$$\begin{cases} a + \frac{1}{15}b = \frac{23}{1125} \\ b = \frac{11}{30} \end{cases}$$

$$a + \frac{1}{15}\left(\frac{11}{30}\right) = \frac{23}{1125} \Rightarrow a = -\frac{1}{250} = -0.004$$

Equation of parabola: $y = -0.004x^2 + 0.367x + 5$

(b)



(c) The maximum height is approximately 13 feet and the ball strikes the ground at approximately 104 feet.

(d) The maximum height occurs at the vertex.

$$x = -\frac{b}{2a} = \frac{-0.367}{2(-0.004)} = 45.875$$

$$y = -0.004(45.875)^2 + 0.367(45.875) + 5$$
$$= 13.418 \text{ feet}$$

The ball strikes the ground when $y = 0$.

$$-0.004x^2 + 0.367x + 5 = 0$$

By the Quadratic Formula and using the positive value for x, you have $x \approx 103.793$ feet.

(e) The values found in part (d) are more accurate, but still very close to the estimates found in part (c).

97. False. It is a 2×4 matrix.

98. False. Gaussian elimination reduces a matrix until a row-echelon form is obtained and Gauss-Jordan elimination reduces a matrix until a reduced row-echelon form is obtained.

99. They are the same.

100. $\begin{bmatrix} 1 & b \\ c & 1 \end{bmatrix}$

(a) If $b = 0$ and $c = 0$, then $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. So, the matrix is in reduced row-echelon form.

(b) If $b \neq 0$ and $c = 0$, then $\begin{bmatrix} 1 & \text{non-zero} \\ 0 & 1 \end{bmatrix}$. So, the matrix is in row-echelon form.

(c) If $b = 0$ and $c \neq 0$, then $\begin{bmatrix} 1 & 0 \\ \text{non-zero} & 1 \end{bmatrix}$. So, the matrix is neither in row-echelon nor reduced row-echelon form.

(d) If $b \neq 0$ and $c \neq 0$, then $\begin{bmatrix} 1 & \text{non-zero} \\ \text{non-zero} & 1 \end{bmatrix}$. So, the matrix is in neither row-echelon nor reduced row-echelon form.

Section 8.2 Operations with Matrices

1. equal

2. scalars

3. zero; O

4. identity

5. $\begin{bmatrix} x & -2 \\ 7 & 23 \end{bmatrix} = \begin{bmatrix} -4 & -2 \\ 7 & y \end{bmatrix}$

$x = -4$
$y = 23$

6. $\begin{bmatrix} -5x & x \\ 3y & 8 \end{bmatrix} = \begin{bmatrix} -5 & 13 \\ 12 & 8 \end{bmatrix}$

$x = 13$

$3y = 12 \Rightarrow y = 4$

7. $\begin{bmatrix} 16 & 4 & x & 4 \\ 0 & 2 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 16 & 4 & 2x+1 & 4 \\ 0 & 2 & 3y-5 & 0 \end{bmatrix}$

$x = 2x + 1 \Rightarrow x = -1$

$4 = 3y - 5 \Rightarrow y = 3$

8. $\begin{bmatrix} x+2 & 8 & -3 \\ 1 & 18 & -8 \\ 7 & -2 & y+2 \end{bmatrix} = \begin{bmatrix} 2x+6 & 8 & -3 \\ 1 & 18 & -8 \\ 7 & -2 & x \end{bmatrix}$

$x + 2 = 2x + 6 \Rightarrow x = -4$

$y + 2 = x \Rightarrow y = -6$

9. (a) $A + B = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ -1 & 8 \end{bmatrix} = \begin{bmatrix} 1+2 & -1-1 \\ 2-1 & -1+8 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 1 & 7 \end{bmatrix}$

(b) $A - B = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ -1 & 8 \end{bmatrix} = \begin{bmatrix} 1-2 & -1+1 \\ 2+1 & -1-8 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 3 & -9 \end{bmatrix}$

(c) $3A = 3\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 3(1) & 3(-1) \\ 3(2) & 3(-1) \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ 6 & -3 \end{bmatrix}$

(d) $3A - 2B = \begin{bmatrix} 3 & -3 \\ 6 & -3 \end{bmatrix} - 2\begin{bmatrix} 2 & -1 \\ -1 & 8 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ 6 & -3 \end{bmatrix} + \begin{bmatrix} -4 & 2 \\ 2 & -16 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 8 & -19 \end{bmatrix}$

10. (a) $A + B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} -3 & -2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1-3 & 2-2 \\ 2+4 & 1+2 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 6 & 3 \end{bmatrix}$

(b) $A - B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} -3 & -2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1+3 & 2+2 \\ 2-4 & 1-2 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ -2 & -1 \end{bmatrix}$

(c) $3A = 3\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 3(1) & 3(2) \\ 3(2) & 3(1) \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 6 & 3 \end{bmatrix}$

(d) $3A - 2B = \begin{bmatrix} 3 & 6 \\ 6 & 3 \end{bmatrix} - 2\begin{bmatrix} -3 & -2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 3+6 & 6+4 \\ 6-8 & 3-4 \end{bmatrix} = \begin{bmatrix} 9 & 10 \\ -2 & -1 \end{bmatrix}$

11. $A = \begin{bmatrix} 6 & 0 & 3 \\ -1 & -4 & 0 \end{bmatrix}, B = \begin{bmatrix} 8 & -1 \\ 4 & -3 \end{bmatrix}$

(a) $A + B$ is not possible. A and B do not have the same order.

(b) $A - B$ is not possible. A and B do not have the same order.

(c) $3A = \begin{bmatrix} 18 & 0 & 9 \\ -3 & -12 & 0 \end{bmatrix}$

(d) $3A - 2B$ is not possible. A and B do not have the same order.

12. (a) $A + B$ is not possible. A and B do not have the same order.

(b) $A - B$ is not possible. A and B do not have the same order.

(c) $3A = 3\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ -3 \end{bmatrix}$

(d) $3A - 2B$ is not possible. A and B do not have the same order.

13. $A = \begin{bmatrix} 8 & -1 \\ 2 & 3 \\ -4 & 5 \end{bmatrix}, B = \begin{bmatrix} 1 & 6 \\ -1 & -5 \\ 1 & 10 \end{bmatrix}$

(a) $A + B = \begin{bmatrix} 8 & -1 \\ 2 & 3 \\ -4 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ -1 & -5 \\ 1 & 10 \end{bmatrix} = \begin{bmatrix} 8+1 & -1+6 \\ 2-1 & 3-5 \\ -4+1 & 5+10 \end{bmatrix} = \begin{bmatrix} 9 & 5 \\ 1 & -2 \\ -3 & 15 \end{bmatrix}$

(b) $A - B = \begin{bmatrix} 8 & -1 \\ 2 & 3 \\ -4 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 6 \\ -1 & -5 \\ 1 & 10 \end{bmatrix} = \begin{bmatrix} 8-1 & -1-6 \\ 2-(-1) & 3-(-5) \\ -4-1 & 5-10 \end{bmatrix} = \begin{bmatrix} 7 & -7 \\ 3 & 8 \\ -5 & -5 \end{bmatrix}$

(c) $3A = 3\begin{bmatrix} 8 & -1 \\ 2 & 3 \\ -4 & 5 \end{bmatrix} = \begin{bmatrix} 3(8) & 3(-1) \\ 3(2) & 3(3) \\ 3(-4) & 3(5) \end{bmatrix} = \begin{bmatrix} 24 & -3 \\ 6 & 9 \\ -12 & 15 \end{bmatrix}$

(d) $3A - 2B = \begin{bmatrix} 24 & -3 \\ 6 & 9 \\ -12 & 15 \end{bmatrix} - 2\begin{bmatrix} 1 & 6 \\ -1 & -5 \\ 1 & 10 \end{bmatrix} = \begin{bmatrix} 24-2 & -3-12 \\ 6+2 & 9+10 \\ -12-2 & 15-20 \end{bmatrix} = \begin{bmatrix} 22 & -15 \\ 8 & 19 \\ -14 & -5 \end{bmatrix}$

14. $A = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 6 & 9 \end{bmatrix}, B = \begin{bmatrix} -2 & 0 & -5 \\ -3 & 4 & -7 \end{bmatrix}$

(a) $A + B = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 6 & 9 \end{bmatrix} + \begin{bmatrix} -2 & 0 & -5 \\ -3 & 4 & -7 \end{bmatrix} = \begin{bmatrix} 1-2 & -1+0 & 3-5 \\ 0-3 & 6+4 & 9-7 \end{bmatrix} = \begin{bmatrix} -1 & -1 & -2 \\ -3 & 10 & 2 \end{bmatrix}$

(b) $A - B = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 6 & 9 \end{bmatrix} - \begin{bmatrix} -2 & 0 & -5 \\ -3 & 4 & -7 \end{bmatrix} = \begin{bmatrix} 1-(-2) & -1-0 & 3-(-5) \\ 0-(-3) & 6-4 & 9-(-7) \end{bmatrix} = \begin{bmatrix} 3 & -1 & 8 \\ 3 & 2 & 16 \end{bmatrix}$

(c) $3A = 3\begin{bmatrix} 1 & -1 & 3 \\ 0 & 6 & 9 \end{bmatrix} = \begin{bmatrix} 3(1) & 3(-1) & 3(3) \\ 3(0) & 3(6) & 3(9) \end{bmatrix} = \begin{bmatrix} 3 & -3 & 9 \\ 0 & 18 & 27 \end{bmatrix}$

(d) $3A - 2B = \begin{bmatrix} 3 & -3 & 9 \\ 0 & 18 & 27 \end{bmatrix} - 2\begin{bmatrix} -2 & 0 & -5 \\ -3 & 4 & -7 \end{bmatrix} = \begin{bmatrix} 3+4 & -3-0 & 9+10 \\ 0+6 & 18-8 & 27+14 \end{bmatrix} = \begin{bmatrix} 7 & -3 & 19 \\ 6 & 10 & 41 \end{bmatrix}$

15. $A = \begin{bmatrix} 4 & 5 & -1 & 3 & 4 \\ 1 & 2 & -2 & -1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & -1 & 1 & 0 \\ -6 & 8 & 2 & -3 & -7 \end{bmatrix}$

(a) $A + B = \begin{bmatrix} 4 & 5 & -1 & 3 & 4 \\ 1 & 2 & -2 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -1 & 1 & 0 \\ -6 & 8 & 2 & -3 & -7 \end{bmatrix} = \begin{bmatrix} 4+1 & 5+0 & -1-1 & 3+1 & 4+0 \\ 1-6 & 2+8 & -2+2 & -1-3 & 0-7 \end{bmatrix}$

$= \begin{bmatrix} 5 & 5 & -2 & 4 & 4 \\ -5 & 10 & 0 & -4 & -7 \end{bmatrix}$

(b) $A - B = \begin{bmatrix} 4 & 5 & -1 & 3 & 4 \\ 1 & 2 & -2 & -1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -1 & 1 & 0 \\ -6 & 8 & 2 & -3 & -7 \end{bmatrix} = \begin{bmatrix} 4-1 & 5-0 & -1-(-1) & 3-1 & 4-0 \\ 1-(-6) & 2-8 & -2-2 & -1-(-3) & 0-(-7) \end{bmatrix}$

$= \begin{bmatrix} 3 & 5 & 0 & 2 & 4 \\ 7 & -6 & -4 & 2 & 7 \end{bmatrix}$

(c) $3A = 3\begin{bmatrix} 4 & 5 & -1 & 3 & 4 \\ 1 & 2 & -2 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 3(4) & 3(5) & 3(-1) & 3(3) & 3(4) \\ 3(1) & 3(2) & 3(-2) & 3(-1) & 3(0) \end{bmatrix} = \begin{bmatrix} 12 & 15 & -3 & 9 & 12 \\ 3 & 6 & -6 & -3 & 0 \end{bmatrix}$

(d) $3A - 2B = \begin{bmatrix} 12 & 15 & -3 & 9 & 12 \\ 3 & 6 & -6 & -3 & 0 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 & -1 & 1 & 0 \\ -6 & 8 & 2 & -3 & -7 \end{bmatrix} = \begin{bmatrix} 12-2 & 15+0 & -3+2 & 9-2 & 12-0 \\ 3+12 & 6-16 & -6-4 & -3+6 & 0+14 \end{bmatrix}$

$= \begin{bmatrix} 10 & 15 & -1 & 7 & 12 \\ 15 & -10 & -10 & 3 & 14 \end{bmatrix}$

16. (a) $A + B = \begin{bmatrix} -1 & 4 & 0 \\ 3 & -2 & 2 \\ 5 & 4 & -1 \\ 0 & 8 & -6 \\ -4 & -1 & 0 \end{bmatrix} + \begin{bmatrix} -3 & 5 & 1 \\ 2 & -4 & -7 \\ 10 & -9 & -1 \\ 3 & 2 & -4 \\ 0 & 1 & -2 \end{bmatrix} = \begin{bmatrix} -1-3 & 4+5 & 0+1 \\ 3+2 & -2-4 & 2-7 \\ 5+10 & 4-9 & -1-1 \\ 0+3 & 8+2 & -6-4 \\ -4+0 & -1+1 & 0-2 \end{bmatrix} = \begin{bmatrix} -4 & 9 & 1 \\ 5 & -6 & -5 \\ 15 & -5 & -2 \\ 3 & 10 & -10 \\ -4 & 0 & -2 \end{bmatrix}$

(b) $A - B = \begin{bmatrix} -1 & 4 & 0 \\ 3 & -2 & 2 \\ 5 & 4 & -1 \\ 0 & 8 & -6 \\ -4 & -1 & 0 \end{bmatrix} - \begin{bmatrix} -3 & 5 & 1 \\ 2 & -4 & -7 \\ 10 & -9 & -1 \\ 3 & 2 & -4 \\ 0 & 1 & -2 \end{bmatrix} = \begin{bmatrix} -1+3 & 4-5 & 0-1 \\ 3-2 & -2+4 & 2+7 \\ 5-10 & 4+9 & -1+1 \\ 0-3 & 8-2 & -6+4 \\ -4-0 & -1-1 & 0+2 \end{bmatrix} = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 2 & 9 \\ -5 & 13 & 0 \\ -3 & 6 & -2 \\ -4 & -2 & 2 \end{bmatrix}$

(c) $3A = 3\begin{bmatrix} -1 & 4 & 0 \\ 3 & -2 & 2 \\ 5 & 4 & -1 \\ 0 & 8 & -6 \\ -4 & -1 & 0 \end{bmatrix} = \begin{bmatrix} -3 & 12 & 0 \\ 9 & -6 & 6 \\ 15 & 12 & -3 \\ 0 & 24 & -18 \\ -12 & -3 & 0 \end{bmatrix}$

(d) $3A - 2B = \begin{bmatrix} -3 & 12 & 0 \\ 9 & -6 & 6 \\ 15 & 12 & -3 \\ 0 & 24 & -18 \\ -12 & -3 & 0 \end{bmatrix} - 2\begin{bmatrix} -3 & 5 & 1 \\ 2 & -4 & -7 \\ 10 & -9 & -1 \\ 3 & 2 & -4 \\ 0 & 1 & -2 \end{bmatrix} = \begin{bmatrix} -3 & 12 & 0 \\ 9 & -6 & 6 \\ 15 & 12 & -3 \\ 0 & 24 & -18 \\ -12 & -3 & 0 \end{bmatrix} + \begin{bmatrix} 6 & -10 & -2 \\ -4 & 8 & 14 \\ -20 & 18 & 2 \\ -6 & -4 & 8 \\ 0 & -2 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 2 & -2 \\ 5 & 2 & 20 \\ -5 & 30 & -1 \\ -6 & 20 & -10 \\ -12 & -5 & 4 \end{bmatrix}$

17. $\begin{bmatrix} -5 & 0 \\ 3 & -6 \end{bmatrix} + \begin{bmatrix} 7 & 1 \\ -2 & -1 \end{bmatrix} + \begin{bmatrix} -10 & -8 \\ 14 & 6 \end{bmatrix} = \begin{bmatrix} -5+7+(-10) & 0+1+(-8) \\ 3+(-2)+14 & -6+(-1)+6 \end{bmatrix} = \begin{bmatrix} -8 & -7 \\ 15 & -1 \end{bmatrix}$

18. $\begin{bmatrix} 6 & 8 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 5 \\ -3 & -1 \end{bmatrix} + \begin{bmatrix} -11 & -7 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 6+0+(-11) & 8+5+(-7) \\ -1+(-3)+2 & 0+(-1)+(-1) \end{bmatrix} = \begin{bmatrix} -5 & 6 \\ -2 & -2 \end{bmatrix}$

19. $4\left(\begin{bmatrix} -4 & 0 & 1 \\ 0 & 2 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 1 & -2 \\ 3 & -6 & 0 \end{bmatrix}\right) = 4\begin{bmatrix} -6 & -1 & 3 \\ -3 & 8 & 3 \end{bmatrix} = \begin{bmatrix} -24 & -4 & 12 \\ -12 & 32 & 12 \end{bmatrix}$

20. $\frac{1}{2}\left(\begin{bmatrix} 5 & -2 & 4 & 0 \end{bmatrix} + \begin{bmatrix} 14 & 6 & -18 & 9 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 5+14 & -2+6 & 4+(-18) & 0+9 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 19 & 4 & -14 & 9 \end{bmatrix} = \begin{bmatrix} \frac{19}{2} & 2 & -7 & \frac{9}{2} \end{bmatrix}$

21. $-3\left(\begin{bmatrix} 0 & -3 \\ 7 & 2 \end{bmatrix} + \begin{bmatrix} -6 & 3 \\ 8 & 1 \end{bmatrix}\right) - 2\begin{bmatrix} 4 & -4 \\ 7 & -9 \end{bmatrix} = -3\begin{bmatrix} -6 & 0 \\ 15 & 3 \end{bmatrix} - \begin{bmatrix} 8 & -8 \\ 14 & -18 \end{bmatrix} = \begin{bmatrix} 18 & 0 \\ -45 & -9 \end{bmatrix} - \begin{bmatrix} 8 & -8 \\ 14 & -18 \end{bmatrix} = \begin{bmatrix} 10 & 8 \\ -59 & 9 \end{bmatrix}$

22. $-1\begin{bmatrix} 4 & 11 \\ -2 & -1 \\ 9 & 3 \end{bmatrix} + \frac{1}{6}\left(\begin{bmatrix} -5 & -1 \\ 3 & 4 \\ 0 & 13 \end{bmatrix} + \begin{bmatrix} 7 & 5 \\ -9 & -1 \\ 6 & -1 \end{bmatrix}\right) = \begin{bmatrix} -4 & -11 \\ 2 & 1 \\ -9 & -3 \end{bmatrix} + \frac{1}{6}\begin{bmatrix} -5+7 & -1+5 \\ 3+(-9) & 4+(-1) \\ 0+6 & 13+(-1) \end{bmatrix}$

$$= \begin{bmatrix} -4 & -11 \\ 2 & 1 \\ -9 & -3 \end{bmatrix} + \frac{1}{6}\begin{bmatrix} 2 & 4 \\ -6 & 3 \\ 6 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & -11 \\ 2 & 1 \\ -9 & -3 \end{bmatrix} + \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ -1 & \frac{1}{2} \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -4+\frac{1}{3} & -11+\frac{2}{3} \\ 2+(-1) & 1+\frac{1}{2} \\ -9+1 & -3+2 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{11}{3} & -\frac{31}{3} \\ 1 & \frac{3}{2} \\ -8 & -1 \end{bmatrix}$$

23. $\frac{11}{25}\begin{bmatrix} 2 & 5 \\ -1 & -4 \end{bmatrix} + 6\begin{bmatrix} -3 & 0 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} -17.12 & 2.2 \\ 11.56 & 10.24 \end{bmatrix}$

24. $55\left(\begin{bmatrix} 14 & -11 \\ -22 & 19 \end{bmatrix} - \begin{bmatrix} -8 & 20 \\ 13 & 6 \end{bmatrix}\right) = \begin{bmatrix} 1210 & -1705 \\ -1925 & 715 \end{bmatrix}$

25. $-2\begin{bmatrix} 1.23 & 4.19 & -3.85 \\ 7.21 & -2.60 & 6.54 \end{bmatrix} - \begin{bmatrix} 8.35 & -3.02 & 7.30 \\ -0.38 & -5.49 & 1.68 \end{bmatrix} = \begin{bmatrix} -10.81 & -5.39 & 0.4 \\ -14.04 & 10.69 & -14.76 \end{bmatrix}$

26. $-\begin{bmatrix} 10 & 15 \\ -20 & 10 \\ 12 & 4 \end{bmatrix} + \frac{1}{8}\left(\begin{bmatrix} -13 & 11 \\ 7 & 0 \\ 6 & 9 \end{bmatrix} + \begin{bmatrix} -3 & 13 \\ -3 & 8 \\ -14 & 15 \end{bmatrix}\right) = \begin{bmatrix} -12 & -12 \\ 20.5 & -9 \\ -13 & -1 \end{bmatrix}$

In Exercises 27-34, $A = \begin{bmatrix} -2 & 1 & 3 \\ -1 & 0 & 4 \end{bmatrix}$ **and** $B = \begin{bmatrix} 0 & 2 & -4 \\ 3 & 0 & 1 \end{bmatrix}$

27. $X = 2A + 2B = 2\begin{bmatrix} -2 & 1 & 3 \\ -1 & 0 & 4 \end{bmatrix} + 2\begin{bmatrix} 0 & 2 & -4 \\ 3 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -4 & 2 & 6 \\ -2 & 0 & 8 \end{bmatrix} + \begin{bmatrix} 0 & 4 & -8 \\ 6 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -4 & 6 & -2 \\ 4 & 0 & 10 \end{bmatrix}$

28. $X = 3A - 2B = 3\begin{bmatrix} -2 & 1 & 3 \\ -1 & 0 & 4 \end{bmatrix} - 2\begin{bmatrix} 0 & 2 & -4 \\ 3 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -6 & 3 & 9 \\ -3 & 0 & 12 \end{bmatrix} - \begin{bmatrix} 0 & 4 & -8 \\ 6 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -6 & -1 & 17 \\ -9 & 0 & 10 \end{bmatrix}$

29. $2X = 2A - B$

$$X = \frac{1}{2}(2A - B)$$

$$= \frac{1}{2}\left(2\begin{bmatrix} -2 & 1 & 3 \\ -1 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 0 & 2 & -4 \\ 3 & 0 & 1 \end{bmatrix}\right)$$

$$= \frac{1}{2}\left(\begin{bmatrix} -4 & 2 & 6 \\ -2 & 0 & 8 \end{bmatrix} - \begin{bmatrix} 0 & 2 & -4 \\ 3 & 0 & 1 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} -4 & 0 & 10 \\ -5 & 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 0 & 5 \\ -\frac{5}{2} & 0 & \frac{7}{2} \end{bmatrix}$$

30. $2X = A + B$

$$X = \frac{1}{2}(A + B)$$

$$= \frac{1}{2}\left(\begin{bmatrix} -2 & 1 & 3 \\ -1 & 0 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 2 & -4 \\ 3 & 0 & 1 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} -2 & 3 & -1 \\ 2 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & \frac{3}{2} & -\frac{1}{2} \\ 1 & 0 & \frac{5}{2} \end{bmatrix}$$

31. $$\begin{aligned} 2X + 3A &= B \\ 2X &= B - 3A \\ X &= \tfrac{1}{2}(B - 3A) \\ &= \tfrac{1}{2}\left(\begin{bmatrix} 0 & 2 & -4 \\ 3 & 0 & 1 \end{bmatrix} - 3\begin{bmatrix} -2 & 1 & 3 \\ -1 & 0 & 4 \end{bmatrix}\right) \\ &= \tfrac{1}{2}\left(\begin{bmatrix} 0 & 2 & -4 \\ 3 & 0 & 1 \end{bmatrix} - \begin{bmatrix} -6 & 3 & 9 \\ -3 & 0 & 12 \end{bmatrix}\right) \\ &= \tfrac{1}{2}\begin{bmatrix} 6 & -1 & -13 \\ 6 & 0 & -11 \end{bmatrix} \\ &= \begin{bmatrix} 3 & -\frac{1}{2} & -\frac{13}{2} \\ 3 & 0 & -\frac{11}{2} \end{bmatrix} \end{aligned}$$

32. $$\begin{aligned} 3X - 4A &= 2B \\ 3X &= 2B + 4A \\ X &= \tfrac{1}{3}(2B + 4A) \\ &= \tfrac{1}{3}\left(2\begin{bmatrix} 0 & 2 & -4 \\ 3 & 0 & 1 \end{bmatrix} + 4\begin{bmatrix} -2 & 1 & 3 \\ -1 & 0 & 4 \end{bmatrix}\right) \\ &= \tfrac{1}{3}\left(\begin{bmatrix} 0 & 4 & -8 \\ 6 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -8 & 4 & 12 \\ -4 & 0 & 16 \end{bmatrix}\right) \\ &= \tfrac{1}{3}\begin{bmatrix} -8 & 8 & 4 \\ 2 & 0 & 18 \end{bmatrix} \\ &= \begin{bmatrix} -\frac{8}{3} & \frac{8}{3} & \frac{4}{3} \\ \frac{2}{3} & 0 & 6 \end{bmatrix} \end{aligned}$$

33. $$\begin{aligned} 4B &= -2X - 2A \\ 2X &= -2A - 4B \\ X &= \tfrac{1}{2}(-2A - 4B) \\ &= \tfrac{1}{2}\left(-2\begin{bmatrix} -2 & 1 & 3 \\ -1 & 0 & 4 \end{bmatrix} - 4\begin{bmatrix} 0 & 2 & -4 \\ 3 & 0 & 1 \end{bmatrix}\right) \\ &= \tfrac{1}{2}\left(\begin{bmatrix} 4 & -2 & -6 \\ 2 & 0 & -8 \end{bmatrix} - \begin{bmatrix} 0 & 8 & -16 \\ 12 & 0 & 4 \end{bmatrix}\right) \\ &= \tfrac{1}{2}\begin{bmatrix} 4 & -10 & 10 \\ -10 & 0 & -12 \end{bmatrix} \\ &= \begin{bmatrix} 2 & -5 & 5 \\ -5 & 0 & -6 \end{bmatrix} \end{aligned}$$

34. $$\begin{aligned} 5A &= 6B - 3X \\ 3X &= 6B - 5A \\ X &= \tfrac{1}{3}(6B - 5A) \\ &= \tfrac{1}{3}\left(6\begin{bmatrix} 0 & 2 & -4 \\ 3 & 0 & 1 \end{bmatrix} - 5\begin{bmatrix} -2 & 1 & 3 \\ -1 & 0 & 4 \end{bmatrix}\right) \\ &= \tfrac{1}{3}\left(\begin{bmatrix} 0 & 12 & -24 \\ 18 & 0 & 6 \end{bmatrix} - \begin{bmatrix} -10 & 5 & 15 \\ -5 & 0 & 20 \end{bmatrix}\right) \\ &= \tfrac{1}{3}\begin{bmatrix} 10 & 7 & -39 \\ 23 & 0 & -14 \end{bmatrix} \\ &= \begin{bmatrix} \frac{10}{3} & \frac{7}{3} & -13 \\ \frac{23}{3} & 0 & -\frac{14}{3} \end{bmatrix} \end{aligned}$$

35. A is 3×2, B is $2 \times 2 \Rightarrow AB$ is 3×2.

$$A = \begin{bmatrix} -1 & 6 \\ -4 & 5 \\ 0 & 3 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ 0 & 9 \end{bmatrix}$$

$$AB = \begin{bmatrix} -1 & 6 \\ -4 & 5 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 0 & 9 \end{bmatrix} = \begin{bmatrix} (-1)(2) + (6)(0) & (-1)(3) + (6)(9) \\ (-4)(2) + (5)(0) & (-4)(3) + (5)(9) \\ (0)(2) + (3)(0) & (0)(3) + (3)(9) \end{bmatrix} = \begin{bmatrix} -2 & 51 \\ -8 & 33 \\ 0 & 27 \end{bmatrix}$$

36. A is 3×3, B is $3 \times 2 \Rightarrow AB$ is 3×2.

$$A = \begin{bmatrix} 0 & -1 & 2 \\ 6 & 0 & 3 \\ 7 & -1 & 8 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 \\ 4 & -5 \\ 1 & 6 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & -1 & 2 \\ 6 & 0 & 3 \\ 7 & -1 & 8 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 4 & -5 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} -2 & 17 \\ 15 & 12 \\ 18 & 46 \end{bmatrix}$$

37. A is 3×2 and B is 3×3. AB is not possible.

38. A is 2×4, B is 2×2. AB is not possible.

39. A is 3×3, B is $3 \times 3 \Rightarrow AB$ is 3×3.

$$AB = \begin{bmatrix} 5 & 0 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & 7 \end{bmatrix}\begin{bmatrix} \frac{1}{5} & 0 & 0 \\ 0 & -\frac{1}{8} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{7}{2} \end{bmatrix}$$

40. A is 3×3, B is $3 \times 3 \Rightarrow AB$ is 3×3.

$$\begin{bmatrix} 0 & 0 & 5 \\ 0 & 0 & -3 \\ 0 & 0 & 4 \end{bmatrix}\begin{bmatrix} 6 & -11 & 4 \\ 8 & 16 & 4 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} (0)(6)+(0)(8)+(5)(0) & (0)(-11)+(0)(16)+(5)(0) & (0)(4)+(0)(4)+(5)(0) \\ (0)(6)+(0)(8)+(-3)(0) & (0)(-11)+(0)(16)+(-3)(0) & (0)(4)+(0)(4)+(-3)(0) \\ (0)(6)+(0)(8)+(4)(0) & (0)(-11)+(0)(16)+(4)(0) & (0)(4)+(0)(4)+(4)(0) \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

41. $A = \begin{bmatrix} 7 & 5 & -4 \\ -2 & 5 & 1 \\ 10 & -4 & -7 \end{bmatrix}, B = \begin{bmatrix} 2 & -2 & 3 \\ 8 & 1 & 4 \\ -4 & 2 & -8 \end{bmatrix}$

$$AB = \begin{bmatrix} 7 & 5 & -4 \\ -2 & 5 & 1 \\ 10 & -4 & -7 \end{bmatrix}\begin{bmatrix} 2 & -2 & 3 \\ 8 & 1 & 4 \\ -4 & 2 & -8 \end{bmatrix} = \begin{bmatrix} 70 & -17 & 73 \\ 32 & 11 & 6 \\ 16 & -38 & 70 \end{bmatrix}$$

42. $\begin{bmatrix} 11 & -12 & 4 \\ 14 & 10 & 12 \\ 6 & -2 & 9 \end{bmatrix}\begin{bmatrix} 12 & 10 \\ -5 & 12 \\ 15 & 16 \end{bmatrix} = \begin{bmatrix} 252 & 30 \\ 298 & 452 \\ 217 & 180 \end{bmatrix}$

43. $\begin{bmatrix} -3 & 8 & -6 & 8 \\ -12 & 15 & 9 & 6 \\ 5 & -1 & 1 & 5 \end{bmatrix}\begin{bmatrix} 3 & 1 & 6 \\ 24 & 15 & 14 \\ 16 & 10 & 21 \\ 8 & -4 & 10 \end{bmatrix} = \begin{bmatrix} 151 & 25 & 48 \\ 516 & 279 & 387 \\ 47 & -20 & 87 \end{bmatrix}$

44. A is 3×3, B is 4×2. AB is not possible.

45. (a) $AB = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -1 & 8 \end{bmatrix} = \begin{bmatrix} (1)(2)+(2)(-1) & (1)(-1)+(2)(8) \\ (4)(2)+(2)(-1) & (4)(-1)+(2)(8) \end{bmatrix} = \begin{bmatrix} 0 & 15 \\ 6 & 12 \end{bmatrix}$

(b) $BA = \begin{bmatrix} 2 & -1 \\ -1 & 8 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} (2)(1)+(-1)(4) & (2)(2)+(-1)(2) \\ (-1)(1)+(8)(4) & (-1)(2)+(8)(2) \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 31 & 14 \end{bmatrix}$

(c) $A^2 = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} (1)(1)+(2)(4) & (1)(2)+(2)(2) \\ (4)(1)+(2)(4) & (4)(2)+(2)(2) \end{bmatrix} = \begin{bmatrix} 9 & 6 \\ 12 & 12 \end{bmatrix}$

46. $A = \begin{bmatrix} 6 & 3 \\ -2 & -4 \end{bmatrix}, B = \begin{bmatrix} -2 & 0 \\ 2 & 4 \end{bmatrix}$

(a) $AB = \begin{bmatrix} 6 & 3 \\ -2 & -4 \end{bmatrix}\begin{bmatrix} -2 & 0 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} (6)(-2)+(3)(2) & (6)(0)+(3)(4) \\ (-2)(-2)+(-4)(2) & (-2)(0)+(-4)(4) \end{bmatrix} = \begin{bmatrix} -6 & 12 \\ -4 & -16 \end{bmatrix}$

(b) $BA = \begin{bmatrix} -2 & 0 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 6 & 3 \\ -2 & -4 \end{bmatrix} = \begin{bmatrix} (-2)(6)+(0)(-2) & (-2)(3)+(0)(-4) \\ (2)(6)+(4)(-2) & (2)(3)+(4)(-4) \end{bmatrix} = \begin{bmatrix} -12 & -6 \\ 4 & -10 \end{bmatrix}$

(c) $A^2 = \begin{bmatrix} 6 & 3 \\ -2 & -4 \end{bmatrix}\begin{bmatrix} 6 & 3 \\ -2 & -4 \end{bmatrix} = \begin{bmatrix} (6)(6)+(3)(-2) & (6)(3)+(3)(-4) \\ (-2)(6)+(-4)(-2) & (-2)(3)+(-4)(-4) \end{bmatrix} = \begin{bmatrix} 30 & 6 \\ -4 & 10 \end{bmatrix}$

47. (a) $AB = \begin{bmatrix} 5 & -9 & 0 \\ 3 & 0 & -8 \\ -1 & 4 & 11 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} (5)(1) + (-9)(0) + (0)(0) & (5)(0) + (-9)(1) + (0)(0) & (5)(0) + (-9)(0) + (0)(1) \\ (3)(1) + (0)(0) + (-8)(0) & (3)(0) + (0)(1) + (-8)(0) & (3)(0) + (0)(0) + (-8)(1) \\ (-1)(1) + (4)(0) + (11)(0) & (-1)(0) + (4)(1) + (11)(0) & (-1)(0) + (4)(0) + (11)(1) \end{bmatrix}$

$= \begin{bmatrix} 5 & -9 & 0 \\ 3 & 0 & -8 \\ -1 & 4 & 11 \end{bmatrix}$

(b) $BA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 5 & -9 & 0 \\ 3 & 0 & -8 \\ -1 & 4 & 11 \end{bmatrix} = \begin{bmatrix} (1)(5) + (0)(3) + (0)(-1) & (1)(-9) + (0)(0) + (0)(4) & (1)(0) + (0)(-8) + (0)(11) \\ (0)(5) + (1)(3) + (0)(-1) & (0)(-9) + (1)(0) + (0)(4) & (0)(0) + (1)(-8) + (0)(11) \\ (0)(5) + (0)(3) + (1)(-1) & (0)(-9) + (0)(0) + (1)(4) & (0)(0) + (0)(-8) + (1)(11) \end{bmatrix}$

$= \begin{bmatrix} 5 & -9 & 0 \\ 3 & 0 & -8 \\ -1 & 4 & 11 \end{bmatrix}$

(c) $AA = \begin{bmatrix} 5 & -9 & 0 \\ 3 & 0 & -8 \\ -1 & 4 & 11 \end{bmatrix}\begin{bmatrix} 5 & -9 & 0 \\ 3 & 0 & -8 \\ -1 & 4 & 11 \end{bmatrix} = \begin{bmatrix} (5)(5) + (-9)(3) + (0)(-1) & (5)(-9) + (-9)(0) + (0)(4) & (5)(0) + (-9)(-8) + (0)(11) \\ (3)(5) + (0)(3) + (-8)(-1) & (3)(-9) + (0)(0) + (-8)(4) & (3)(0) + (0)(-8) + (-8)(11) \\ (-1)(5) + (4)(3) + (11)(-1) & (-1)(-9) + (4)(0) + (11)(4) & (-1)(0) + (4)(-8) + (11)(11) \end{bmatrix}$

$= \begin{bmatrix} -2 & -45 & 72 \\ 23 & -59 & -88 \\ -4 & 53 & 89 \end{bmatrix}$

48. (a) $AB = \begin{bmatrix} 2 & -2 \\ -3 & 0 \\ 7 & 6 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} (2)(1) + (-2)(0) & (2)(0) + (-2)(1) \\ (-3)(1) + (0)(0) & (-3)(0) + (0)(1) \\ (7)(1) + (6)(0) & (7)(0) + (6)(1) \end{bmatrix}$

$= \begin{bmatrix} 2 & -2 \\ -3 & 0 \\ 7 & 6 \end{bmatrix}$

(b) BA is not possible, B is 2×2 and A is 3×2.

(c) AA is not possible, A is 3×2.

49. (a) $AB = \begin{bmatrix} -4 & -1 \\ 2 & 12 \end{bmatrix}\begin{bmatrix} -6 \\ 5 \end{bmatrix} = \begin{bmatrix} (-4)(-6) + (-1)(5) \\ (2)(-6) + (12)(5) \end{bmatrix} = \begin{bmatrix} 19 \\ 48 \end{bmatrix}$

(b) BA is not possible, B is 2×1 and A is 2×2.

(c) $A^2 = \begin{bmatrix} -4 & -1 \\ 2 & 12 \end{bmatrix}\begin{bmatrix} -4 & -1 \\ 2 & 12 \end{bmatrix} = \begin{bmatrix} (-4)(-4) + (-1)(2) & (-4)(-1) + (-1)(12) \\ (2)(-4) + (12)(2) & (2)(-1) + (12)(12) \end{bmatrix} = \begin{bmatrix} 14 & -8 \\ 16 & 142 \end{bmatrix}$

50. (a) $AB = \begin{bmatrix} 1 & 3 & -2 \\ -5 & 10 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} = \begin{bmatrix} (1)(3) + (3)(3) + (-2)(3) \\ (-5)(3) + (10)(3) + (1)(3) \end{bmatrix} = \begin{bmatrix} 6 \\ 18 \end{bmatrix}$

(b) BA is not possible, B is 3×1 and A is 2×3.

(c) AA is not possible, A is 2×3.

51. (a) $AB = \begin{bmatrix} 7 \\ 8 \\ -1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 7(1) & 7(1) & 7(2) \\ 8(1) & 8(1) & 8(2) \\ -1(1) & -1(1) & -1(2) \end{bmatrix} = \begin{bmatrix} 7 & 7 & 14 \\ 8 & 8 & 16 \\ -1 & -1 & -2 \end{bmatrix}$

(b) $BA = \begin{bmatrix} 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} 7 \\ 8 \\ -1 \end{bmatrix} = \begin{bmatrix} (1)(7) + (1)(8) + (2)(-1) \end{bmatrix} = \begin{bmatrix} 13 \end{bmatrix}$

(c) A^2 is not possible.

52. (a) $\begin{bmatrix} 3 & 2 & 1 & 4 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3(2) & 2(3) & 1(0) & 4(0) \end{bmatrix} = \begin{bmatrix} 16 \end{bmatrix}$

(b) $BA = \begin{bmatrix} 2 \\ 3 \\ 0 \\ 1 \end{bmatrix}\begin{bmatrix} 3 & 2 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 2(3) & 2(2) & 2(1) & 2(4) \\ 3(3) & 3(2) & 3(1) & 3(4) \\ 0(3) & 0(2) & 0(1) & 0(4) \\ 1(3) & 1(2) & 1(1) & 1(4) \end{bmatrix} = \begin{bmatrix} 6 & 4 & 2 & 8 \\ 9 & 6 & 3 & 12 \\ 0 & 0 & 0 & 0 \\ 3 & 2 & 1 & 4 \end{bmatrix}$

(c) The number of columns of A does not equal the number of rows of A; the multiplication is not possible.

53. $\begin{bmatrix} 3 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 4 & -4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 8 \\ -4 & -16 \end{bmatrix}$

54. $3\left(\begin{bmatrix} 6 & 5 & -1 \\ 1 & -2 & 0 \end{bmatrix}\begin{bmatrix} 0 & 3 \\ -1 & -3 \\ 4 & 1 \end{bmatrix}\right) = -3\left(\begin{bmatrix} 6(0) + 5(-1) + (-1)(4) & 6(3) + 5(-3) + (-1)(1) \\ 1(0) + (-2)(-1) + (0)(4) & 1(3) + (-2)(-3) + (0)(1) \end{bmatrix}\right) = -3\begin{bmatrix} -9 & 2 \\ 2 & 9 \end{bmatrix} = \begin{bmatrix} 27 & -6 \\ -6 & -27 \end{bmatrix}$

55. $\begin{bmatrix} 0 & 2 & -2 \\ 4 & 1 & 2 \end{bmatrix}\left(\begin{bmatrix} 4 & 0 \\ 0 & -1 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 3 \\ -3 & 5 \\ 0 & -3 \end{bmatrix}\right) = \begin{bmatrix} 0 & 2 & -2 \\ 4 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -3 & 4 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} -4 & 10 \\ 3 & 14 \end{bmatrix}$

56. $\begin{bmatrix} 3 \\ -1 \\ 5 \\ 7 \end{bmatrix}\left(\begin{bmatrix} 5 & -6 \end{bmatrix} + \begin{bmatrix} 7 & -1 \end{bmatrix} + \begin{bmatrix} -8 & 9 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ -1 \\ 5 \\ 7 \end{bmatrix}\begin{bmatrix} 4 & 2 \end{bmatrix} = \begin{bmatrix} 3(4) & 3(2) \\ (-1)(4) & (-1)(2) \\ 5(4) & 5(2) \\ 7(4) & 7(2) \end{bmatrix} = \begin{bmatrix} 12 & 6 \\ -4 & -2 \\ 20 & 10 \\ 28 & 14 \end{bmatrix}$

57. $\mathbf{u} = \langle 1, 5 \rangle, \mathbf{v} = \langle 3, 2 \rangle$

(a) $\mathbf{u} + \mathbf{v} = \begin{bmatrix} 1 \\ 5 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix} = \langle 4, 7 \rangle$

(b) $\mathbf{u} - \mathbf{v} = \begin{bmatrix} 1 \\ 5 \end{bmatrix} - \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \langle -2, 3 \rangle$

(c) $3\mathbf{v} - \mathbf{u} = 3\begin{bmatrix} 3 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \end{bmatrix} = \langle 8, 1 \rangle$

58. $\mathbf{u} = \langle 4, 2 \rangle, \mathbf{v} = \langle 6, -3 \rangle$

(a) $\mathbf{u} + \mathbf{v} = \begin{bmatrix} 4 \\ 2 \end{bmatrix} + \begin{bmatrix} 6 \\ -3 \end{bmatrix} = \begin{bmatrix} 10 \\ -1 \end{bmatrix} = \langle 10, -1 \rangle$

(b) $\mathbf{u} - \mathbf{v} = \begin{bmatrix} 4 \\ 2 \end{bmatrix} - \begin{bmatrix} 6 \\ -3 \end{bmatrix} = \begin{bmatrix} -2 \\ 5 \end{bmatrix} = \langle -2, 5 \rangle$

(c) $3\mathbf{v} - \mathbf{u} = 3\begin{bmatrix} 6 \\ -3 \end{bmatrix} - \begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 14 \\ -11 \end{bmatrix} = \langle 14, -11 \rangle$

59. $\mathbf{u} = \langle -2, 2\rangle, \mathbf{v} = \langle 5, 4\rangle$

(a) $\mathbf{u} + \mathbf{v} = \begin{bmatrix} -2 \\ 2 \end{bmatrix} + \begin{bmatrix} 5 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix} = \langle 3, 6\rangle$

(b) $\mathbf{u} - \mathbf{v} = \begin{bmatrix} -2 \\ 2 \end{bmatrix} - \begin{bmatrix} 5 \\ 4 \end{bmatrix} = \begin{bmatrix} -7 \\ -2 \end{bmatrix} = \langle -7, -2\rangle$

(c) $3\mathbf{v} - \mathbf{u} = 3\begin{bmatrix} 5 \\ 4 \end{bmatrix} - \begin{bmatrix} -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 17 \\ 10 \end{bmatrix} = \langle 17, 10\rangle$

60. $\mathbf{u} = \langle 7, -4\rangle, \mathbf{v} = \langle 2, 1\rangle$

(a) $\mathbf{u} + \mathbf{v} = \begin{bmatrix} 7 \\ -4 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 9 \\ -3 \end{bmatrix} = \langle 9, -3\rangle$

(b) $\mathbf{u} - \mathbf{v} = \begin{bmatrix} 7 \\ -4 \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ -5 \end{bmatrix} = \langle 5, -5\rangle$

(c) $3\mathbf{v} - \mathbf{u} = 3\begin{bmatrix} 2 \\ 1 \end{bmatrix} - \begin{bmatrix} 7 \\ -4 \end{bmatrix} = \begin{bmatrix} -1 \\ 7 \end{bmatrix} = \langle -1, 7\rangle$

In Exercises 61–66, $\mathbf{v} = \langle 4, 2\rangle = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

61. $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, A\mathbf{v} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix} = \langle 4, -2\rangle$ is a reflection in the x-axis.

62. $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, A\mathbf{v} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} -4 \\ 2 \end{bmatrix} = \langle -4, 2\rangle$ is a reflection in the y-axis.

63. $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, A\mathbf{v} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \langle 2, 4\rangle$ is a reflection in the line $y = x$.

64. $A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}, A\mathbf{v} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ -4 \end{bmatrix} = \langle -2, -4\rangle$ is a reflection in the line $y = -x$.

65. $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, A\mathbf{v} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ 2 \end{bmatrix} = \langle 8, 2\rangle$ is a horizontal stretch.

66. $A = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}, A\mathbf{v} = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix} = \langle 4, 6\rangle$ is a vertical stretch.

67. (a) $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$

(b)
$$\begin{matrix} R_2 \\ R_1 \end{matrix}\begin{bmatrix} 1 & 4 & \vdots & 10 \\ 2 & 3 & \vdots & 5 \end{bmatrix}$$

$$-2R_1 + R_2 \rightarrow \begin{bmatrix} 1 & 4 & \vdots & 10 \\ 0 & -5 & \vdots & -15 \end{bmatrix}$$

$$-\tfrac{1}{5}R_2 \rightarrow \begin{bmatrix} 1 & 4 & \vdots & 10 \\ 0 & 1 & \vdots & 3 \end{bmatrix}$$

$$\begin{matrix} -4R_2 + R_1 \rightarrow \\ -\tfrac{1}{5}R_2 \rightarrow \end{matrix}\begin{bmatrix} 1 & 0 & \vdots & -2 \\ 0 & 1 & \vdots & 3 \end{bmatrix}$$

$$X = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

68. (a) $\begin{bmatrix} -2 & -3 \\ 6 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -4 \\ -36 \end{bmatrix}$

(b)
$$\begin{bmatrix} -2 & -3 & \vdots & -4 \\ 6 & 1 & \vdots & -36 \end{bmatrix}$$

$$3R_1 + R_2 \rightarrow \begin{bmatrix} -2 & -3 & \vdots & -4 \\ 0 & -8 & \vdots & -48 \end{bmatrix}$$

$$\begin{matrix} -\tfrac{1}{2}R_1 \rightarrow \\ -\tfrac{1}{8}R_2 \rightarrow \end{matrix}\begin{bmatrix} 1 & \tfrac{3}{2} & \vdots & 2 \\ 0 & 1 & \vdots & 6 \end{bmatrix}$$

$$-\tfrac{3}{2}R_2 + R_1 \rightarrow \begin{bmatrix} 1 & 0 & \vdots & -7 \\ 0 & 1 & \vdots & 6 \end{bmatrix}$$

$$X = \begin{bmatrix} -7 \\ 6 \end{bmatrix}$$

69. (a) $\begin{bmatrix} 1 & -2 & 3 \\ -1 & 3 & -1 \\ 2 & -5 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ -6 \\ 17 \end{bmatrix}$

(b)
$$\begin{bmatrix} 1 & -2 & 3 & \vdots & 9 \\ -1 & 3 & -1 & \vdots & -6 \\ 2 & -5 & 5 & \vdots & 17 \end{bmatrix}$$

$$\begin{matrix} \\ R_1 + R_2 \rightarrow \\ -2R_1 + R_3 \rightarrow \end{matrix}\begin{bmatrix} 1 & -2 & 3 & \vdots & 9 \\ 0 & 1 & 2 & \vdots & 3 \\ 0 & -1 & -1 & \vdots & -1 \end{bmatrix}$$

$$\begin{matrix} 2R_2 + R_1 \rightarrow \\ \\ R_2 + R_3 \rightarrow \end{matrix}\begin{bmatrix} 1 & 0 & 7 & \vdots & 15 \\ 0 & 1 & 2 & \vdots & 3 \\ 0 & 0 & 1 & \vdots & 2 \end{bmatrix}$$

$$\begin{matrix} -7R_3 + R_1 \rightarrow \\ -2R_3 + R_2 \rightarrow \\ \\ \end{matrix}\begin{bmatrix} 1 & 0 & 0 & \vdots & 1 \\ 0 & 1 & 0 & \vdots & -1 \\ 0 & 0 & 1 & \vdots & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

70. (a) $\begin{bmatrix} 1 & 1 & -3 \\ -1 & 2 & 0 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$

(b)

$$\left[\begin{array}{ccc:c} 1 & 1 & -3 & -1 \\ -1 & 2 & 0 & 1 \\ 1 & -1 & 1 & 2 \end{array}\right]$$

$$\begin{array}{r} \\ \\ R_2 + R_3 \rightarrow \end{array}\left[\begin{array}{ccc:c} 1 & 1 & -3 & -1 \\ -1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 3 \end{array}\right]$$

$$\begin{array}{r} \\ R_1 + R_2 \rightarrow \\ \end{array}\left[\begin{array}{ccc:c} 1 & 1 & -3 & -1 \\ 0 & 3 & -3 & 0 \\ 0 & 1 & 1 & 3 \end{array}\right]$$

$$\begin{array}{r} \\ \frac{1}{3}R_2 \rightarrow \\ R_2 + (-3)R_3 \rightarrow \end{array}\left[\begin{array}{ccc:c} 1 & 1 & -3 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -6 & -9 \end{array}\right]$$

$$\begin{array}{r} -R_2 + R_1 \rightarrow \\ \\ -\frac{1}{6}R_3 \rightarrow \end{array}\left[\begin{array}{ccc:c} 1 & 0 & -2 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & \frac{3}{2} \end{array}\right]$$

$$\begin{array}{r} \\ R_3 + R_2 \rightarrow \\ \end{array}\left[\begin{array}{ccc:c} 1 & 0 & -2 & -1 \\ 0 & 1 & 0 & \frac{3}{2} \\ 0 & 0 & 1 & \frac{3}{2} \end{array}\right]$$

$$\begin{array}{r} 2R_3 + R_1 \rightarrow \\ \\ \end{array}\left[\begin{array}{ccc:c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & \frac{3}{2} \\ 0 & 0 & 1 & \frac{3}{2} \end{array}\right]$$

$$X = \begin{bmatrix} 2 \\ \frac{3}{2} \\ \frac{3}{2} \end{bmatrix}$$

71. (a) $\begin{bmatrix} 1 & -5 & 2 \\ -3 & 1 & -1 \\ 0 & -2 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -20 \\ 8 \\ -16 \end{bmatrix}$

(b)

$$\left[\begin{array}{ccc:c} 1 & -5 & 2 & -20 \\ -3 & 1 & -1 & 8 \\ 0 & -2 & 5 & -16 \end{array}\right]$$

$$\begin{array}{r} \\ 3R_1 + R_2 \rightarrow \\ \end{array}\left[\begin{array}{ccc:c} 1 & -5 & 2 & -20 \\ 0 & -14 & 5 & -52 \\ 0 & -2 & 5 & -16 \end{array}\right]$$

$$\begin{array}{r} \\ -R_3 + R_2 \rightarrow \\ \end{array}\left[\begin{array}{ccc:c} 1 & -5 & 2 & -20 \\ 0 & -12 & 0 & -36 \\ 0 & -2 & 5 & -16 \end{array}\right]$$

$$\begin{array}{r} \\ -\frac{1}{12}R_2 \rightarrow \\ \end{array}\left[\begin{array}{ccc:c} 1 & -5 & 2 & -20 \\ 0 & 1 & 0 & 3 \\ 0 & -2 & 5 & -16 \end{array}\right]$$

$$\begin{array}{r} 5R_2 + R_1 \rightarrow \\ \\ 2R_2 + R_3 \rightarrow \end{array}\left[\begin{array}{ccc:c} 1 & 0 & 2 & -5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 5 & -10 \end{array}\right]$$

$$\begin{array}{r} \\ \\ \frac{1}{5}R_3 \rightarrow \end{array}\left[\begin{array}{ccc:c} 1 & 0 & 2 & -5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$\begin{array}{r} -2R_3 + R_1 \rightarrow \\ \\ \end{array}\left[\begin{array}{ccc:c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$X = \begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix}$$

72. (a) $\begin{bmatrix} 1 & -1 & 4 \\ 1 & 3 & 0 \\ 0 & -6 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 17 \\ -11 \\ 40 \end{bmatrix}$

(b) $\left[\begin{array}{rrr:r} 1 & -1 & 4 & 17 \\ 1 & 3 & 0 & -11 \\ 0 & -6 & 5 & 40 \end{array}\right]$

$-R_1 + R_2 \rightarrow \left[\begin{array}{rrr:r} 1 & -1 & 4 & 17 \\ 0 & 4 & -4 & -28 \\ 0 & -6 & 5 & 40 \end{array}\right]$

$\frac{1}{4}R_2 \rightarrow \left[\begin{array}{rrr:r} 1 & -1 & 4 & 17 \\ 0 & 1 & -1 & -7 \\ 0 & -6 & 5 & 40 \end{array}\right]$

$6R_2 + R_3 \rightarrow \left[\begin{array}{rrr:r} 1 & -1 & 4 & 17 \\ 0 & 1 & -1 & -7 \\ 0 & 0 & -1 & -2 \end{array}\right]$

$\begin{array}{r} R_2 + R_1 \rightarrow \\ \\ -R_3 \rightarrow \end{array} \left[\begin{array}{rrr:r} 1 & 0 & 3 & 10 \\ 0 & 1 & -1 & -7 \\ 0 & 0 & 1 & 2 \end{array}\right]$

$\begin{array}{r} -3R_3 + R_1 \rightarrow \\ R_3 + R_2 \rightarrow \\ \\ \end{array} \left[\begin{array}{rrr:r} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 2 \end{array}\right]$

$X = \begin{bmatrix} 4 \\ -5 \\ 2 \end{bmatrix}$

73. $1.10\begin{bmatrix} 100 & 90 & 70 & 30 \\ 40 & 20 & 60 & 60 \end{bmatrix} = \begin{bmatrix} 110 & 99 & 77 & 33 \\ 44 & 22 & 66 & 66 \end{bmatrix}$

74. $A + 0.12A = 1.12A$

$= 1.12\begin{bmatrix} 615 & 670 & 740 & 990 \\ 995 & 1030 & 1180 & 1105 \end{bmatrix}$

$= \begin{bmatrix} 688.80 & 750.40 & 828.80 & 1108.80 \\ 1114.40 & 1153.60 & 1321.60 & 1237.60 \end{bmatrix}$

The entries represent the room rates for two different rooms at four hotels.

75. $BA = \begin{bmatrix} 3.50 & 6.00 \end{bmatrix}\begin{bmatrix} 125 & 100 & 75 \\ 100 & 175 & 125 \end{bmatrix}$

$= \begin{bmatrix} \$1037.50 & \$1400 & \$1012.50 \end{bmatrix}$

The entries represent the profits from both crops at each of the three outlets.

76. $BA = \begin{bmatrix} \$699.95 & \$899.95 & \$1099.95 \end{bmatrix}\begin{bmatrix} 5{,}000 & 4{,}000 \\ 6{,}000 & 10{,}000 \\ 8{,}000 & 5{,}000 \end{bmatrix} = \begin{bmatrix} \$17{,}699{,}050 & \$17{,}299{,}050 \end{bmatrix}$

The entries represent the cost of the three models of the LCD televisions at the two warehouses.

77. $ST = \begin{bmatrix} 1.0 & 0.5 & 0.2 \\ 1.6 & 1.0 & 0.2 \\ 2.5 & 2.0 & 1.4 \end{bmatrix}\begin{bmatrix} 15 & 13 \\ 12 & 11 \\ 11 & 10 \end{bmatrix} = \begin{bmatrix} \$23.20 & \$20.50 \\ \$38.20 & \$33.80 \\ \$76.90 & \$68.50 \end{bmatrix}$ The entries represent the labor costs at each plant for each size of boat.

78. (a)

$AB = \begin{bmatrix} 40 & 64 & 52 \\ 60 & 82 & 76 \\ 76 & 96 & 84 \end{bmatrix}\begin{bmatrix} 3.45 & 1.20 \\ 3.65 & 1.30 \\ 3.85 & 1.45 \end{bmatrix} = \begin{array}{c} \text{Sales \$} \quad \text{Profit} \\ \begin{bmatrix} \$571.80 & \$206.60 \\ \$798.90 & \$288.80 \\ \$936 & \$337.80 \end{bmatrix} \end{array}$

The entries represent the total sales (in dollars) and the profit (in dollars) for milk on Friday, Saturday, and Sunday.

(b) Total profit = \$206.60 + \$288.80 + \$337.80 = \$833.20

79. $P^2 = \begin{bmatrix} 0.6 & 0.1 & 0.1 \\ 0.2 & 0.7 & 0.1 \\ 0.2 & 0.2 & 0.8 \end{bmatrix}\begin{bmatrix} 0.6 & 0.1 & 0.1 \\ 0.2 & 0.7 & 0.1 \\ 0.2 & 0.2 & 0.8 \end{bmatrix} = \begin{bmatrix} 0.40 & 0.15 & 0.15 \\ 0.28 & 0.53 & 0.17 \\ 0.32 & 0.32 & 0.68 \end{bmatrix}$

The P^2 matrix gives the proportion of the voting population that changed parties or remained loyal to their parties from the first election to the third.

80. (a)

	basketball	jumping rope	weight lifting
$B =$	$[2$	0.25	$0.5]$

(b) $BA = \begin{bmatrix} 2 & 0.25 & 0.5 \end{bmatrix}\begin{bmatrix} 472 & 563 \\ 590 & 704 \\ 177 & 211 \end{bmatrix}$

$= \begin{bmatrix} 1180 & 1407.5 \end{bmatrix}$

The resulting matrix BA represents the total calories burned playing basketball for 2 hours, jumping rope for 15 minutes and lifting weights for 30 minutes by each person. So, the 130-lb person burned 1180 calories, and the 155-lb person burned 1407.5 calories.

81. True.

The sum of two matrices of different orders is undefined.

82. False. For most matrices, $AB \neq BA$.

Consider

$$A = \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 1 & 2 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

So, $AB \neq BA$.

83. Answers will vary. *Sample answer:*

$$(A + B)^2 = \left(\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}\right)^2 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \neq$$

$$A^2 + 2AB + B^2 = \left(\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}\right)^2 + 2\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix} + \left(\begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}\right)^2 = \begin{bmatrix} 0 & 0 \\ 3 & 2 \end{bmatrix}$$

84. Answers will vary. *Sample answer:*

$$(A - B)^2 = \left(\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}\right)^2 = \begin{bmatrix} 7 & -16 \\ 8 & 23 \end{bmatrix} \neq$$

$$A^2 - 2AB + B^2 = \left(\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}\right)^2 - 2\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix} + \left(\begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}\right)^2 = \begin{bmatrix} 8 & -16 \\ 7 & 22 \end{bmatrix}$$

85. Answers will vary. *Sample answer:*

$$(A + B)(A - B) = \left(\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}\right)\left(\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}\right) = \begin{bmatrix} 3 & -2 \\ 4 & 3 \end{bmatrix} \neq$$

$$A^2 - B^2 = \left(\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}\right)^2 - \left(\begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}\right)^2 = \begin{bmatrix} 2 & -2 \\ 5 & 4 \end{bmatrix}$$

86. Answers will vary. *Sample answer:*

$$(A + B)^2 = \left(\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} =$$

$$A^2 + AB + BA + B^2 = \left(\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}\right)^2 + \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} + \left(\begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}\right)^2 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

87. $AC = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$

$BC = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$

So, $AC = BC$ even though $A \neq B$.

88. $AB = \begin{bmatrix} 3 & 3 \\ 4 & 4 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$AB = O$ and neither A nor B is O.

89. Answer will vary. *Sample answer*:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} (1)(0) + (0)(1) & (1)(1) + (0)(0) \\ (0)(0) + (1)(1) & (0)(1) + (1)(0) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$BA = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} (0)(1) + (1)(0) & (0)(0) + (1)(1) \\ (1)(1) + (0)(0) & (1)(0) + (0)(1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

So, $AB = BA$.

90. (a) The entry a_{22} indicates that Factory B produces 100 electric guitars.

(b) To determine production levels if production is increased by 20%, multiply matrix A by the scalar 1.2.

(c) Use the sales prices to create a 1×2 matrix B, where $B = [80 \quad 120]$. Then compute BA to find the total sales value of the guitars produced at each factory.

91. The product of two diagonal matrices of the same order is a diagonal matrix whose entries are the products of the corresponding diagonal entries of A and B.

92. (a) $A^2 = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} = \begin{bmatrix} (i)(i) + (0)(0) & (i)(0) + (0)(i) \\ (0)(i) + (i)(0) & (0)(0) + (i)(i) \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ and $i^2 = -1$

$A^3 = A^2A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} = \begin{bmatrix} (-1)(i) + (0)(0) & (-1)(0) + (0)(i) \\ (0)(i) + (-1)(0) & (0)(0) + (-1)(i) \end{bmatrix} = \begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix}$ and $i^3 = -i$

$A^4 = A^3A = \begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix}\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} = \begin{bmatrix} (-i)(i) + (0)(0) & (i)(0) + (0)(i) \\ (0)(i) + (-i)(0) & (0)(0) + (-i)(i) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $i^4 = 1$

(b) $B = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

$B^2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = \begin{bmatrix} (0)(0) + (-i)(i) & (0)(-i) + (-i)(0) \\ (i)(0) + (0)(i) & (i)(-i) + (0)(0) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$, the identity matrix

Section 8.3 The Inverse of a Square Matrix

1. inverse

2. nonsingular; singular

3. determinant

4. $A^{-1}B$

5. $AB = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} 6 - 5 & -2 + 2 \\ 15 - 15 & -5 + 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$BA = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 6 - 5 & 3 - 3 \\ -10 + 10 & -5 + 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

6. $AB = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 - 1 & 1 - 1 \\ -2 + 2 & -1 + 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$BA = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 - 1 & -2 + 2 \\ 1 - 1 & -1 + 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

7. $AB = \frac{1}{10}\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 12 - 2 & -6 + 6 \\ 4 - 4 & -2 + 12 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$BA = \frac{1}{10}\begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 12 - 2 & 8 - 8 \\ -3 + 3 & -2 + 12 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

8. $AB = \frac{1}{5}\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 3+2 & 1-1 \\ 6-6 & 2+3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$BA = \frac{1}{5}\begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 3+2 & -3+3 \\ -2+2 & 2+3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

9. $AB = \begin{bmatrix} 2 & -17 & 11 \\ -1 & 11 & -7 \\ 0 & 3 & -2 \end{bmatrix}\begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix} = \begin{bmatrix} 2-34+33 & 2-68+66 & 4+51-55 \\ -1+22-21 & -1+44-42 & -2-33+35 \\ 6-6 & 12-12 & -9+10 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$BA = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix}\begin{bmatrix} 2 & -17 & 11 \\ -1 & 11 & -7 \\ 0 & 3 & -2 \end{bmatrix} = \begin{bmatrix} 2-1 & -17+11+6 & 11-7-4 \\ 4-4 & -34+44-9 & 22-28+6 \\ 6-6 & -51+66-15 & 33-42+10 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

10. $AB = \frac{1}{4}\begin{bmatrix} -4 & 1 & 5 \\ -1 & 2 & 4 \\ 0 & -1 & -1 \end{bmatrix}\begin{bmatrix} -2 & 4 & 6 \\ 1 & -4 & -11 \\ -1 & 4 & 7 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 8+1-5 & -16-4+20 & -24-11+35 \\ 2+2-4 & -4-8+16 & -6-22+28 \\ 0-1+1 & 0+4-4 & 0+11-7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$BA = \frac{1}{4}\begin{bmatrix} -2 & 4 & 6 \\ 1 & -4 & -11 \\ -1 & 4 & 7 \end{bmatrix}\begin{bmatrix} -4 & 1 & 5 \\ -1 & 2 & 4 \\ 0 & -1 & -1 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 8-4+0 & -2+8-6 & -10+16-6 \\ -4+4+0 & 1-8+11 & 5-16+11 \\ 4-4+0 & -1+8-7 & -5+16-7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

11. $AB = \frac{1}{3}\begin{bmatrix} 2 & 0 & 2 & 1 \\ 3 & 0 & 0 & 1 \\ -1 & 1 & -2 & 1 \\ 3 & -1 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 3 & -2 & -2 \\ -2 & 9 & -7 & -10 \\ 1 & 0 & -1 & -1 \\ 3 & -6 & 6 & 6 \end{bmatrix}$

$= \frac{1}{3}\begin{bmatrix} -2+0+2+3 & 6+0+0-6 & -4+0-2+6 & -4+0-2+6 \\ -3+0+0+3 & 9+0+0-6 & -6+0+0+6 & -6+0+0+6 \\ 1-2-2+3 & -3+9+0-6 & 2-7+2+6 & 2-10+2+6 \\ -3+2+1+0 & 9-9+0+0 & -6+7-1+0 & -6+10-1+0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$BA = \frac{1}{3}\begin{bmatrix} -1 & 3 & -2 & -2 \\ -2 & 9 & -7 & -10 \\ 1 & 0 & -1 & -1 \\ 3 & -6 & 6 & 6 \end{bmatrix}\begin{bmatrix} 2 & 0 & 2 & 1 \\ 3 & 0 & 0 & 1 \\ -1 & 1 & -2 & 1 \\ 3 & -1 & 1 & 0 \end{bmatrix}$

$= \frac{1}{3}\begin{bmatrix} -2+9+2-6 & 0+0-2+2 & -2+0+4-2 & -1+3-2+0 \\ -4+27+7-30 & 0+0-7+10 & -4+0+14-10 & -2+9-7+0 \\ 2+0+1-3 & 0+0-1+1 & 2+0+2-1 & 1+0-1+0 \\ 6-18-6+18 & 0+0+6-6 & 6+0-12+6 & 3-6+6+0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

12. $AB = \frac{1}{3}\begin{bmatrix} -1 & 1 & 0 & -1 \\ 1 & -1 & 1 & 0 \\ -1 & 1 & 2 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix}\begin{bmatrix} -3 & 1 & 1 & -3 \\ -3 & -1 & 2 & -3 \\ 0 & 1 & 1 & 0 \\ -3 & -2 & 1 & 0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 3-3+3 & -1-1+2 & -1+2-1 & 3-3 \\ -3+3 & 1+1+1 & 1-2+1 & -3+3 \\ 3-3 & -1-1+2 & -1+2+2 & 3-3 \\ 3-3 & 1+1-2 & -2+1+1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$BA = \frac{1}{3}\begin{bmatrix} -3 & 1 & 1 & -3 \\ -3 & -1 & 2 & -3 \\ 0 & 1 & 1 & 0 \\ -3 & -2 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 & -1 \\ 1 & -1 & 1 & 0 \\ -1 & 1 & 2 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 3+1-1 & -3-1+1+3 & 1+2-3 & 3-3 \\ 3-1-2 & -3+1+2+3 & -1+4-3 & 3-3 \\ 1-1 & -1+1 & 1+2 & 0 \\ 3-2-1 & -3+2+1 & -2+2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

13. $[A \; \vdots \; I] = \begin{bmatrix} 2 & 1 & \vdots & 1 & 0 \\ 5 & 3 & \vdots & 0 & 1 \end{bmatrix}$

$$\begin{array}{r} \\ -5R_1 + 2R_2 \rightarrow \end{array} \begin{bmatrix} 2 & 1 & \vdots & 1 & 0 \\ 0 & 1 & \vdots & -5 & 2 \end{bmatrix}$$

$$\begin{array}{r} -R_2 + R_1 \rightarrow \\ \end{array} \begin{bmatrix} 2 & 0 & \vdots & 6 & -2 \\ 0 & 1 & \vdots & -5 & 2 \end{bmatrix}$$

$$\begin{array}{r} \frac{1}{2}R_1 \rightarrow \\ \end{array} \begin{bmatrix} 1 & 0 & \vdots & 3 & -1 \\ 0 & 1 & \vdots & -5 & 2 \end{bmatrix} = [I \; \vdots \; A^{-1}]$$

$$A^{-1} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

14. $[A \; \vdots \; I] = \begin{bmatrix} 1 & 2 & \vdots & 1 & 0 \\ 3 & 7 & \vdots & 0 & 1 \end{bmatrix}$

$$\begin{array}{r} \\ -3R_1 + R_2 \rightarrow \end{array} \begin{bmatrix} 1 & 2 & \vdots & 1 & 0 \\ 0 & 1 & \vdots & -3 & 1 \end{bmatrix}$$

$$\begin{array}{r} -2R_2 + R_1 \rightarrow \\ \end{array} \begin{bmatrix} 1 & 0 & \vdots & 7 & -2 \\ 0 & 1 & \vdots & -3 & 1 \end{bmatrix} = [I \; \vdots \; A^{-1}]$$

$$A^{-1} = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$$

15. $[A \; \vdots \; I] = \begin{bmatrix} 1 & -2 & \vdots & 1 & 0 \\ 2 & -3 & \vdots & 0 & 1 \end{bmatrix}$

$$\begin{array}{r} \\ -2R_1 + R_2 \rightarrow \end{array} \begin{bmatrix} 1 & -2 & \vdots & 1 & 0 \\ 0 & 1 & \vdots & -2 & 1 \end{bmatrix}$$

$$\begin{array}{r} 2R_2 + R_1 \rightarrow \\ \end{array} \begin{bmatrix} 1 & 0 & \vdots & -3 & 2 \\ 0 & 1 & \vdots & -2 & 1 \end{bmatrix} = [I \; \vdots \; A^{-1}]$$

$$A^{-1} = \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}$$

16. $[A \; \vdots \; I] = \begin{bmatrix} -7 & 33 & \vdots & 1 & 0 \\ 4 & -19 & \vdots & 0 & 1 \end{bmatrix}$

$$\begin{array}{r} 2R_2 + R_1 \rightarrow \\ \end{array} \begin{bmatrix} 1 & -5 & \vdots & 1 & 2 \\ 4 & -19 & \vdots & 0 & 1 \end{bmatrix}$$

$$\begin{array}{r} \\ -4R_1 + R_2 \rightarrow \end{array} \begin{bmatrix} 1 & -5 & \vdots & 1 & 2 \\ 0 & 1 & \vdots & -4 & -7 \end{bmatrix}$$

$$\begin{array}{r} 5R_2 + R_1 \rightarrow \\ \end{array} \begin{bmatrix} 1 & 0 & \vdots & -19 & -33 \\ 0 & 1 & \vdots & -4 & -7 \end{bmatrix} = [I \; \vdots \; A^{-1}]$$

$$A^{-1} = \begin{bmatrix} -19 & -33 \\ -4 & -7 \end{bmatrix}$$

17. $[A \; \vdots \; I] = \begin{bmatrix} 3 & 1 & \vdots & 1 & 0 \\ 4 & 2 & \vdots & 0 & 1 \end{bmatrix}$

$$\begin{array}{r} \\ \frac{1}{2}R_2 \rightarrow \end{array} \begin{bmatrix} 3 & 1 & \vdots & 1 & 0 \\ 2 & 1 & \vdots & 0 & \frac{1}{2} \end{bmatrix}$$

$$\begin{array}{r} -R_2 + R_1 \rightarrow \\ \end{array} \begin{bmatrix} 1 & 0 & \vdots & 1 & -\frac{1}{2} \\ 2 & 1 & \vdots & 0 & \frac{1}{2} \end{bmatrix}$$

$$\begin{array}{r} \\ -2R_1 + R_2 \rightarrow \end{array} \begin{bmatrix} 1 & 0 & \vdots & 1 & -\frac{1}{2} \\ 0 & 1 & \vdots & -2 & \frac{3}{2} \end{bmatrix} = [I \; \vdots \; A^{-1}]$$

$$A^{-1} = \begin{bmatrix} 1 & -\frac{1}{2} \\ -2 & \frac{3}{2} \end{bmatrix}$$

18. $[A \; \vdots \; I] = \begin{bmatrix} 4 & -1 & \vdots & 1 & 0 \\ -3 & 1 & \vdots & 0 & 1 \end{bmatrix}$

$$\begin{array}{r} R_2 + R_1 \rightarrow \\ \end{array} \begin{bmatrix} 1 & 0 & \vdots & 1 & 1 \\ -3 & 1 & \vdots & 0 & 1 \end{bmatrix}$$

$$\begin{array}{r} \\ 3R_1 + R_2 \rightarrow \end{array} \begin{bmatrix} 1 & 0 & \vdots & 1 & 1 \\ 0 & 1 & \vdots & 3 & 4 \end{bmatrix} = [I \; \vdots \; A^{-1}]$$

$$A^{-1} = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}$$

19. $[A \; \vdots \; I] = \begin{bmatrix} 1 & 1 & 1 & \vdots & 1 & 0 & 0 \\ 3 & 5 & 4 & \vdots & 0 & 1 & 0 \\ 3 & 6 & 5 & \vdots & 0 & 0 & 1 \end{bmatrix}$

$$\begin{array}{r} \\ -3R_1 + R_2 \rightarrow \\ -3R_1 + R_3 \rightarrow \end{array} \begin{bmatrix} 1 & 1 & 1 & \vdots & 1 & 0 & 0 \\ 0 & 2 & 1 & \vdots & -3 & 1 & 0 \\ 0 & 3 & 2 & \vdots & -3 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{r} \\ \frac{1}{2}R_2 \rightarrow \\ \\ \end{array} \begin{bmatrix} 1 & 1 & 1 & \vdots & 1 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & \vdots & -\frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 3 & 2 & \vdots & -3 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{r} -R_2 + R_1 \rightarrow \\ \\ -3R_2 + R_3 \rightarrow \end{array} \begin{bmatrix} 1 & 0 & \frac{1}{2} & \vdots & \frac{5}{2} & -\frac{1}{2} & 0 \\ 0 & 1 & \frac{1}{2} & \vdots & -\frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \vdots & \frac{3}{2} & -\frac{3}{2} & 1 \end{bmatrix}$$

$$\begin{array}{r} -R_3 + R_1 \rightarrow \\ -R_3 + R_2 \rightarrow \\ \\ \end{array} \begin{bmatrix} 1 & 0 & 0 & \vdots & 1 & 1 & -1 \\ 0 & 1 & 0 & \vdots & -3 & 2 & -1 \\ 0 & 0 & \frac{1}{2} & \vdots & \frac{3}{2} & -\frac{3}{2} & 1 \end{bmatrix}$$

$$\begin{array}{r} \\ \\ 2R_3 \rightarrow \end{array} \begin{bmatrix} 1 & 0 & 0 & \vdots & 1 & 1 & -1 \\ 0 & 1 & 0 & \vdots & -3 & 2 & -1 \\ 0 & 0 & 1 & \vdots & 3 & -3 & 2 \end{bmatrix} = [I \; \vdots \; A^{-1}]$$

$$A^{-1} = \begin{bmatrix} 1 & 1 & -1 \\ -3 & 2 & -1 \\ 3 & -3 & 2 \end{bmatrix}$$

20. $$[A \,\vdots\, I] = \left[\begin{array}{rrr:rrr} 1 & 2 & 2 & 1 & 0 & 0 \\ 3 & 7 & 9 & 0 & 1 & 0 \\ -1 & -4 & -7 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{r} \\ -3R_1 + R_2 \to \\ R_1 + R_3 \to \end{array} \left[\begin{array}{rrr:rrr} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & -3 & 1 & 0 \\ 0 & -2 & -5 & 1 & 0 & 1 \end{array}\right]$$

$$\begin{array}{r} -2R_2 + R_1 \to \\ \\ 2R_2 + R_3 \to \end{array} \left[\begin{array}{rrr:rrr} 1 & 0 & -4 & 7 & -2 & 0 \\ 0 & 1 & 3 & -3 & 1 & 0 \\ 0 & 0 & 1 & -5 & 2 & 1 \end{array}\right]$$

$$\begin{array}{r} 4R_3 + R_1 \to \\ -3R_3 + R_2 \to \\ \\ \end{array} \left[\begin{array}{rrr:rrr} 1 & 0 & 0 & -13 & 6 & 4 \\ 0 & 1 & 0 & 12 & -5 & -3 \\ 0 & 0 & 1 & -5 & 2 & 1 \end{array}\right] = [I \,\vdots\, A^{-1}]$$

$$A^{-1} = \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix}$$

21. $$[A \,\vdots\, I] = \left[\begin{array}{rrr:rrr} -5 & 0 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 & 0 \\ -1 & 5 & 7 & 0 & 0 & 1 \end{array}\right] \begin{array}{r} \\ \\ R_2 + 2R_3 \to \end{array} \left[\begin{array}{rrr:rrr} -5 & 0 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 10 & 14 & 0 & 1 & 2 \end{array}\right] \begin{array}{r} \\ 2R_1 + 5R_2 \to \\ \\ \end{array} \left[\begin{array}{rrr:rrr} -5 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 5 & 0 \\ 0 & 10 & 14 & 0 & 1 & 2 \end{array}\right]$$

Because the first three entries of row 2 are all zeros, the inverse of A does not exist.

22. $$[A \,\vdots\, I] = \left[\begin{array}{rrr:rrr} 1 & 0 & 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 0 & 1 & 0 \\ 2 & 5 & 5 & 0 & 0 & 1 \end{array}\right] \begin{array}{r} \\ -3R_1 + R_2 \to \\ -2R_1 + R_3 \to \end{array} \left[\begin{array}{rrr:rrr} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -3 & 1 & 0 \\ 0 & 5 & 5 & -2 & 0 & 1 \end{array}\right]$$

Because the first three entries of row 2 are all zeros, the inverse of A does not exist.

23. $$[A \,\vdots\, I] = \left[\begin{array}{rrrr:rrrr} -8 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -5 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{r} -\frac{1}{8}R_1 \to \\ \\ \frac{1}{4}R_3 \to \\ -\frac{1}{5}R_4 \to \end{array} \left[\begin{array}{rrrr:rrrr} 1 & 0 & 0 & 0 & -\frac{1}{8} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -\frac{1}{5} \end{array}\right] = [I \,\vdots\, A^{-1}]$$

$$A^{-1} = \begin{bmatrix} -\frac{1}{8} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & -\frac{1}{5} \end{bmatrix}$$

24. $$[A \;\vdots\; I] = \left[\begin{array}{cccc|cccc} 1 & 3 & -2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 4 & 6 & 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{r} \\ \frac{1}{2}R_2 \rightarrow \\ \\ \frac{1}{5}R_4 \rightarrow \end{array} \left[\begin{array}{cccc|cccc} 1 & 3 & -2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -2 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{5} \end{array}\right]$$

$$\begin{array}{r} -3R_2 + R_1 \rightarrow \\ R_3 + R_2 \rightarrow \\ -R_4 + R_3 \rightarrow \\ \\ \end{array} \left[\begin{array}{cccc|cccc} 1 & 0 & -8 & -9 & 1 & -\frac{3}{2} & 0 & 0 \\ 0 & 1 & 0 & 4 & 0 & \frac{1}{2} & 1 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 1 & -\frac{1}{5} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{5} \end{array}\right]$$

$$\begin{array}{r} -4R_3 + R_1 \rightarrow \\ -4R_4 + R_2 \rightarrow \\ -\frac{1}{2}R_3 \rightarrow \\ \\ \end{array} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & -9 & 1 & -\frac{3}{2} & -4 & \frac{4}{5} \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{2} & 1 & -\frac{4}{5} \\ 0 & 0 & 1 & 0 & 0 & 0 & -\frac{1}{2} & \frac{1}{10} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{5} \end{array}\right]$$

$$\begin{array}{r} 9R_4 + R_1 \rightarrow \\ \\ \\ \\ \end{array} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & -\frac{3}{2} & -4 & \frac{13}{5} \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{2} & 1 & -\frac{4}{5} \\ 0 & 0 & 1 & 0 & 0 & 0 & -\frac{1}{2} & \frac{1}{10} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{5} \end{array}\right] = [I \;\vdots\; A^{-1}]$$

$$A^{-1} = \begin{bmatrix} 1 & -\frac{3}{2} & -4 & \frac{13}{5} \\ 0 & \frac{1}{2} & 1 & -\frac{4}{5} \\ 0 & 0 & -\frac{1}{2} & \frac{1}{10} \\ 0 & 0 & 0 & \frac{1}{5} \end{bmatrix}$$

25. $$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 7 & -10 \\ -5 & -7 & -15 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -175 & 37 & -13 \\ 95 & -20 & 7 \\ 14 & -3 & 1 \end{bmatrix}$$

26. $$A = \begin{bmatrix} 10 & 5 & -7 \\ -5 & 1 & 4 \\ 3 & 2 & -2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -10 & -4 & 27 \\ 2 & 1 & -5 \\ -13 & -5 & 35 \end{bmatrix}$$

27. $$A = \begin{bmatrix} -\frac{1}{2} & \frac{3}{4} & \frac{1}{4} \\ 1 & 0 & -\frac{3}{2} \\ 0 & -1 & \frac{1}{2} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -12 & -5 & -9 \\ -4 & -2 & -4 \\ -8 & -4 & -6 \end{bmatrix}$$

28. $$\begin{bmatrix} -\frac{5}{6} & \frac{1}{3} & \frac{11}{6} \\ 0 & \frac{2}{3} & 2 \\ 1 & -\frac{1}{2} & -\frac{5}{2} \end{bmatrix}$$

A^{-1} does not exist.

29. $A = \begin{bmatrix} 0.1 & 0.2 & 0.3 \\ -0.3 & 0.2 & 0.2 \\ 0.5 & 0.4 & 0.4 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 0 & -1.\overline{81} & 0.\overline{90} \\ -10 & 5 & 5 \\ 10 & -2.\overline{72} & -3.\overline{63} \end{bmatrix}$

30. $A = \begin{bmatrix} 0.6 & 0 & -0.3 \\ 0.7 & -1 & 0.2 \\ 1 & 0 & -0.9 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 3.75 & 0 & -1.25 \\ 3.458\overline{3} & -1 & -1.375 \\ 4.1\overline{6} & 0 & -2.5 \end{bmatrix}$

31. $A = \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & 2 & 0 & -1 \\ 2 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}$

32. $A = \begin{bmatrix} 1 & -2 & -1 & -2 \\ 3 & -5 & -2 & -3 \\ 2 & -5 & -2 & -5 \\ -1 & 4 & 4 & 11 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} -24 & 7 & 1 & -2 \\ -10 & 3 & 0 & -1 \\ -29 & 7 & 3 & -2 \\ 12 & -3 & -1 & 1 \end{bmatrix}$

33. $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, A^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$A = \begin{bmatrix} 2 & 3 \\ -1 & 5 \end{bmatrix}$

$ad - bc = (2)(5) - (3)(-1) = 13$

$A^{-1} = \frac{1}{13}\begin{bmatrix} 5 & -3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{5}{13} & -\frac{3}{13} \\ \frac{1}{13} & \frac{2}{13} \end{bmatrix}$

34. $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, A^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$A = \begin{bmatrix} 1 & -2 \\ -3 & 2 \end{bmatrix}$

$ad - bc = (1)(2) - (-2)(-3) = -4$

$A^{-1} = -\frac{1}{4}\begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} \\ -\frac{3}{4} & -\frac{1}{4} \end{bmatrix}$

35. $A = \begin{bmatrix} -4 & -6 \\ 2 & 3 \end{bmatrix}$

$ad - bc = (-4)(3) - (-2)(-6) = 0$

Because $ad - bc = 0$, A^{-1} does not exist.

36. $A = \begin{bmatrix} -12 & 3 \\ 5 & -2 \end{bmatrix}$

$ad - bc = (-12)(-2) - 3(5) = 24 - 15 = 9$

$A^{-1} = \frac{1}{9}\begin{bmatrix} -2 & -3 \\ -5 & -12 \end{bmatrix} = \begin{bmatrix} -\frac{2}{9} & -\frac{1}{3} \\ -\frac{5}{9} & -\frac{4}{3} \end{bmatrix}$

37. $A = \begin{bmatrix} 0.5 & 0.3 \\ 1.5 & 0.6 \end{bmatrix}$

$ad - bc = (0.5)(0.6) - (0.3)(1.5) = 0.3 - 0.45 = -0.15$

$A^{-1} = -\frac{1}{0.15}\begin{bmatrix} 0.6 & -0.3 \\ -1.5 & 0.5 \end{bmatrix} = \begin{bmatrix} -4 & 2 \\ 10 & -\frac{10}{3} \end{bmatrix}$

38. $A = \begin{bmatrix} -1.25 & 0.625 \\ 0.16 & 0.32 \end{bmatrix}$

$$\begin{aligned} ad - bc &= (-1.25)(0.32) - (0.625)(0.16) \\ &= -0.4 - 0.1 \\ &= -0.5 \end{aligned}$$

$A^{-1} = -\frac{1}{0.5}\begin{bmatrix} 0.32 & -0.625 \\ -0.16 & -1.25 \end{bmatrix} = \begin{bmatrix} -0.64 & 1.25 \\ 0.32 & 2.5 \end{bmatrix}$

39. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 10 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$

Solution: $(5, 0)$

40. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$

Solution: $(6, 3)$

41. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} -8 \\ -6 \end{bmatrix}$

Solution: $(-8, -6)$

42. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} -7 \\ -4 \end{bmatrix}$

Solution: $(-7, -4)$

43. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ -3 & 2 & -1 \\ 3 & -3 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \\ -11 \end{bmatrix}$

Solution: $(3, 8, -11)$

44. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ -3 & 2 & -1 \\ 3 & -3 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ -9 \end{bmatrix}$

Solution: $(1, 7, -9)$

45. $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -24 & 7 & 1 & -2 \\ -10 & 3 & 0 & -1 \\ -29 & 7 & 3 & -2 \\ 12 & -3 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

Solution: $(2, 1, 0, 0)$

46. $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -24 & 7 & 1 & -2 \\ -10 & 3 & 0 & -1 \\ -29 & 7 & 3 & -2 \\ 12 & -3 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 0 \\ -3 \end{bmatrix} = \begin{bmatrix} -32 \\ -13 \\ -37 \\ 15 \end{bmatrix}$

Solution: $(-32, -13, -37, 15)$

47. $A = \begin{bmatrix} 5 & 4 \\ 2 & 5 \end{bmatrix}$

$A^{-1} = \dfrac{1}{25 - 8} \begin{bmatrix} 5 & -4 \\ -2 & 5 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = -\dfrac{1}{17} \begin{bmatrix} 5 & -4 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = -\dfrac{1}{17} \begin{bmatrix} -17 \\ 17 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

Solution: $(-1, 1)$

48. $A = \begin{bmatrix} 18 & 12 \\ 30 & 24 \end{bmatrix}$

$A^{-1} = \dfrac{1}{432 - 360} \begin{bmatrix} 24 & -12 \\ -30 & 18 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \dfrac{1}{72} \begin{bmatrix} 24 & -12 \\ -30 & 18 \end{bmatrix} \begin{bmatrix} 13 \\ 23 \end{bmatrix} = \dfrac{1}{72} \begin{bmatrix} 36 \\ 24 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \end{bmatrix}$

Solution: $\left(\dfrac{1}{2}, \dfrac{1}{3}\right)$

49. $A = \begin{bmatrix} -0.4 & 0.8 \\ 2 & -4 \end{bmatrix}$

$A^{-1} = \dfrac{1}{1.6 - 1.6} \begin{bmatrix} -4 & -0.8 \\ -2 & -0.4 \end{bmatrix}$

A^{-1} does not exist.

This implies that there is no unique solution; that is, either the system is inconsistent *or* there are infinitely many solutions.

Find the reduced row-echelon form of the matrix corresponding to the system.

$$\begin{bmatrix} -0.4 & 0.8 & \vdots & 1.6 \\ 2 & -4 & \vdots & 5 \end{bmatrix}$$

$$-2.5R_1 \rightarrow \begin{bmatrix} 1 & -2 & \vdots & -4 \\ 2 & -4 & \vdots & 5 \end{bmatrix}$$

$$-2R_1 + R_2 \rightarrow \begin{bmatrix} 1 & -2 & \vdots & -4 \\ 0 & 0 & \vdots & 13 \end{bmatrix}$$

The given system is inconsistent and there is no solution.

50. $A = \begin{bmatrix} 0.2 & -0.6 \\ -1 & 1.4 \end{bmatrix}$

$A^{-1} = \dfrac{1}{0.28 - 0.6} \begin{bmatrix} 1.4 & 0.6 \\ 1 & 0.2 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = -\dfrac{1}{0.32} \begin{bmatrix} 1.4 & 0.6 \\ 1 & 0.2 \end{bmatrix} \begin{bmatrix} 2.4 \\ -8.8 \end{bmatrix} = -\dfrac{1}{0.32} \begin{bmatrix} -1.92 \\ 0.64 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix}$

Solution: $(6, -2)$

51. $A = \begin{bmatrix} -\frac{1}{4} & \frac{3}{8} \\ \frac{3}{2} & \frac{3}{4} \end{bmatrix}$

$$A^{-1} = \frac{1}{-\frac{3}{16} - \frac{9}{16}} \begin{bmatrix} \frac{3}{4} & -\frac{3}{8} \\ -\frac{3}{2} & -\frac{1}{4} \end{bmatrix}$$

$$= -\frac{4}{3} \begin{bmatrix} \frac{3}{4} & -\frac{3}{8} \\ -\frac{3}{2} & -\frac{1}{4} \end{bmatrix}$$

$$= \begin{bmatrix} -1 & \frac{1}{2} \\ 2 & \frac{1}{3} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & \frac{1}{2} \\ 2 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} -2 \\ -12 \end{bmatrix} = \begin{bmatrix} -4 \\ -8 \end{bmatrix}$$

Solution: $(-4, -8)$

52. $A = \begin{bmatrix} 5.1 & -3.4 \\ 0.9 & -0.6 \end{bmatrix}$

$$A^{-1} = \frac{1}{-3.06 + 3.06} \begin{bmatrix} -0.6 & 3.4 \\ -0.9 & 5.1 \end{bmatrix}$$

A^{-1} does not exist.

This implies that there is no unique solution; that is, either the system is inconsistent *or* there are infinitely many solutions.

Find the reduced row-echelon form of matrix corresponding to the system.

$$\begin{bmatrix} 5.1 & -3.4 & \vdots & -20 \\ 0.9 & -0.6 & \vdots & -51 \end{bmatrix}$$

$$\tfrac{1}{5.1}R_1 \rightarrow \begin{bmatrix} 1 & -\frac{2}{3} & \vdots & -\frac{200}{51} \\ \frac{9}{10} & -\frac{3}{5} & \vdots & -51 \end{bmatrix}$$

$$-R_1 + \tfrac{10}{9}R_2 \rightarrow \begin{bmatrix} 1 & -\frac{2}{3} & \vdots & -\frac{200}{51} \\ 0 & 0 & \vdots & -\frac{310}{9} \end{bmatrix}$$

The given system is inconsistent and there is no solution.

53. $A = \begin{bmatrix} 4 & -1 & 1 \\ 2 & 2 & 3 \\ 5 & -2 & 6 \end{bmatrix}$

Find A^{-1}.

$$[A \ \vdots \ I] = \left[\begin{array}{ccc:ccc} 4 & -1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 3 & 0 & 1 & 0 \\ 5 & -2 & 6 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{l} R_1 \\ \\ R_3 \end{array} \left[\begin{array}{ccc:ccc} 5 & -2 & 6 & 0 & 0 & 1 \\ 2 & 2 & 3 & 0 & 1 & 0 \\ 4 & -1 & 1 & 1 & 0 & 0 \end{array}\right]$$

$$\begin{array}{r} -R_3 + R_1 \rightarrow \\ \\ \\ \end{array} \left[\begin{array}{ccc:ccc} 1 & -1 & 5 & -1 & 0 & 1 \\ 2 & 2 & 3 & 0 & 1 & 0 \\ 4 & -1 & 1 & 1 & 0 & 0 \end{array}\right]$$

$$\begin{array}{r} \\ -2R_1 + R_2 \rightarrow \\ -4R_1 + R_3 \rightarrow \end{array} \left[\begin{array}{ccc:ccc} 1 & -1 & 5 & -1 & 0 & 1 \\ 0 & 4 & -7 & 2 & 1 & -2 \\ 0 & 3 & -19 & 5 & 0 & -4 \end{array}\right]$$

$$\begin{array}{r} \\ -R_3 + R_2 \rightarrow \\ \\ \end{array} \left[\begin{array}{ccc:ccc} 1 & -1 & 5 & -1 & 0 & 1 \\ 0 & 1 & 12 & -3 & 1 & 2 \\ 0 & 3 & -19 & 5 & 0 & -4 \end{array}\right]$$

$$\begin{array}{r} R_2 + R_1 \rightarrow \\ \\ -3R_2 + R_3 \rightarrow \end{array} \left[\begin{array}{ccc:ccc} 1 & 0 & 17 & -4 & 1 & 3 \\ 0 & 1 & 12 & -3 & 1 & 2 \\ 0 & 0 & -55 & 14 & -3 & -10 \end{array}\right]$$

$$\begin{array}{r} \\ \\ -\frac{1}{55}R_3 \rightarrow \end{array} \left[\begin{array}{ccc:ccc} 1 & 0 & 17 & -4 & 1 & 3 \\ 0 & 1 & 12 & -3 & 1 & 2 \\ 0 & 0 & 1 & -\frac{14}{55} & \frac{3}{55} & \frac{2}{11} \end{array}\right]$$

$$\begin{array}{r} -17R_3 + R_1 \rightarrow \\ -12R_3 + R_2 \rightarrow \\ \\ \end{array} \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & \frac{18}{55} & \frac{4}{55} & -\frac{1}{11} \\ 0 & 1 & 0 & \frac{3}{55} & \frac{19}{55} & -\frac{2}{11} \\ 0 & 0 & 1 & -\frac{14}{55} & \frac{3}{55} & \frac{2}{11} \end{array}\right] = [I \ \vdots \ A^{-1}]$$

$$A^{-1} = \frac{1}{55}\begin{bmatrix} 18 & 4 & -5 \\ 3 & 19 & -10 \\ -14 & 3 & 10 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{55}\begin{bmatrix} 18 & 4 & -5 \\ 3 & 19 & -10 \\ -14 & 3 & 10 \end{bmatrix}\begin{bmatrix} -5 \\ 10 \\ 1 \end{bmatrix} = \frac{1}{55}\begin{bmatrix} -55 \\ 165 \\ 110 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$$

Solution: $(-1, 3, 2)$

54. $A = \begin{bmatrix} 4 & -2 & 3 \\ 2 & 2 & 5 \\ 8 & -5 & -2 \end{bmatrix}$

$$A^{-1} = \frac{1}{82}\begin{bmatrix} -21 & 19 & 16 \\ -44 & 32 & 14 \\ 26 & -4 & -12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{82}\begin{bmatrix} -21 & 19 & 16 \\ -44 & 32 & 14 \\ 26 & -4 & -12 \end{bmatrix}\begin{bmatrix} -2 \\ 16 \\ 4 \end{bmatrix} = \frac{1}{82}\begin{bmatrix} 410 \\ 656 \\ -164 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ -2 \end{bmatrix}$$

Solution: $(5, 8, -2)$

55. $\begin{cases} 5x - 3y + 2z = 2 \\ 2x + 2y - 3z = 3 \\ x - 7y + 7z = -4 \end{cases}$

Using a graphing utility

$(0.8125, 0.6875, 0) = \left(\frac{13}{16}, \frac{11}{16}, 0\right)$

56. $\begin{cases} 2x + 3y + 5z = 4 \\ 3x + 5y + 9z = 7 \\ 5x + 9y + 16z = 13 \end{cases}$

Using a graphing utility $(-1, 2, 0)$

57. $A = \begin{bmatrix} 1 & 1 & 1 \\ 0.045 & 0.05 & 0.09 \\ 0 & 2 & -1 \end{bmatrix}$

$$[A \ \vdots \ I] = \begin{bmatrix} 1 & 1 & 1 & \vdots & 1 & 0 & 0 \\ 0.045 & 0.05 & 0.09 & \vdots & 0 & 1 & 0 \\ 0 & 2 & -1 & \vdots & 0 & 0 & 1 \end{bmatrix}$$

$$200R_2 \rightarrow \begin{bmatrix} 1 & 1 & 1 & \vdots & 1 & 0 & 0 \\ 9 & 10 & 18 & \vdots & 0 & 200 & 0 \\ 0 & 2 & -1 & \vdots & 0 & 0 & 1 \end{bmatrix}$$

$$-9R_1 + R_2 \rightarrow \begin{bmatrix} 1 & 1 & 1 & \vdots & 1 & 0 & 0 \\ 0 & 1 & 9 & \vdots & -9 & 200 & 0 \\ 0 & 2 & -1 & \vdots & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{matrix} -R_2 + R_1 \rightarrow \\ \\ -2R_2 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 0 & -8 & \vdots & 10 & -200 & 0 \\ 0 & 1 & 9 & \vdots & -9 & 200 & 0 \\ 0 & 0 & -19 & \vdots & 18 & -400 & 1 \end{bmatrix}$$

$$-\tfrac{1}{19}R_3 \rightarrow \begin{bmatrix} 1 & 0 & -8 & \vdots & 10 & -200 & 0 \\ 0 & 1 & 9 & \vdots & -9 & 200 & 0 \\ 0 & 0 & 1 & \vdots & -\frac{18}{19} & \frac{400}{19} & -\frac{1}{19} \end{bmatrix}$$

$$\begin{matrix} 8R_3 + R_1 \rightarrow \\ -9R_3 + R_2 \rightarrow \\ \\ \end{matrix} \begin{bmatrix} 1 & 0 & 0 & \vdots & \frac{46}{19} & -\frac{600}{19} & -\frac{8}{19} \\ 0 & 1 & 0 & \vdots & -\frac{9}{19} & \frac{200}{19} & \frac{9}{19} \\ 0 & 0 & 1 & \vdots & -\frac{18}{19} & \frac{400}{19} & -\frac{1}{19} \end{bmatrix} = [I \ \vdots \ A^{-1}]$$

$$X = A^{-1}B = \frac{1}{19}\begin{bmatrix} 46 & -600 & -8 \\ -9 & 200 & 9 \\ -18 & 400 & -1 \end{bmatrix}\begin{bmatrix} 10{,}000 \\ 650 \\ 0 \end{bmatrix} \approx \begin{bmatrix} 3684.21 \\ 2105.26 \\ 4210.53 \end{bmatrix}$$

Solution: \$3684.21 in AAA-rated bonds, \$2105.26 in A-rated bonds, \$4210.53 in B-rated bonds

58. $A = \begin{bmatrix} 1 & 1 & 1 \\ 0.045 & 0.05 & 0.09 \\ 0 & 2 & -1 \end{bmatrix}$

$$[A \ \vdots \ I] = \left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0.045 & 0.05 & 0.09 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$200R_2 \rightarrow \left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 9 & 10 & 18 & 0 & 200 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$-9R_1 + R_2 \rightarrow \left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 9 & -9 & 200 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{l} -R_2 + R_1 \rightarrow \\ \\ -2R_2 + R_3 \rightarrow \end{array} \left[\begin{array}{ccc:ccc} 1 & 0 & -8 & 10 & -200 & 0 \\ 0 & 1 & 9 & -9 & 200 & 0 \\ 0 & 0 & -19 & 18 & -400 & 1 \end{array}\right]$$

$$-\tfrac{1}{19}R_3 \rightarrow \left[\begin{array}{ccc:ccc} 1 & 0 & -8 & 10 & -200 & 0 \\ 0 & 1 & 9 & -9 & 200 & 0 \\ 0 & 0 & 1 & -\frac{18}{19} & \frac{400}{19} & -\frac{1}{19} \end{array}\right]$$

$$\begin{array}{l} 8R_3 + R_1 \rightarrow \\ -9R_3 + R_2 \rightarrow \\ \\ \end{array} \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & \frac{46}{19} & -\frac{600}{19} & -\frac{8}{19} \\ 0 & 1 & 0 & -\frac{9}{19} & \frac{200}{19} & \frac{9}{19} \\ 0 & 0 & 1 & -\frac{18}{19} & \frac{400}{19} & -\frac{1}{19} \end{array}\right] = [I \ \vdots \ A^{-1}]$$

$$X = A^{-1}B = \frac{1}{19}\begin{bmatrix} 46 & -600 & -8 \\ -9 & 200 & 9 \\ -18 & 400 & -1 \end{bmatrix}\begin{bmatrix} 12{,}000 \\ 835 \\ 0 \end{bmatrix} \approx \begin{bmatrix} 2684.21 \\ 3105.26 \\ 6210.53 \end{bmatrix}$$

Solution: \$2684.21 in AAA-rated bonds, \$3105.26 in A-rated bonds, \$6210.53 in B-rated bonds.

59. $\begin{cases} 2I_1 \qquad + 4I_3 = 15 \\ \qquad I_2 + 4I_3 = 17 \\ I_1 + I_2 - \ I_3 = \ 0 \end{cases}$

$$\underset{A}{\begin{bmatrix} 2 & 0 & 4 \\ 0 & 1 & 4 \\ 1 & 1 & -1 \end{bmatrix}}\underset{X}{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}} = \underset{B}{\begin{bmatrix} 15 \\ 17 \\ 0 \end{bmatrix}}$$

$$X = A^{-1}B = \begin{bmatrix} \frac{5}{14} & -\frac{2}{7} & \frac{2}{7} \\ -\frac{2}{7} & \frac{3}{7} & \frac{4}{7} \\ \frac{1}{14} & \frac{1}{7} & -\frac{1}{7} \end{bmatrix}\begin{bmatrix} 15 \\ 17 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ 3 \\ \frac{7}{2} \end{bmatrix}$$

So, $I_1 = 0.5$ ampere, $I_2 = 3.0$ ampere, and $I_3 = 3.5$ ampere.

60. $\begin{cases} 2I_1 + 4I_3 = 10 \\ I_2 + 4I_3 = 10 \\ I_1 + I_2 - I_3 = 0 \end{cases}$

$$\underbrace{\begin{bmatrix} 2 & 0 & 4 \\ 0 & 1 & 4 \\ 1 & 1 & -1 \end{bmatrix}}_{A}\underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 10 \\ 10 \\ 0 \end{bmatrix}}_{B}$$

$$X = A^{-1}B = \begin{bmatrix} \frac{5}{14} & -\frac{2}{7} & \frac{2}{7} \\ -\frac{2}{7} & \frac{3}{7} & \frac{4}{7} \\ \frac{1}{14} & \frac{1}{7} & -\frac{1}{7} \end{bmatrix}\begin{bmatrix} 10 \\ 10 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{5}{7} \\ \frac{10}{7} \\ \frac{15}{7} \end{bmatrix}$$

So, $I_1 \approx 0.71$ ampere, $I_2 \approx 1.43$ ampere, and $I_3 \approx 2.14$ ampere.

61. $\begin{cases} 2I_1 + 4I_3 = 28 \\ I_2 + 4I_3 = 21 \\ I_1 + I_2 - I_3 = 0 \end{cases}$

$$\underbrace{\begin{bmatrix} 2 & 0 & 4 \\ 0 & 1 & 4 \\ 1 & 1 & -1 \end{bmatrix}}_{A}\underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 28 \\ 21 \\ 0 \end{bmatrix}}_{B}$$

$$X = A^{-1}B = \begin{bmatrix} \frac{5}{14} & -\frac{2}{7} & \frac{2}{7} \\ -\frac{2}{7} & \frac{3}{7} & \frac{4}{7} \\ \frac{1}{14} & \frac{1}{7} & -\frac{1}{7} \end{bmatrix}\begin{bmatrix} 28 \\ 21 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 5 \end{bmatrix}$$

So, $I_1 = 4$ ampere, $I_2 = 1$ ampere, and $I_3 = 5$ ampere.

62. $\begin{cases} 2I_1 + 4I_3 = 24 \\ I_2 + 4I_3 = 23 \\ I_1 + I_2 - I_3 = 0 \end{cases}$

$$\underbrace{\begin{bmatrix} 2 & 0 & 4 \\ 0 & 1 & 4 \\ 1 & 1 & -1 \end{bmatrix}}_{A}\underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 24 \\ 23 \\ 0 \end{bmatrix}}_{B}$$

$$X = A^{-1}B = \begin{bmatrix} \frac{5}{14} & -\frac{2}{7} & \frac{2}{7} \\ -\frac{2}{7} & \frac{3}{7} & \frac{4}{7} \\ \frac{1}{14} & \frac{1}{7} & -\frac{1}{7} \end{bmatrix}\begin{bmatrix} 24 \\ 23 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

So, $I_1 = 2$ ampere, $I_2 = 3$ ampere, and $I_3 = 5$ ampere.

In Exercises 63–64, use the following:

Let x = bags of potting soil for seedlings,

y = bags of potting soil for general potting, and

z = bags of potting soil for hardwood plants.

$$AX = B = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \text{Sand} \\ \text{Loam} \\ \text{Peat Moss} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ -\frac{1}{2} & -\frac{1}{2} & \frac{3}{2} \end{bmatrix}$$

63. $A^{-1}\begin{bmatrix} 500 \\ 500 \\ 400 \end{bmatrix} = \begin{bmatrix} 100 \\ 100 \\ 100 \end{bmatrix}$

Solution:

x = 100 bags of potting soil for seedlings,

y = 100 bags of potting soil for general potting,

z = 100 bags of potting soil for hardwood plants.

64. $A^{-1}\begin{bmatrix} 500 \\ 750 \\ 450 \end{bmatrix} = \begin{bmatrix} 50 \\ 300 \\ 50 \end{bmatrix}$

Solution:

x = 50 bags of potting soil for seedlings,

y = 300 bags of potting soil for general potting,

z = 50 bags of potting soil for hardwood plants.

65. Let r = number of roses, l = number of lilies, and i = number of irises.

(a) $\begin{cases} r + l + i = 120 \\ 2.5r + 4l + 2i = 300 \\ -r + 2l + 2i = 0 \end{cases}$

$$\underset{A}{\begin{bmatrix} 1 & 1 & 1 \\ 2.5 & 4 & 2 \\ -1 & 2 & 2 \end{bmatrix}}\underset{X}{\begin{bmatrix} r \\ l \\ i \end{bmatrix}} = \underset{B}{\begin{bmatrix} 120 \\ 300 \\ 0 \end{bmatrix}}$$

(b) $A^{-1} = \begin{bmatrix} \frac{2}{3} & 0 & -\frac{1}{3} \\ -\frac{7}{6} & \frac{1}{2} & \frac{1}{12} \\ \frac{3}{2} & -\frac{1}{2} & -\frac{1}{4} \end{bmatrix}$

$$X = A^{-1}B = \begin{bmatrix} \frac{2}{3} & 0 & -\frac{1}{3} \\ -\frac{7}{6} & \frac{1}{2} & \frac{1}{12} \\ \frac{3}{2} & -\frac{1}{2} & \frac{1}{4} \end{bmatrix}\begin{bmatrix} 120 \\ 300 \\ 0 \end{bmatrix} = \begin{bmatrix} 80 \\ 10 \\ 30 \end{bmatrix}$$

So, 80 roses, 10 lilies, and 30 irises will create 40 centerpieces.

66. $(12, 1474), (13, 1807), (14, 2188)$

(a) $\begin{cases} 144a + 12b + c = 1474 \\ 169a + 13b + c = 1807 \\ 196a + 14b + c = 2188 \end{cases}$

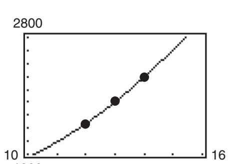
(b) $AX = B = \begin{bmatrix} 144 & 12 & 1 \\ 169 & 13 & 1 \\ 196 & 14 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix}\begin{bmatrix} 1474 \\ 1807 \\ 2188 \end{bmatrix}$

$$A^{-1} = \begin{bmatrix} 0.5 & -1 & 0.5 \\ -13.5 & 26 & -12.5 \\ 91 & -168 & 78 \end{bmatrix}$$

(c) $X = A^{-1}B = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0.5 & -1 & 0.5 \\ -13.5 & 26 & -12.5 \\ 91 & -168 & 78 \end{bmatrix}\begin{bmatrix} 1474 \\ 1807 \\ 2188 \end{bmatrix} = \begin{bmatrix} 24 \\ -267 \\ 1222 \end{bmatrix}$

$y = 24t^2 - 267t + 1222$

(d)



67. True. If B is the inverse of A, then $AB = I = BA$.

68. True. If A and B are both square matrices and $AB = I_n$, it can be shown that $BA = I_n$.

69. If the determinant of a 2×2 matrix is not equal to 0, then the inverse exists.

To find the inverse, take 1 divided by the determinant and multiply it by the matrix which has a diagonal from top left to bottom right that has the terms from the original matrix flipped and the other diagonal is the negative of the terms from the original matrix.

70. Write the linear equations so that the variable terms are on the left and the constant term is on the right. Then you can write this as a matrix equation $AX = B$. For matrix A, each row represents the variable terms and each column represents a variable. For matrix X, write the variables vertically, so that the first column of matrix A represents the variable put in the first row, and so on. For matrix B, each row represents the constant term from an equation.

To solve the matrix equation $AX = B$, multiply each side by the inverse of A, A^{-1}, to obtain $X = A^{-1}B$.

The product $A^{-1}B$ will be a matrix that will have the same dimensions as matrix X and will represent the solution of the system of linear equations.

71. If A^{-1} does not exist, it is singular.

$$\begin{bmatrix} 4 & 3 \\ -2 & k \end{bmatrix}, (4)(k) - (-2)(3) = 0 \Rightarrow 4k + 6 = 0 \Rightarrow k = -\tfrac{3}{2}.$$

When $k \neq -\frac{3}{2}$, A^{-1} exists because the determinant does not equal zero.

72. If A^{-1} does not exist, it is singular.

$$\begin{bmatrix} 2k+1 & 3 \\ -7 & 1 \end{bmatrix}, (2k+1)(1) - (-7)(3) = 0 \Rightarrow 2k + 1 + 21 = 0 \Rightarrow k = -11.$$

When $k \neq -11$, A^{-1} exists because the determinant does not equal zero.

73. (a) Given $A = \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix}$, $A^{-1} = \begin{bmatrix} \frac{1}{a_{11}} & 0 \\ 0 & \frac{1}{a_{22}} \end{bmatrix}$.

Given $A = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}$, $A^{-1} = \begin{bmatrix} \frac{1}{a_{11}} & 0 & 0 \\ 0 & \frac{1}{a_{22}} & 0 \\ 0 & 0 & \frac{1}{a_{33}} \end{bmatrix}$.

(b) In general, the inverse of a matrix in the form of A is

$$\begin{bmatrix} \frac{1}{a_{11}} & 0 & 0 & \cdots & 0 \\ 0 & \frac{1}{a_{22}} & 0 & \cdots & 0 \\ 0 & 0 & \frac{1}{a_{33}} & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & \frac{1}{a_{nn}} \end{bmatrix}.$$

74. (a) A^{-1} exists when the determinant does not equal zero. The $\det A = (xz) - (0 \cdot y) = xz$.

So, if $x \neq 0$ and $z \neq 0$, then $\det A \neq 0$.

(b) $A^{-1} = \dfrac{1}{xz}\begin{bmatrix} z & -y \\ 0 & x \end{bmatrix}$

So, $A^{-1} = A \Rightarrow \dfrac{1}{xz}\begin{bmatrix} z & -y \\ 0 & x \end{bmatrix} = \begin{bmatrix} x & y \\ 0 & z \end{bmatrix}$

When $y = 0$, $x = \pm 1$, and $z = \pm 1$, or when $y \neq 0$, $|x| = |z| = 1$ and $xz = -1$.

75. Answers will vary. *Sample Answer.* $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, A^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$A \cdot A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\left(\frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\right) = \frac{1}{ad - bc}\begin{bmatrix} ad - bc & -ab + ab \\ cd - cd & -bc + ad \end{bmatrix} = \frac{1}{ad - bc}\begin{bmatrix} ad - bc & 0 \\ 0 & ad - bc \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$A^{-1} \cdot A = \left(\frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\right)\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \frac{1}{ad - bc}\begin{bmatrix} ad - bc & -ab + ab \\ cd - cd & -bc + ad \end{bmatrix} = \frac{1}{ad - bc}\begin{bmatrix} ad - bc & 0 \\ 0 & ad - bc \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Section 8.4 The Determinant of a Square Matrix

1. determinant

2. minor

3. cofactor

4. expanding; cofactors

5. 4

6. −10

7. $\begin{vmatrix} 8 & 4 \\ 2 & 3 \end{vmatrix} = (8)(3) - (4)(2) = 16$

8. $\begin{vmatrix} -9 & 0 \\ 6 & -2 \end{vmatrix} = (-9)(-2) - (0)(6) = 18$

9. $\begin{vmatrix} 6 & -3 \\ -5 & 2 \end{vmatrix} = (6)(2) - (-3)(-5) = -3$

10. $\begin{vmatrix} 3 & -3 \\ 4 & -8 \end{vmatrix} = (3)(-8) - (-3)(4) = -12$

11. $\begin{vmatrix} -7 & 0 \\ 3 & 0 \end{vmatrix} = -7(0) - 0(3) = 0$

12. $\begin{vmatrix} 4 & -3 \\ 0 & 0 \end{vmatrix} = (4)(0) - (0)(-3) = 0$

13. $\begin{vmatrix} 2 & 6 \\ 0 & 3 \end{vmatrix} = 2(3) - 6(0) = 6$

14. $\begin{vmatrix} 2 & -3 \\ -6 & 9 \end{vmatrix} = (2)(9) - (-6)(-3) = 0$

15. $\begin{vmatrix} -3 & -2 \\ -6 & -4 \end{vmatrix} = (-3)(-4) - (-2)(-6) = 12 - 12 = 0$

16. $\begin{vmatrix} 4 & 7 \\ -2 & 5 \end{vmatrix} = (4)(5) - (-2)(7) = 34$

17. $\begin{vmatrix} -2 & -7 \\ -3 & 1 \end{vmatrix} = (-2)(1) - (-3)(-7) = -23$

18. $\begin{vmatrix} 2 & -5 \\ -4 & -1 \end{vmatrix} = (2)(-1) - (-4)(-5) = -22$

19. $\begin{vmatrix} -7 & 6 \\ \frac{1}{2} & 3 \end{vmatrix} = (-7)(3) - (6)\left(\frac{1}{2}\right) = -24$

20. $\begin{vmatrix} 0 & 2.5 \\ -3 & 2 \end{vmatrix} = 0(2) - 2.5(-3) = 0 + 7.5 = 7.5$

21. $\begin{vmatrix} -\frac{1}{2} & \frac{1}{3} \\ -6 & \frac{1}{3} \end{vmatrix} = -\frac{1}{2}\left(\frac{1}{3}\right) - \frac{1}{3}(-6) = -\frac{1}{2} + 2 = \frac{11}{6}$

22. $\begin{vmatrix} \frac{2}{3} & -\frac{4}{3} \\ -1 & \frac{1}{3} \end{vmatrix} = \left(\frac{2}{3}\right)\left(\frac{1}{3}\right) - (-1)\left(-\frac{4}{3}\right) = -\frac{10}{9}$

23. $\begin{vmatrix} 3 & 4 \\ -2 & 1 \end{vmatrix} = 11$

24. $\begin{vmatrix} 5 & -9 \\ 7 & 16 \end{vmatrix} = 143$

25. $\begin{vmatrix} 19 & 20 \\ 43 & -56 \end{vmatrix} = -1924$

26. $\begin{vmatrix} 101 & 197 \\ -253 & 172 \end{vmatrix} = 67{,}213$

27. $\begin{vmatrix} \frac{1}{10} & \frac{1}{5} \\ -\frac{3}{10} & \frac{1}{5} \end{vmatrix} = 0.08$

28. $\begin{vmatrix} 0.1 & 0.1 \\ 7.5 & 6.2 \end{vmatrix} = -0.13$

29. $\begin{bmatrix} 4 & 5 \\ 3 & -6 \end{bmatrix}$

(a) $M_{11} = -6$
$M_{12} = 3$
$M_{21} = 5$
$M_{22} = 4$

(b) $C_{11} = M_{11} = -6$
$C_{12} = -M_{12} = -3$
$C_{21} = -M_{21} = -5$
$C_{22} = M_{22} = 4$

30. $\begin{bmatrix} 0 & 10 \\ 3 & -4 \end{bmatrix}$

(a) $M_{11} = -4$
$M_{12} = 3$
$M_{21} = 10$
$M_{22} = 0$

(b) $C_{11} = M_{11} = -4$
$C_{12} = -M_{12} = -3$
$C_{21} = -M_{21} = -10$
$C_{22} = M_{22} = 0$

31. $\begin{bmatrix} 4 & 0 & 2 \\ -3 & 2 & 1 \\ 1 & -1 & 1 \end{bmatrix}$

(a) $M_{11} = \begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} = 2 - (-1) = 3$

$M_{12} = \begin{vmatrix} -3 & 1 \\ 1 & 1 \end{vmatrix} = -3 - 1 = -4$

$M_{13} = \begin{vmatrix} -3 & 2 \\ 1 & -1 \end{vmatrix} = 3 - 2 = 1$

$M_{21} = \begin{vmatrix} 0 & 2 \\ -1 & 1 \end{vmatrix} = 0 - (-2) = 2$

$M_{22} = \begin{vmatrix} 4 & 2 \\ 1 & 1 \end{vmatrix} = 4 - 2 = 2$

$M_{23} = \begin{vmatrix} 4 & 0 \\ 1 & -1 \end{vmatrix} = -4 - 0 = -4$

$M_{31} = \begin{vmatrix} 0 & 2 \\ 2 & 1 \end{vmatrix} = 0 - 4 = -4$

$M_{32} = \begin{vmatrix} 4 & 2 \\ -3 & 1 \end{vmatrix} = 4 - (-6) = 10$

$M_{33} = \begin{vmatrix} 4 & 0 \\ -3 & 2 \end{vmatrix} = 8 - 0 = 8$

(b) $C_{11} = (-1)^2 M_{11} = 3$
$C_{12} = (-1)^3 M_{12} = 4$
$C_{13} = (-1)^4 M_{13} = 1$
$C_{21} = (-1)^3 M_{21} = -2$
$C_{22} = (-1)^4 M_{22} = 2$
$C_{23} = (-1)^5 M_{23} = 4$
$C_{31} = (-1)^4 M_{31} = -4$
$C_{32} = (-1)^5 M_{32} = -10$
$C_{33} = (-1)^6 M_{33} = 8$

32. $\begin{bmatrix} 1 & -1 & 0 \\ 3 & 2 & 5 \\ 4 & -6 & 4 \end{bmatrix}$

(a) $M_{11} = \begin{vmatrix} 2 & 5 \\ -6 & 4 \end{vmatrix} = 8 - (-30) = 38$

$M_{12} = \begin{vmatrix} 3 & 5 \\ 4 & 4 \end{vmatrix} = 12 - 20 = -8$

$M_{13} = \begin{vmatrix} 3 & 2 \\ 4 & -6 \end{vmatrix} = -18 - 8 = -26$

$M_{21} = \begin{vmatrix} -1 & 0 \\ -6 & 4 \end{vmatrix} = -4 - 0 = -4$

$M_{22} = \begin{vmatrix} 1 & 0 \\ 4 & 4 \end{vmatrix} = 4 - 0 = 4$

$M_{23} = \begin{vmatrix} 1 & -1 \\ 4 & -6 \end{vmatrix} = -6 - (-4) = -2$

$M_{31} = \begin{vmatrix} -1 & 0 \\ 2 & 5 \end{vmatrix} = -5 - 0 = -5$

$M_{32} = \begin{vmatrix} 1 & 0 \\ 3 & 5 \end{vmatrix} = 5 - 0 = 5$

$M_{33} = \begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} = 2 - (-3) = 5$

(b) $C_{11} = (-1)^2 M_{11} = 38$
$C_{12} = (-1)^3 M_{12} = 8$
$C_{13} = (-1)^4 M_{13} = -26$
$C_{21} = (-1)^3 M_{21} = 4$
$C_{22} = (-1)^4 M_{22} = 4$
$C_{23} = (-1)^5 M_{23} = 2$
$C_{31} = (-1)^4 M_{31} = -5$
$C_{32} = (-1)^5 M_{32} = -5$
$C_{33} = (-1)^6 M_{33} = 5$

33. $\begin{bmatrix} -4 & 6 & 3 \\ 7 & -2 & 8 \\ 1 & 0 & -5 \end{bmatrix}$

(a) $M_{11} = \begin{vmatrix} -2 & 8 \\ 0 & -5 \end{vmatrix} = (-2)(-5) - (8)(0) = 10$

$M_{12} = \begin{vmatrix} 7 & 8 \\ 1 & -5 \end{vmatrix} = (7)(-5) - (8)(1) = -43$

$M_{13} = \begin{vmatrix} 7 & -2 \\ 1 & 0 \end{vmatrix} = (7)(0) - (-2)(1) = 2$

$M_{21} = \begin{vmatrix} 6 & 3 \\ 0 & -5 \end{vmatrix} = (6)(-5) - (3)(0) = -30$

$M_{22} = \begin{vmatrix} -4 & 3 \\ 1 & -5 \end{vmatrix} = (-4)(-5) - (3)(1) = 17$

$M_{23} = \begin{vmatrix} -4 & 6 \\ 1 & 0 \end{vmatrix} = (-4)(0) - (6)(1) = -6$

$M_{31} = \begin{vmatrix} 6 & 3 \\ -2 & 8 \end{vmatrix} = (6)(8) - (3)(-2) = 54$

$M_{32} = \begin{vmatrix} -4 & 3 \\ 7 & 8 \end{vmatrix} = (-4)(8) - (3)(7) = -53$

$M_{33} = \begin{vmatrix} -4 & 6 \\ 7 & -2 \end{vmatrix} = (-4)(-2) - (6)(7) = -34$

(b) $C_{11} = (-1)^2 M_{11} = 10$

$C_{12} = (-1)^3 M_{12} = 43$

$C_{13} = (-1)^4 M_{13} = 2$

$C_{21} = (-1)^3 M_{21} = 30$

$C_{22} = (-1)^4 M_{22} = 17$

$C_{23} = (-1)^5 M_{23} = 6$

$C_{31} = (-1)^4 M_{31} = 54$

$C_{32} = (-1)^5 M_{32} = 53$

$C_{33} = (-1)^6 M_{33} = -34$

34. $\begin{bmatrix} -2 & 9 & 4 \\ 7 & -6 & 0 \\ 6 & 7 & -6 \end{bmatrix}$

(a) $M_{11} = \begin{bmatrix} -6 & 0 \\ 7 & -6 \end{bmatrix} = (-6)(-6) - (0)(7) = 36$

$M_{12} = \begin{bmatrix} 7 & 0 \\ 6 & -6 \end{bmatrix} = (7)(-6) - (0)(6) = -42$

$M_{13} = \begin{bmatrix} 7 & -6 \\ 6 & 7 \end{bmatrix} = (7)(7) - (-6)(6) = 85$

$M_{21} = \begin{bmatrix} 9 & 4 \\ 7 & -6 \end{bmatrix} = (9)(-6) - (4)(7) = -82$

$M_{22} = \begin{bmatrix} -2 & 4 \\ 6 & -6 \end{bmatrix} = (-2)(-6) - (4)(6) = -12$

$M_{23} = \begin{bmatrix} -2 & 9 \\ 6 & 7 \end{bmatrix} = (-2)(7) - (9)(6) = -68$

$M_{31} = \begin{bmatrix} 9 & 4 \\ -6 & 0 \end{bmatrix} = (9)(0) - (4)(-6) = 24$

$M_{32} = \begin{bmatrix} -2 & 4 \\ 7 & 0 \end{bmatrix} = (-2)(0) - (4)(7) = -28$

$M_{33} = \begin{bmatrix} -2 & 9 \\ 7 & -6 \end{bmatrix} = (-2)(-6) - (9)(7) = -51$

(b) $C_{11} = (-1)^2 M_{11} = 36$

$C_{12} = (-1)^3 M_{12} = 42$

$C_{13} = (-1)^4 M_{13} = 85$

$C_{21} = (-1)^3 M_{21} = 82$

$C_{22} = (-1)^4 M_{22} = -12$

$C_{23} = (-1)^5 M_{23} = 68$

$C_{31} = (-1)^4 M_{31} = 24$

$C_{32} = (-1)^5 M_{32} = 28$

$C_{33} = (-1)^6 M_{33} = -51$

35. (a) $\begin{vmatrix} 2 & 5 \\ 6 & -3 \end{vmatrix} = 2(-3) - 5(6) = -36$

(b) $\begin{vmatrix} 2 & 5 \\ 6 & -3 \end{vmatrix} = 2(-3) - 6(5) = -36$

36. (a) $\begin{vmatrix} 7 & -1 \\ -4 & 10 \end{vmatrix} = -4(-1) + 10(7) = 66$

(b) $\begin{vmatrix} 7 & -1 \\ -4 & 10 \end{vmatrix} = -(-1)(-4) + 10(7) = 66$

37. (a) $\begin{vmatrix} 5 & 0 & -3 \\ 0 & 12 & 4 \\ 1 & 6 & 3 \end{vmatrix} = 0\begin{vmatrix} 0 & -3 \\ 6 & 3 \end{vmatrix} + 12\begin{vmatrix} 5 & -3 \\ 1 & 3 \end{vmatrix} - 4\begin{vmatrix} 5 & 0 \\ 1 & 6 \end{vmatrix} = 0(18) + 12(18) - 4(30) = 96$

(b) $\begin{vmatrix} 5 & 0 & -3 \\ 0 & 12 & 4 \\ 1 & 6 & 3 \end{vmatrix} = 0\begin{vmatrix} 0 & 4 \\ 1 & 3 \end{vmatrix} + 12\begin{vmatrix} 5 & -3 \\ 1 & 3 \end{vmatrix} - 6\begin{vmatrix} 5 & -3 \\ 0 & 4 \end{vmatrix} = 0(-4) + 12(18) - 6(20) = 96$

38. (a) $\begin{vmatrix} 3 & -2 & 5 \\ 1 & 0 & 3 \\ 0 & 4 & -1 \end{vmatrix} = 0\begin{vmatrix} 0 & 3 \\ 4 & -1 \end{vmatrix} - 4\begin{vmatrix} 3 & 5 \\ 1 & 3 \end{vmatrix} + (-1)\begin{vmatrix} 3 & -2 \\ 1 & 0 \end{vmatrix} = 0(-12) - 4(4) - (2) = -18$

(b) $\begin{vmatrix} 3 & -2 & 5 \\ 1 & 0 & 3 \\ 0 & 4 & -1 \end{vmatrix} = 3\begin{vmatrix} 0 & 3 \\ 4 & -1 \end{vmatrix} - 1\begin{vmatrix} -2 & 5 \\ 4 & -1 \end{vmatrix} + 0\begin{vmatrix} -2 & 5 \\ 0 & 3 \end{vmatrix} = 3(-12) - (-18) + 0(-6) = -18$

39. (a) $\begin{vmatrix} -3 & 2 & 1 \\ 4 & 5 & 6 \\ 2 & -3 & 1 \end{vmatrix} = -3\begin{vmatrix} 5 & 6 \\ -3 & 1 \end{vmatrix} - 2\begin{vmatrix} 4 & 6 \\ 2 & 1 \end{vmatrix} + \begin{vmatrix} 4 & 5 \\ 2 & -3 \end{vmatrix} = -3(23) - 2(-8) - 22 = -75$

(b) $\begin{vmatrix} -3 & 2 & 1 \\ 4 & 5 & 6 \\ 2 & -3 & 1 \end{vmatrix} = -2\begin{vmatrix} 4 & 6 \\ 2 & 1 \end{vmatrix} + 5\begin{vmatrix} -3 & 1 \\ 2 & 1 \end{vmatrix} + 3\begin{vmatrix} -3 & 1 \\ 4 & 6 \end{vmatrix} = -2(-8) + 5(-5) + 3(-22) = -75$

40. (a) $\begin{vmatrix} -3 & 4 & 2 \\ 6 & 3 & 1 \\ 4 & -7 & -8 \end{vmatrix} = -6\begin{vmatrix} 4 & 2 \\ -7 & -8 \end{vmatrix} + 3\begin{vmatrix} -3 & 2 \\ 4 & -8 \end{vmatrix} - 1\begin{vmatrix} -3 & 4 \\ 4 & -7 \end{vmatrix} = -6(-18) + 3(16) - (5) = 151$

(b) $\begin{vmatrix} -3 & 4 & 2 \\ 6 & 3 & 1 \\ 4 & -7 & -8 \end{vmatrix} = 2\begin{vmatrix} 6 & 3 \\ 4 & -7 \end{vmatrix} - \begin{vmatrix} -3 & 4 \\ 4 & -7 \end{vmatrix} - 8\begin{vmatrix} -3 & 4 \\ 6 & 3 \end{vmatrix} = 2(-54) - 5 - 8(-33) = 151$

41. (a) $\begin{vmatrix} 6 & 0 & -3 & 5 \\ 4 & 0 & 6 & -8 \\ -1 & 0 & 7 & 4 \\ 8 & 0 & 0 & 2 \end{vmatrix} = -8\begin{vmatrix} 0 & -3 & 5 \\ 0 & 6 & -8 \\ 0 & 7 & 4 \end{vmatrix} + 0\begin{vmatrix} 6 & -3 & 5 \\ 4 & 6 & -8 \\ -1 & 7 & 4 \end{vmatrix} - 0\begin{vmatrix} 6 & 0 & 5 \\ 4 & 0 & -8 \\ -1 & 0 & 4 \end{vmatrix} + 2\begin{vmatrix} 6 & 0 & -3 \\ 4 & 0 & 6 \\ -1 & 0 & 0 \end{vmatrix}$

$= -8(0) + 2(0) = 0$

(b) $\begin{vmatrix} 6 & 0 & -3 & 5 \\ 4 & 0 & 6 & -8 \\ -1 & 0 & 7 & 4 \\ 8 & 0 & 0 & 2 \end{vmatrix} = -0\begin{vmatrix} 4 & 6 & -8 \\ -1 & 7 & 4 \\ 8 & 0 & 2 \end{vmatrix} + 0\begin{vmatrix} 6 & -3 & 5 \\ -1 & 7 & 4 \\ 8 & 0 & 2 \end{vmatrix} - 0\begin{vmatrix} 6 & -3 & 5 \\ 4 & 6 & -8 \\ 8 & 0 & 2 \end{vmatrix} + 0\begin{vmatrix} 6 & -3 & 5 \\ 4 & 6 & -8 \\ -1 & 7 & 4 \end{vmatrix}$

$= 0$

42. (a) $\begin{vmatrix} 10 & 8 & 3 & -7 \\ 4 & 0 & 5 & -6 \\ 0 & 3 & 2 & 7 \\ 0 & 0 & 0 & 0 \end{vmatrix} = 0\begin{vmatrix} 8 & 3 & -7 \\ 0 & 5 & -6 \\ 0 & -3 & 2 \end{vmatrix} - 0\begin{vmatrix} 10 & 3 & -7 \\ 4 & 5 & -6 \\ 1 & -3 & 2 \end{vmatrix} + 0\begin{vmatrix} 10 & 8 & -7 \\ 4 & 0 & -6 \\ 1 & 0 & 2 \end{vmatrix} - 0\begin{vmatrix} 10 & 8 & 3 \\ 4 & 0 & 5 \\ 1 & 0 & 2 \end{vmatrix}$

$= 0$

(b) $\begin{vmatrix} 10 & 8 & 3 & -7 \\ 4 & 0 & 5 & -6 \\ 0 & 3 & 2 & 7 \\ 0 & 0 & 0 & 0 \end{vmatrix} = 10\begin{vmatrix} 0 & 5 & -6 \\ 3 & 2 & 7 \\ 0 & 0 & 0 \end{vmatrix} - 4\begin{vmatrix} 8 & 3 & -7 \\ 3 & 2 & 7 \\ 0 & 0 & 0 \end{vmatrix} + 0\begin{vmatrix} 8 & 3 & -7 \\ 0 & 5 & -6 \\ 0 & 0 & 0 \end{vmatrix} - 0\begin{vmatrix} 8 & 3 & -7 \\ 0 & 5 & -6 \\ 3 & 2 & 7 \end{vmatrix}$

$= 10(0) - 4(0) + 0(0) - 0(427) = 0$

43. (a) $\begin{vmatrix} -2 & 4 & 7 & 1 \\ 3 & 0 & 0 & 0 \\ 8 & 5 & 10 & 5 \\ 6 & 0 & 5 & 0 \end{vmatrix} = -3\begin{vmatrix} 4 & 7 & 1 \\ 5 & 10 & 5 \\ 0 & 5 & 0 \end{vmatrix} + 0\begin{vmatrix} -2 & 7 & 1 \\ 8 & 10 & 5 \\ 6 & 5 & 0 \end{vmatrix} - 0\begin{vmatrix} -2 & 4 & 1 \\ 8 & 5 & 5 \\ 6 & 0 & 0 \end{vmatrix} + 0\begin{vmatrix} -2 & 4 & 7 \\ 8 & 5 & 10 \\ 6 & 0 & 5 \end{vmatrix}$

$= -3(-75) = 225$

(b) $\begin{vmatrix} -2 & 4 & 7 & 1 \\ 3 & 0 & 0 & 0 \\ 8 & 5 & 10 & 5 \\ 6 & 0 & 5 & 0 \end{vmatrix} = -1\begin{vmatrix} 3 & 0 & 0 \\ 8 & 5 & 10 \\ 6 & 0 & 5 \end{vmatrix} + 0\begin{vmatrix} -2 & 4 & 7 \\ 8 & 5 & 10 \\ 6 & 0 & 5 \end{vmatrix} - 5\begin{vmatrix} -2 & 4 & 7 \\ 3 & 0 & 0 \\ 6 & 0 & 5 \end{vmatrix} + 0\begin{vmatrix} -2 & 4 & 7 \\ 3 & 0 & 0 \\ 8 & 5 & 10 \end{vmatrix}$

$= (-1)(75) - 5(-60) = 225$

44. (a) $\begin{vmatrix} 7 & 0 & 0 & -6 \\ 6 & 0 & 1 & -2 \\ 1 & -2 & 3 & 2 \\ -3 & 0 & -1 & 4 \end{vmatrix} = -(-3)\begin{vmatrix} 0 & 0 & -6 \\ 0 & 1 & -2 \\ -2 & 3 & 2 \end{vmatrix} + 0\begin{vmatrix} 7 & 0 & -6 \\ 6 & 1 & -2 \\ 1 & 3 & 2 \end{vmatrix} - (-1)\begin{vmatrix} 7 & 0 & -6 \\ 6 & 0 & -2 \\ 1 & -2 & 2 \end{vmatrix} + 4\begin{vmatrix} 7 & 0 & 0 \\ 6 & 0 & 1 \\ 1 & -2 & 3 \end{vmatrix}$

$= (3)(-12) + (44) + 4(14) = 64$

(b) $\begin{vmatrix} 7 & 0 & 0 & -6 \\ 6 & 0 & 1 & -2 \\ 1 & -2 & 3 & 2 \\ -3 & 0 & -1 & 4 \end{vmatrix} = 0\begin{vmatrix} 6 & 1 & -2 \\ 1 & 3 & 2 \\ -3 & -1 & 4 \end{vmatrix} + 0\begin{vmatrix} 7 & 0 & -6 \\ 1 & 3 & 2 \\ -3 & -1 & 4 \end{vmatrix} - (-2)\begin{vmatrix} 7 & 0 & -6 \\ 6 & 1 & -2 \\ -3 & -1 & 4 \end{vmatrix} + 0\begin{vmatrix} 7 & 0 & -6 \\ 6 & 1 & -2 \\ 1 & 3 & 2 \end{vmatrix}$

$= (2)(32) = 64$

45. Expand along Column 1.

$\begin{vmatrix} -1 & 2 & -5 \\ 0 & 3 & 4 \\ 0 & 0 & 3 \end{vmatrix} = -1\begin{vmatrix} 3 & 4 \\ 0 & 3 \end{vmatrix} - 0\begin{vmatrix} 2 & -5 \\ 0 & 3 \end{vmatrix} + 0\begin{vmatrix} 2 & -5 \\ 3 & 4 \end{vmatrix}$

$= -1(9) - 0(6) + 0(23) = -9$

46. Expand along Row 1.

$\begin{vmatrix} 1 & 0 & 0 \\ -4 & -1 & 0 \\ 5 & 1 & 5 \end{vmatrix} = 1\begin{vmatrix} -1 & 0 \\ 1 & 5 \end{vmatrix} - 0\begin{vmatrix} -4 & 0 \\ 5 & 5 \end{vmatrix} + 0\begin{vmatrix} -4 & -1 \\ 5 & 1 \end{vmatrix}$

$= 1(-5) - 0(-20) + 0(1) = -5$

47. Expand along Row 2.

$\begin{vmatrix} 6 & 3 & -7 \\ 0 & 0 & 0 \\ 4 & -6 & 3 \end{vmatrix} = 0\begin{vmatrix} 3 & -7 \\ -6 & 3 \end{vmatrix} - 0\begin{vmatrix} 6 & -7 \\ 4 & 3 \end{vmatrix} + 0\begin{vmatrix} 6 & 3 \\ 4 & -6 \end{vmatrix} = 0$

48. Expand along Row 1.

$\begin{vmatrix} 0 & 1 & 2 \\ 3 & 1 & 0 \\ -2 & 0 & 3 \end{vmatrix} = 0\begin{vmatrix} 1 & 0 \\ 0 & 3 \end{vmatrix} - (1)\begin{vmatrix} 3 & 0 \\ -2 & 3 \end{vmatrix} + 2\begin{vmatrix} 3 & 1 \\ -2 & 0 \end{vmatrix}$

$= -(9) + 2(2) = -5$

49. Expand along Column 1.

$$\begin{vmatrix} 2 & -1 & 0 \\ 4 & 2 & 1 \\ 4 & 2 & 1 \end{vmatrix} = 2\begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix} - 4\begin{vmatrix} -1 & 0 \\ 2 & 1 \end{vmatrix} + 4\begin{vmatrix} -1 & 0 \\ 2 & 1 \end{vmatrix}$$

$$= 2(0) - 4(-1) + 4(-1) = 0$$

50. Expand along Row 3.

$$\begin{vmatrix} -2 & 2 & 3 \\ 1 & -1 & 0 \\ 0 & 1 & 4 \end{vmatrix} = 0\begin{vmatrix} 2 & 3 \\ -1 & 0 \end{vmatrix} - 1\begin{vmatrix} -2 & 3 \\ 1 & 0 \end{vmatrix} + 4\begin{vmatrix} -2 & 2 \\ 1 & -1 \end{vmatrix}$$

$$= 0(3) - 1(-3) + 4(0) = 3$$

51. Expand along Column 3.

$$\begin{vmatrix} 1 & 4 & -2 \\ 3 & 2 & 0 \\ -1 & 4 & 3 \end{vmatrix} = -2\begin{vmatrix} 3 & 2 \\ -1 & 4 \end{vmatrix} + 3\begin{vmatrix} 1 & 4 \\ 3 & 2 \end{vmatrix}$$

$$= -2(14) + 3(-10) = -58$$

52. Expand along Column 1.

$$\begin{vmatrix} 2 & -1 & 3 \\ 4 & 2 & 1 \\ 4 & 2 & 1 \end{vmatrix} = 2\begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix} - 4\begin{vmatrix} -1 & 3 \\ 2 & 1 \end{vmatrix} + 4\begin{vmatrix} -1 & 3 \\ 2 & 1 \end{vmatrix}$$

$$= 2(0) - 4(-7) + 4(-7) = 0$$

53. Expand along Column 3.

$$\begin{vmatrix} 2 & 6 & 0 & 2 \\ 2 & 7 & 3 & 6 \\ 1 & 0 & 0 & 1 \\ 3 & 7 & 0 & 7 \end{vmatrix} = 0\begin{vmatrix} 2 & 7 & 6 \\ 1 & 0 & 1 \\ 3 & 7 & 7 \end{vmatrix} - 3\begin{vmatrix} 2 & 6 & 2 \\ 1 & 0 & 1 \\ 3 & 7 & 7 \end{vmatrix} + 0\begin{vmatrix} 2 & 6 & 2 \\ 2 & 7 & 6 \\ 3 & 7 & 7 \end{vmatrix} - 0\begin{vmatrix} 2 & 6 & 2 \\ 2 & 7 & 6 \\ 1 & 0 & 1 \end{vmatrix}$$

$$= -3(-24) = 72$$

54. Expand along Row 3.

$$\begin{vmatrix} 1 & 4 & 3 & 2 \\ -5 & 6 & 2 & 1 \\ 0 & 0 & 0 & 0 \\ 3 & -2 & 1 & 5 \end{vmatrix} = 0$$

55. Expand along Column 1.

$$\begin{vmatrix} 5 & 3 & 0 & 6 \\ 4 & 6 & 4 & 12 \\ 0 & 2 & -3 & 4 \\ 0 & 1 & -2 & 2 \end{vmatrix} = 5\begin{vmatrix} 6 & 4 & 12 \\ 2 & -3 & 4 \\ 1 & -2 & 2 \end{vmatrix} - 4\begin{vmatrix} 3 & 0 & 6 \\ 2 & -3 & 4 \\ 1 & -2 & 2 \end{vmatrix}$$

$$= 5(0) - 4(0) = 0$$

56. Expand along Row 2.

$$\begin{vmatrix} 3 & 6 & -5 & 4 \\ -2 & 0 & 6 & 0 \\ 1 & 1 & 2 & 2 \\ 0 & 3 & -1 & -1 \end{vmatrix} = -(-2)\begin{vmatrix} 6 & -5 & 4 \\ 1 & 2 & 2 \\ 3 & -1 & -1 \end{vmatrix} - 6\begin{vmatrix} 3 & 6 & 4 \\ 1 & 1 & 2 \\ 0 & 3 & -1 \end{vmatrix}$$

$$= 2(-63) - 6(-3) = -108$$

57. Expand along Column 2, then along Column 4.

$$\begin{vmatrix} 3 & 2 & 4 & -1 & 5 \\ -2 & 0 & 1 & 3 & 2 \\ 1 & 0 & 0 & 4 & 0 \\ 6 & 0 & 2 & -1 & 0 \\ 3 & 0 & 5 & 1 & 0 \end{vmatrix} = -2\begin{vmatrix} -2 & 1 & 3 & 2 \\ 1 & 0 & 4 & 0 \\ 6 & 2 & -1 & 0 \\ 3 & 5 & 1 & 0 \end{vmatrix} = (-2)(-2)\begin{vmatrix} 1 & 0 & 4 \\ 6 & 2 & -1 \\ 3 & 5 & 1 \end{vmatrix} = 4(103) = 412$$

58. Expand along Column 1, then along Column 1.

$$\begin{vmatrix} 5 & 2 & 0 & 0 & -2 \\ 0 & 1 & 4 & 3 & \frac{1}{2} \\ 0 & 0 & 2 & 6 & 3 \\ 0 & 0 & 3 & \frac{3}{2} & 1 \\ 0 & 0 & 0 & 0 & 2 \end{vmatrix} = 5\begin{vmatrix} 1 & 4 & 3 & \frac{1}{2} \\ 0 & 2 & 6 & 3 \\ 0 & 3 & \frac{3}{2} & 1 \\ 0 & 0 & 0 & 2 \end{vmatrix} = 5 \cdot 1\begin{vmatrix} 2 & 6 & 3 \\ 3 & \frac{3}{2} & 1 \\ 0 & 0 & 2 \end{vmatrix} = 5(-30) = -150$$

59. $\begin{vmatrix} 3 & 8 & -7 \\ 0 & -5 & 4 \\ 8 & 1 & 6 \end{vmatrix} = -126$

60. $\begin{vmatrix} 5 & -8 & 0 \\ 9 & 7 & 4 \\ -8 & 7 & 1 \end{vmatrix} = 223$

61. $\begin{vmatrix} 1 & -1 & 8 & 4 \\ 2 & 6 & 0 & -4 \\ 2 & 0 & 2 & 6 \\ 0 & 2 & 8 & 0 \end{vmatrix} = -336$

62. $\begin{vmatrix} 0 & -3 & 8 & 2 \\ 8 & 1 & -1 & 6 \\ -4 & 6 & 0 & 9 \\ -7 & 0 & 0 & 14 \end{vmatrix} = 7441$

63. (a) $\begin{vmatrix} -1 & 0 \\ 0 & 3 \end{vmatrix} = -3$

(b) $\begin{vmatrix} 2 & 0 \\ 0 & -1 \end{vmatrix} = -2$

(c) $\begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix}$

(d) $\begin{vmatrix} -2 & 0 \\ 0 & -3 \end{vmatrix} = 6$

64. (a) $|A| = \begin{vmatrix} -2 & 1 \\ 4 & -2 \end{vmatrix} = 0$

(b) $|B| = \begin{vmatrix} 1 & 2 \\ 0 & -1 \end{vmatrix} = -1$

(c) $AB = \begin{bmatrix} -2 & 1 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -2 & -5 \\ 4 & 10 \end{bmatrix}$

(d) $|AB| = \begin{vmatrix} -2 & -5 \\ 4 & 10 \end{vmatrix} = 0$

65. (a) $\begin{vmatrix} 4 & 0 \\ 3 & -2 \end{vmatrix} = -8$

(b) $\begin{vmatrix} -1 & 1 \\ -2 & 2 \end{vmatrix} = 0$

(c) $\begin{bmatrix} 4 & 0 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} -4 & 4 \\ 1 & -1 \end{bmatrix}$

(d) $\begin{vmatrix} -4 & 4 \\ 1 & -1 \end{vmatrix} = 0$

66. (a) $|A| = \begin{vmatrix} 5 & 4 \\ 3 & -1 \end{vmatrix} = -17$

(b) $|B| = \begin{vmatrix} 0 & 6 \\ 1 & -2 \end{vmatrix} = -6$

(c) $AB = \begin{bmatrix} 5 & 4 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} 0 & 6 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 4 & 22 \\ -1 & 20 \end{bmatrix}$

(d) $|AB| = \begin{vmatrix} 4 & 22 \\ -1 & 20 \end{vmatrix} = 102$

67. (a) $\begin{vmatrix} -1 & 2 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = 2$

(b) $\begin{vmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{vmatrix} = -6$

(c) $\begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 3 \\ -1 & 0 & 3 \\ 0 & 2 & 0 \end{bmatrix}$

(d) $\begin{vmatrix} 1 & 4 & 3 \\ -1 & 0 & 3 \\ 0 & 2 & 0 \end{vmatrix} = -12$

68. (a) $|A| = \begin{vmatrix} 2 & 0 & 1 \\ 1 & -1 & 2 \\ 3 & 1 & 0 \end{vmatrix} = 0$

(b) $|B| = \begin{vmatrix} 2 & -1 & 4 \\ 0 & 1 & 3 \\ 3 & -2 & 1 \end{vmatrix} = -7$

(c) $AB = \begin{bmatrix} 2 & 0 & 1 \\ 1 & -1 & 2 \\ 3 & 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 & 4 \\ 0 & 1 & 3 \\ 3 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -4 & 9 \\ 8 & -6 & 3 \\ 6 & -2 & 15 \end{bmatrix}$

(d) $|AB| = \begin{vmatrix} 7 & -4 & 9 \\ 8 & -6 & 3 \\ 6 & -2 & 15 \end{vmatrix} = 0$

69. Answers will vary. *Sample answer*:

$|A| = \begin{vmatrix} 3 & 2 \\ 3 & 3 \end{vmatrix} = 9 - 6 = 3$

70. Answers will vary. *Sample answer*:

$|A| = \begin{vmatrix} 1 & 1 \\ 6 & 1 \end{vmatrix} = 1 - 6 = -5$

71. Answers will vary. *Sample Answer*: $|A| = \begin{vmatrix} 4 & 2 & -1 \\ 2 & 1 & 0 \\ 1 & 1 & 3 \end{vmatrix} = -2\begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix} + 1\begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix} = -2(7) + 13 = -1$

72. Answers will vary. *Sample Answer*: $|A| = \begin{vmatrix} 2 & -4 & 2 \\ 0 & 1 & -1 \\ -1 & 2 & 1 \end{vmatrix} = 1\begin{vmatrix} 2 & 2 \\ -1 & 1 \end{vmatrix} - (-1)\begin{vmatrix} 2 & -4 \\ -1 & 2 \end{vmatrix} = 4$

73. Answers will vary. *Sample Answer*: $|A| = \begin{vmatrix} 2 & 3 \\ 8 & 12 \end{vmatrix} = 24 - 24 = 0$

74. Answers will vary. *Sample Answer*: $|A| = \begin{vmatrix} 6 & 2 & 3 \\ 7 & -5 & 8 \\ -1 & 7 & -5 \end{vmatrix} = 6\begin{vmatrix} -5 & 8 \\ 7 & -5 \end{vmatrix} - 2\begin{vmatrix} 7 & 8 \\ -1 & -5 \end{vmatrix} + 3\begin{vmatrix} 7 & -5 \\ -1 & 7 \end{vmatrix}$

$$= 6(-31) - 2(-27) + 3(44) = 0$$

75. $\begin{vmatrix} w & x \\ y & z \end{vmatrix} = wz - xy$

$-\begin{vmatrix} y & z \\ w & x \end{vmatrix} = -(xy - wz) = wz - xy$

So, $\begin{vmatrix} w & x \\ y & z \end{vmatrix} = -\begin{vmatrix} y & z \\ w & x \end{vmatrix}$.

76. $\begin{vmatrix} w & cx \\ y & cz \end{vmatrix} = cwz - cxy = c(wz - xy)$

$c\begin{vmatrix} w & x \\ y & z \end{vmatrix} = c(wz - xy)$

So, $\begin{vmatrix} w & cx \\ y & cz \end{vmatrix} = c\begin{vmatrix} w & x \\ y & z \end{vmatrix}$.

77. $\begin{vmatrix} w & x \\ y & z \end{vmatrix} = wz - xy$

$\begin{vmatrix} w & x + cw \\ y & z + cy \end{vmatrix} = w(z + cy) - y(x + cw) = wz - xy$

So, $\begin{vmatrix} w & x \\ y & z \end{vmatrix} = \begin{vmatrix} w & x + cw \\ y & z + cy \end{vmatrix}$.

78. $\begin{vmatrix} w & x \\ cw & cx \end{vmatrix} = cxw - cxw = 0$

So, $\begin{vmatrix} w & x \\ cw & cx \end{vmatrix} = 0$.

79. $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = \begin{vmatrix} y & y^2 \\ z & z^2 \end{vmatrix} - \begin{vmatrix} x & x^2 \\ z & z^2 \end{vmatrix} + \begin{vmatrix} x & x^2 \\ y & y^2 \end{vmatrix}$

$$\begin{aligned}
&= (yz^2 - y^2z) - (xz^2 - x^2z) + (xy^2 - x^2y) \\
&= yz^2 - xz^2 - y^2z + x^2z + xy(y - x) \\
&= z^2(y - x) - z(y^2 - x^2) + xy(y - x) \\
&= z^2(y - x) - z(y - x)(y + x) + xy(y - x) \\
&= (y - x)[z^2 - z(y + x) + xy] \\
&= (y - x)[z^2 - zy - zx + xy] \\
&= (y - x)[z^2 - zx - zy + xy] \\
&= (y - x)[z(z - x) - y(z - x)] \\
&= (y - x)(z - x)(z - y)
\end{aligned}$$

80. $\begin{vmatrix} a+b & a & a \\ a & a+b & a \\ a & a & a+b \end{vmatrix} = (a+b)\begin{vmatrix} a+b & a \\ a & a+b \end{vmatrix} - a\begin{vmatrix} a & a \\ a & a+b \end{vmatrix} + a\begin{vmatrix} a & a \\ a+b & a \end{vmatrix}$

$= (a+b)\left[(a+b)^2 - a^2\right] - a\left[a(a+b) - a^2\right] + a\left[a^2 - a(a+b)\right]$

$= (a+b)^3 - a^2(a+b) - a^2(a+b) + a^3 + a^3 - a^2(a+b)$

$= (a+b)^3 - 3a^2(a+b) + 2a^3$

$= a^3 + 3a^2b + 3ab^2 + b^3 - 3a^3 - 3a^2b + 2a^3$

$= 3ab^2 + b^3 = b^2(3a+b)$

81. $\begin{vmatrix} x & 2 \\ 1 & x \end{vmatrix} = 2$

$x^2 - 2 = 2$

$x^2 = 4$

$x = \pm 2$

82. $\begin{vmatrix} x & 4 \\ -1 & x \end{vmatrix} = 20$

$x^2 - (-4) = 20$

$x^2 = 16$

$x = \pm 4$

83. $\begin{vmatrix} x+1 & 2 \\ -1 & x \end{vmatrix} = 4$

$(x+1)(x) - (2)(-1) = 4$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2$ or $x = 1$

84. $\begin{vmatrix} x-2 & -1 \\ -3 & x \end{vmatrix} = 0$

$x(x-2) - (-3)(-1) = 0$

$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$x = -1$ or $x = 3$

85. $\begin{vmatrix} x+3 & 2 \\ 1 & x+2 \end{vmatrix} = 0$

$(x+3)(x+2) - 2 = 0$

$x^2 + 5x + 4 = 0$

$(x+1)(x+4) = 0$

$x = -1$ or $x = -4$

86. $\begin{vmatrix} x+4 & -2 \\ 7 & x-5 \end{vmatrix} = 0$

$(x+4)(x-5) - 7(-2) = 0$

$x^2 - x - 6 = 0$

$(x+2)(x-3) = 0$

$x = -2$ or $x = 3$

87. $\begin{vmatrix} 4u & -1 \\ -1 & 2v \end{vmatrix} = 8uv - 1$

88. $\begin{vmatrix} 3x^2 & -3y^2 \\ 1 & 1 \end{vmatrix} = 3x^2 - \left(-3y^2\right) = 3x^2 + 3y^2$

89. $\begin{vmatrix} e^{2x} & e^{3x} \\ 2e^{2x} & 3e^{3x} \end{vmatrix} = 3e^{5x} - 2e^{5x} = e^{5x}$

90. $\begin{vmatrix} e^{-x} & xe^{-x} \\ -e^{-x} & (1-x)e^{-x} \end{vmatrix} = (1-x)e^{-2x} - \left(-xe^{-2x}\right) = e^{-2x} - xe^{-2x} + xe^{-2x} = e^{-2x}$

91. $\begin{vmatrix} x & \ln x \\ 1 & \dfrac{1}{x} \end{vmatrix} = 1 - \ln x$

92. $\begin{vmatrix} x & x \ln x \\ 1 & 1 + \ln x \end{vmatrix} = x(1 + \ln x) - x \ln x = x + x \ln x - x \ln x = x$

93. True. If an entire row is zero, then each cofactor in the expansion is multiplied by zero.

94. True. If $A = \begin{bmatrix} a & b \\ a & b \end{bmatrix}$, then $|A| = \begin{vmatrix} a & b \\ a & b \end{vmatrix} = ab - ab = 0$.

95. *Sample answer:* Let $A = \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 0 \\ 3 & 5 \end{bmatrix}$.

$|A| = \begin{vmatrix} 1 & 3 \\ -2 & 4 \end{vmatrix} = 10$, $|B| = \begin{vmatrix} -4 & 0 \\ 3 & 5 \end{vmatrix} = -20$, $|A| + |B| = -10$

$A + B = \begin{bmatrix} -3 & 3 \\ 1 & 9 \end{bmatrix}$, $|A + B| = \begin{vmatrix} -3 & 3 \\ 1 & 9 \end{vmatrix} = -30$

So, $|A + B| \neq |A| + |B|$.

96. (a) $\begin{vmatrix} 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12 \end{vmatrix} = 0$

$\begin{vmatrix} 33 & 34 & 35 \\ 36 & 37 & 38 \\ 39 & 40 & 41 \end{vmatrix} = 0$ $\quad\begin{vmatrix} -5 & -4 & -3 \\ -2 & -1 & 0 \\ 1 & 2 & 3 \end{vmatrix} = 0$

$\begin{vmatrix} 19 & 20 & 21 & 22 \\ 23 & 24 & 25 & 26 \\ 27 & 28 & 29 & 30 \\ 31 & 32 & 33 & 34 \end{vmatrix} = 0$ $\quad\begin{vmatrix} 57 & 58 & 59 & 60 \\ 61 & 62 & 63 & 64 \\ 65 & 66 & 67 & 68 \\ 69 & 70 & 71 & 72 \end{vmatrix} = 0$

For an $n \times n$ matrix $(n > 2)$ with consecutive integer entries, the determinant appears to be 0.

(b) $\begin{vmatrix} x & x+1 & x+2 \\ x+3 & x+4 & x+5 \\ x+6 & x+7 & x+8 \end{vmatrix} = x\begin{vmatrix} x+4 & x+5 \\ x+7 & x+8 \end{vmatrix} - (x+1)\begin{vmatrix} x+3 & x+5 \\ x+6 & x+8 \end{vmatrix} + (x+2)\begin{vmatrix} x+3 & x+4 \\ x+6 & x+7 \end{vmatrix}$

$$= x\left[(x+4)(x+8) - (x+7)(x+5)\right] - (x+1)\left[(x+3)(x+8) - (x+6)(x+5)\right] + (x+2)\left[(x+3)(x+7) - (x+6)(x+4)\right]$$

$$= x\left[\left(x^2 + 12x + 32\right) - \left(x^2 + 12x + 35\right)\right] - (x+1)\left[\left(x^2 + 11x + 24\right) - \left(x^2 + 11x + 30\right)\right] + (x+2)\left[\left(x^2 + 10x + 21\right) - \left(x^2 + 10x + 24\right)\right]$$

$$= -3x - (x+1)(-6) + (x+2)(-3)$$

$$= -3x + 6x + 6 - 3x - 6 = 0$$

97. The signs of the cofactors should be $-, +, -$.

$\begin{vmatrix} 1 & 1 & 4 \\ 3 & 2 & 0 \\ 2 & 1 & 3 \end{vmatrix} = 3(-1)\begin{vmatrix} 1 & 4 \\ 1 & 3 \end{vmatrix} + 2(1)\begin{vmatrix} 1 & 4 \\ 2 & 3 \end{vmatrix} + (0)(-1)\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = 3(1) + 2(-5) + 0 = -7.$

98. Let $A = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$ and $|A| = 5$.

$$2A = \begin{bmatrix} 2x_{11} & 2x_{12} & 2x_{13} \\ 2x_{21} & 2x_{22} & 2x_{23} \\ 2x_{31} & 2x_{32} & 2x_{33} \end{bmatrix}$$

$$\begin{aligned} |2A| &= 2x_{11}\begin{vmatrix} 2x_{22} & 2x_{23} \\ 2x_{32} & 2x_{33} \end{vmatrix} - 2x_{12}\begin{vmatrix} 2x_{21} & 2x_{23} \\ 2x_{31} & 2x_{33} \end{vmatrix} + 2x_{13}\begin{vmatrix} 2x_{21} & 2x_{22} \\ 2x_{31} & 2x_{32} \end{vmatrix} \\ &= 2\left[x_{11}(4x_{22}x_{33} - 4x_{32}x_{23}) - x_{12}(4x_{21}x_{33} - 4x_{31}x_{23}) + x_{13}(4x_{21}x_{32} - 4x_{31}x_{22})\right] \\ &= 8\left[x_{11}(x_{22}x_{33} - x_{32}x_{23}) - x_{12}(x_{21}x_{33} - x_{31}x_{23}) + x_{13}(x_{21}x_{32} - x_{31}x_{22})\right] \\ &= 8|A| \end{aligned}$$

So, $|2A| = 8|A| = 8(5) = 40$.

99. (a) $\begin{vmatrix} 1 & 3 & 4 \\ -7 & 2 & -5 \\ 6 & 1 & 2 \end{vmatrix} = -115$

$$-\begin{vmatrix} 1 & 4 & 3 \\ -7 & -5 & 2 \\ 6 & 2 & 1 \end{vmatrix} = -115$$

Column 2 and Column 3 were interchanged.

(b) $\begin{vmatrix} 1 & 3 & 4 \\ -2 & 2 & 0 \\ 1 & 6 & 2 \end{vmatrix} = -40$

$$-\begin{vmatrix} 1 & 6 & 2 \\ -2 & 2 & 0 \\ 1 & 3 & 4 \end{vmatrix} = -40$$

Row 1 and Row 3 were interchanged.

100. (a) Multiplying Row 1 of the matrix $\begin{bmatrix} 1 & -3 \\ 5 & 2 \end{bmatrix}$ by –5 and adding it to Row 2 gives the matrix $\begin{bmatrix} 1 & -3 \\ 0 & 17 \end{bmatrix}$.

$$\begin{vmatrix} 1 & -3 \\ 5 & 2 \end{vmatrix} = 17 = \begin{vmatrix} 1 & -3 \\ 0 & 17 \end{vmatrix}$$

(b) Multiplying Row 2 of the matrix $\begin{bmatrix} 5 & 4 & 2 \\ 2 & -3 & 4 \\ 7 & 6 & 3 \end{bmatrix}$ by –2 and adding it to Row 1 gives the matrix $\begin{bmatrix} 1 & 10 & -6 \\ 2 & -3 & 4 \\ 7 & 6 & 3 \end{bmatrix}$.

$$\begin{vmatrix} 5 & 4 & 2 \\ 2 & -3 & 4 \\ 7 & 6 & 3 \end{vmatrix} = -11 = \begin{vmatrix} 1 & 10 & -6 \\ 2 & -3 & 4 \\ 7 & 6 & 3 \end{vmatrix}$$

101. (a) $A = \begin{bmatrix} 1 & 2 \\ 2 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 10 \\ 2 & -3 \end{bmatrix}$

$$|B| = \begin{vmatrix} 5 & 10 \\ 2 & -3 \end{vmatrix} = -35$$

$$5|A| = 5\begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix} = -35$$

Row 1 was multiplied by 5.

$|B| = 5|A|$

(b) $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -3 & 2 \\ 7 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 8 & -3 \\ 3 & -12 & 6 \\ 7 & 4 & 9 \end{bmatrix}$

$$|B| = \begin{bmatrix} 1 & 8 & -3 \\ 3 & -12 & 6 \\ 7 & 4 & 9 \end{bmatrix} = -300$$

$$12|A| = 12\begin{vmatrix} 1 & 2 & -1 \\ 3 & -3 & 2 \\ 7 & 1 & 3 \end{vmatrix} = -300$$

Column 2 was multiplied by 4 and Column 3 was multiplied by 3.

$|B| = (4)(3)|A| = 12|A|$

102. (a) $\begin{vmatrix} 2 & -4 & 5 \\ 1 & -2 & 3 \\ 0 & 0 & 0 \end{vmatrix} = 0,$ because Row 3 is all zeros.

(b) $\begin{vmatrix} 4 & -4 & 5 & 7 \\ 2 & -2 & 3 & 1 \\ 4 & -4 & 5 & 7 \\ 6 & 1 & -3 & -3 \end{vmatrix} = 0,$ because Row 1 and Row 3 are identical.

103. (a) $\begin{vmatrix} 7 & 0 \\ 0 & 4 \end{vmatrix} = 7(4) - 0 = 28$

(b) $$\begin{vmatrix} -1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 2 \end{vmatrix} = (-1)\begin{vmatrix} 5 & 0 \\ 0 & 2 \end{vmatrix} - 0\begin{vmatrix} 0 & 0 \\ 0 & 2 \end{vmatrix} + 0\begin{vmatrix} 0 & 5 \\ 0 & 0 \end{vmatrix}$$
$$= (-1)(10) = -10$$

(c) $$\begin{vmatrix} 2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{vmatrix} = (-2)\begin{vmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{vmatrix} - 0\begin{vmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{vmatrix} + 0\begin{vmatrix} 0 & -2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{vmatrix} - 0\begin{vmatrix} 0 & -2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{vmatrix}$$
$$= (-2)\left((2)\begin{vmatrix} 1 & 0 \\ 0 & 3 \end{vmatrix} - 0\begin{vmatrix} 0 & 0 \\ 0 & 3 \end{vmatrix} + 0\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}\right)$$
$$= (-2)(2)(3) = -12$$

The determinant of a diagonal matrix is the product of the entries on the main diagonal.

Section 8.5 Applications of Matrices and Determinants

1. Cramer's Rule

2. collinear

3. $A = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

4. cryptogram

5. uncoded; coded

6. A^{-1}

7. $\begin{cases} -5x + 9y = -14 \\ 3x - 7y = 10 \end{cases}$

$$x = \frac{\begin{vmatrix} -14 & 9 \\ 10 & -7 \end{vmatrix}}{\begin{vmatrix} -5 & 9 \\ 3 & -7 \end{vmatrix}} = \frac{8}{8} = 1$$

$$y = \frac{\begin{vmatrix} -5 & -14 \\ 3 & 10 \end{vmatrix}}{\begin{vmatrix} -5 & 9 \\ 3 & -7 \end{vmatrix}} = \frac{-8}{8} = -1$$

Solution: $(1, -1)$

8. $\begin{cases} 4x - 3y = -10 \\ 6x + 9y = 12 \end{cases}$

$$x = \frac{\begin{vmatrix} -10 & -3 \\ 12 & 9 \end{vmatrix}}{\begin{vmatrix} 4 & -3 \\ 6 & 9 \end{vmatrix}} = \frac{-54}{54} = -1$$

$$y = \frac{\begin{vmatrix} 4 & -10 \\ 6 & 12 \end{vmatrix}}{\begin{vmatrix} 4 & -3 \\ 6 & 9 \end{vmatrix}} = \frac{108}{54} = 2$$

Solution: $(-1, 2)$

9. $\begin{cases} 3x + 2y = -2 \\ 6x + 4y = 4 \end{cases}$

Because $\begin{vmatrix} 3 & 2 \\ 6 & 4 \end{vmatrix} = 0$, Cramer's Rule does not apply.

The system is inconsistent in this case and has no solution.

10. $\begin{cases} 12x - 7y = -4 \\ -11x + 8y = 10 \end{cases}$

$$x = \frac{\begin{vmatrix} -4 & -7 \\ 10 & 8 \end{vmatrix}}{\begin{vmatrix} 12 & -7 \\ -11 & 8 \end{vmatrix}} = \frac{38}{19} = 2$$

$$y = \frac{\begin{vmatrix} 12 & -4 \\ -11 & 10 \end{vmatrix}}{\begin{vmatrix} 12 & -7 \\ -11 & 8 \end{vmatrix}} = \frac{76}{19} = 4$$

Solution: $(2, 4)$

11. $\begin{cases} 4x - y + z = -5 \\ 2x + 2y + 3z = 10, \\ 5x - 2y + 6z = 1 \end{cases}$ $\quad D = \begin{vmatrix} 4 & -1 & 1 \\ 2 & 2 & 3 \\ 5 & -1 & 6 \end{vmatrix} = 55$

$$x = \frac{\begin{vmatrix} -5 & -1 & 1 \\ 10 & 2 & 3 \\ 1 & -2 & 6 \end{vmatrix}}{55} = \frac{-55}{55} = -1,\ y = \frac{\begin{vmatrix} 4 & -5 & 1 \\ 2 & 10 & 3 \\ 5 & 1 & 6 \end{vmatrix}}{55} = \frac{165}{55} = 3,\ z = \frac{\begin{vmatrix} 4 & -1 & -5 \\ 2 & 2 & 10 \\ 5 & -2 & 1 \end{vmatrix}}{55} = \frac{110}{55} = 2$$

Solution: $(-1, 3, 2)$

12. $\begin{cases} 4x - 2y + 3z = -2 \\ 2x + 2y + 5z = 16 \\ 8x - 5y - 2z = 4 \end{cases}$

$$D = \begin{vmatrix} 4 & -2 & 3 \\ 2 & 2 & 5 \\ 8 & -5 & -2 \end{vmatrix} = -82$$

$$x = \frac{\begin{vmatrix} -2 & -2 & 3 \\ 16 & 2 & 5 \\ 4 & -5 & -2 \end{vmatrix}}{-82} = \frac{-401}{-82} = 5$$

$$y = \frac{\begin{vmatrix} 4 & -2 & 3 \\ 2 & 16 & 5 \\ 8 & 4 & -2 \end{vmatrix}}{-82} = \frac{-656}{-82} = 8$$

$$z = \frac{\begin{vmatrix} 4 & -2 & -2 \\ 2 & 2 & 16 \\ 8 & -5 & 4 \end{vmatrix}}{-82} = \frac{164}{-82} = -2$$

Solution: $(5, 8, -2)$

13. $\begin{cases} x + 2y + 3z = -3 \\ -2x + y - z = 6, \\ 3x - 3y + 2z = -11 \end{cases}$ $\quad D = \begin{vmatrix} 1 & 2 & 3 \\ -2 & 1 & -1 \\ 3 & -3 & 2 \end{vmatrix} = 10$

$$x = \frac{\begin{vmatrix} -3 & 2 & 3 \\ 6 & 1 & -1 \\ -11 & -3 & 2 \end{vmatrix}}{10} = \frac{-20}{10} = -2$$

$$y = \frac{\begin{vmatrix} 1 & -3 & 3 \\ -2 & 6 & -1 \\ 3 & -11 & 2 \end{vmatrix}}{10} = \frac{10}{10} = 1$$

$$z = \frac{\begin{vmatrix} 1 & 2 & -3 \\ -2 & 1 & 6 \\ 3 & -3 & -11 \end{vmatrix}}{10} = \frac{-10}{10} = -1$$

Solution: $(-2, 1, -1)$

14. $\begin{cases} 5x - 4y + z = -14 \\ -x + 2y - 2z = 10 \\ 3x + y + z = 1 \end{cases}$

$$D = \begin{vmatrix} 5 & -4 & 1 \\ -1 & 2 & -2 \\ 3 & 1 & 1 \end{vmatrix} = 33$$

$$x = \frac{\begin{vmatrix} -14 & -4 & 1 \\ 10 & 2 & -2 \\ 1 & 1 & 1 \end{vmatrix}}{33} = \frac{0}{33} = 0$$

$$y = \frac{\begin{vmatrix} 5 & -14 & 1 \\ -1 & 10 & -2 \\ 3 & 1 & 1 \end{vmatrix}}{33} = \frac{99}{33} = 3$$

$$z = \frac{\begin{vmatrix} 5 & -4 & -14 \\ -1 & 2 & 10 \\ 3 & 1 & 1 \end{vmatrix}}{33} = \frac{-66}{33} = -2$$

Solution: $(0, 3, -2)$

15. Vertices: $(0, 0)\ (3, 1), (1, 5)$

$$\text{Area} = \tfrac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 3 & 1 & 1 \\ 1 & 5 & 1 \end{vmatrix} = \tfrac{1}{2}\begin{vmatrix} 3 & 1 \\ 1 & 5 \end{vmatrix} = 7 \text{ square units}$$

16. Vertices: $(0, 0), (4, 5), (5, -2)$

$$\text{Area} = -\tfrac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 4 & 5 & 1 \\ 5 & -2 & 1 \end{vmatrix} = -\tfrac{1}{2}\begin{vmatrix} 4 & 5 \\ 5 & -2 \end{vmatrix} = \tfrac{33}{2} \text{ square units}$$

17. Vertices: $(-2, -3), (2, -3), (0, 4)$

$$\text{Area} = \tfrac{1}{2}\begin{vmatrix} -2 & -3 & 1 \\ 2 & -3 & 1 \\ 0 & 4 & 1 \end{vmatrix} = \tfrac{1}{2}\left(-2\begin{vmatrix} -3 & 1 \\ 4 & 1 \end{vmatrix} - 2\begin{vmatrix} -3 & 1 \\ 4 & 1 \end{vmatrix}\right) = \tfrac{1}{2}(14 + 14) = 14 \text{ square units}$$

18. Vertices: $(-2, 1), (1, 6), (3, -1)$

$$\text{Area} = -\tfrac{1}{2}\begin{vmatrix} -2 & 1 & 1 \\ 1 & 6 & 1 \\ 3 & -1 & 1 \end{vmatrix} = -\tfrac{1}{2}\left(-2\begin{vmatrix} 6 & 1 \\ -1 & 1 \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 6 \\ 3 & -1 \end{vmatrix}\right) = -\tfrac{1}{2}(-14 + 2 - 19) = \tfrac{31}{2} \text{ square units}$$

19. $4 = \pm\frac{1}{2}\begin{vmatrix} -5 & 1 & 1 \\ 0 & 2 & 1 \\ -2 & y & 1 \end{vmatrix}$

$$\pm 8 = -5\begin{vmatrix} 2 & 1 \\ y & 1 \end{vmatrix} - 2\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}$$

$$\pm 8 = -5(2 - y) - 2(-1)$$

$$\pm 8 = 5y - 8$$

$$y = \frac{8 \pm 8}{5}$$

$$y = \frac{16}{5} \text{ or } y = 0$$

20. $4 = \pm\frac{1}{2}\begin{vmatrix} -4 & 2 & 1 \\ -3 & 5 & 1 \\ -1 & y & 1 \end{vmatrix}$

$$\pm 8 = \begin{vmatrix} -3 & 5 \\ -1 & y \end{vmatrix} - \begin{vmatrix} -4 & 2 \\ -1 & y \end{vmatrix} + \begin{vmatrix} -4 & 2 \\ -3 & 5 \end{vmatrix}$$

$$\pm 8 = -3y + 5 - (-4y + 2) - 20 + 6$$

$$\pm 8 = -3y + 5 + 4y - 2 - 20 + 6$$

$$\pm 8 = y - 11$$

$$y = 11 \pm 8$$

$$y = 19 \text{ or } y = 3$$

21. Vertices: $(0, 25), (10, 0), (28, 5)$

$$\text{Area} = \tfrac{1}{2}\begin{vmatrix} 0 & 25 & 1 \\ 10 & 0 & 1 \\ 28 & 5 & 1 \end{vmatrix} = 250 \text{ square miles}$$

22. Vertices: $(0, 30), (85, 0), (20, -50)$

$$\text{Area} = -\tfrac{1}{2}\begin{vmatrix} 0 & 30 & 1 \\ 85 & 0 & 1 \\ 20 & -50 & 1 \end{vmatrix} = 3100 \text{ square feet}$$

23. Points: $(2, -6), (0, -2), (3, -8)$

$$\begin{vmatrix} 2 & -6 & 1 \\ 0 & -2 & 1 \\ 3 & -8 & 1 \end{vmatrix} = 2\begin{vmatrix} -2 & 1 \\ -8 & 1 \end{vmatrix} + 3\begin{vmatrix} -6 & 1 \\ -2 & 1 \end{vmatrix}$$

$$= 2(6) + 3(-4)$$

$$= 0$$

The points are collinear.

24. Points: $(3, -5), (6, 1), (4, 2)$

$$\begin{vmatrix} 3 & -5 & 1 \\ 6 & 1 & 1 \\ 4 & 2 & 1 \end{vmatrix} = 3\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} + 5\begin{vmatrix} 6 & 1 \\ 4 & 1 \end{vmatrix} + 1\begin{vmatrix} 6 & 1 \\ 4 & 2 \end{vmatrix}$$

$$= 3(-1) + 5(2) + 8$$

$$= 15 \neq 0$$

The points are not collinear.

25. Points: $\left(2, -\tfrac{1}{2}\right), (-4, 4), (6, -3)$

$$\begin{vmatrix} 2 & -\tfrac{1}{2} & 1 \\ -4 & 4 & 1 \\ 6 & -3 & 1 \end{vmatrix} = \begin{vmatrix} -4 & 4 \\ 6 & -3 \end{vmatrix} - \begin{vmatrix} 2 & -\tfrac{1}{2} \\ 6 & -3 \end{vmatrix} + \begin{vmatrix} 2 & -\tfrac{1}{2} \\ -4 & 4 \end{vmatrix}$$

$$= -12 + 3 + 6$$

$$= -3 \neq 0$$

The points are not collinear.

26. Points: $(0, 1), \left(-2, \tfrac{7}{2}\right), \left(1, -\tfrac{1}{4}\right)$

$$\begin{vmatrix} 0 & 1 & 1 \\ -2 & \tfrac{7}{2} & 1 \\ 1 & -\tfrac{1}{4} & 1 \end{vmatrix} = (-1)\begin{vmatrix} -2 & 1 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} -2 & \tfrac{7}{2} \\ 1 & -\tfrac{1}{4} \end{vmatrix}$$

$$= -(-3) + (-3)$$

$$= 0$$

The points are collinear.

27. Points: $(0, 2), (1, 2.4), (-1, 1.6)$

$$\begin{vmatrix} 0 & 2 & 1 \\ 1 & 2.4 & 1 \\ -1 & 1.6 & 1 \end{vmatrix} = -2\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 2.4 \\ -1 & 1.6 \end{vmatrix} = -2(2) + 4 = 0$$

The points are collinear.

28. Points: $(3, 7), (4, 9.5), (-1, -5)$

$$\begin{vmatrix} 3 & 7 & 1 \\ 4 & 9.5 & 1 \\ -1 & -5 & 1 \end{vmatrix} = (3)\begin{vmatrix} 9.5 & 1 \\ -5 & 1 \end{vmatrix} - (7)\begin{vmatrix} 4 & 1 \\ -1 & 1 \end{vmatrix} + (1)\begin{vmatrix} 4 & 9.5 \\ -1 & -5 \end{vmatrix} = 43.5 - 35 - 10.5 = -2$$

The points are not collinear.

29.

$$\begin{vmatrix} 2 & -5 & 1 \\ 4 & y & 1 \\ 5 & -2 & 1 \end{vmatrix} = 0$$

$$2\begin{vmatrix} y & 1 \\ -2 & 1 \end{vmatrix} + 5\begin{vmatrix} 4 & 1 \\ 5 & 1 \end{vmatrix} + \begin{vmatrix} 4 & y \\ 5 & -2 \end{vmatrix} = 0$$

$$2(y + 2) + 5(-1) + (-8 - 5y) = 0$$

$$-3y - 9 = 0$$

$$y = -3$$

30.

$$\begin{vmatrix} -6 & 2 & 1 \\ -5 & y & 1 \\ -3 & 5 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} -5 & y \\ -3 & 5 \end{vmatrix} - \begin{vmatrix} -6 & 2 \\ -3 & 5 \end{vmatrix} + \begin{vmatrix} -6 & 2 \\ -5 & y \end{vmatrix} = 0$$

$$-25 + 3y + 24 - 6y + 10 = 0$$

$$-3y = -9$$

$$y = 3$$

31. Points: $(0, 0), (5, 3)$

$$\text{Equation: } \begin{vmatrix} x & y & 1 \\ 0 & 0 & 1 \\ 5 & 3 & 1 \end{vmatrix} = -\begin{vmatrix} x & y \\ 5 & 3 \end{vmatrix} = 5y - 3x = 0 \Rightarrow 3x - 5y = 0$$

32. Points: $(0, 0), (-2, 2)$

Equation: $\begin{vmatrix} x & y & 1 \\ 0 & 0 & 1 \\ -2 & 2 & 1 \end{vmatrix} = -\begin{vmatrix} x & y \\ -2 & 2 \end{vmatrix} = -(2x + 2y) = 0$ or $x + y = 0$

33. Points: $(-4, 3), (2, 1)$

Equation: $\begin{vmatrix} x & y & 1 \\ -4 & 3 & 1 \\ 2 & 1 & 1 \end{vmatrix} = x\begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} - y\begin{vmatrix} -4 & 1 \\ 2 & 1 \end{vmatrix} + \begin{vmatrix} -4 & 3 \\ 2 & 1 \end{vmatrix} = 2x + 6y - 10 = 0 \Rightarrow x + 3y - 5 = 0$

34. Points: $(10, 7), (-2, -7)$

Equation:

$\begin{vmatrix} x & y & 1 \\ 10 & 7 & 1 \\ -2 & -7 & 1 \end{vmatrix} = \begin{vmatrix} 10 & 7 \\ -2 & -7 \end{vmatrix} - \begin{vmatrix} x & y \\ -2 & -7 \end{vmatrix} + \begin{vmatrix} x & y \\ 10 & 7 \end{vmatrix} = -70 + 14 - (-7x + 2y) + 7x - 10y = 0$ or $7x - 6y - 28 = 0$

35. Points: $\left(-\frac{1}{2}, 3\right), \left(\frac{5}{2}, 1\right)$

Equation: $\begin{vmatrix} x & y & 1 \\ -\frac{1}{2} & 3 & 1 \\ \frac{5}{2} & 1 & 1 \end{vmatrix} = x\begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} - y\begin{vmatrix} -\frac{1}{2} & 1 \\ \frac{5}{2} & 1 \end{vmatrix} + \begin{vmatrix} -\frac{1}{2} & 3 \\ \frac{5}{2} & 1 \end{vmatrix} = 2x + 3y - 8 = 0$

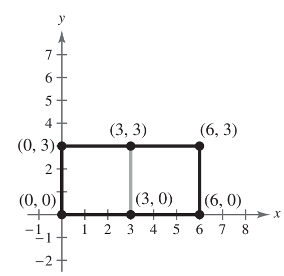
36. Points: $\left(\frac{2}{3}, 4\right), (6, 12)$

Equation: $\begin{vmatrix} x & y & 1 \\ \frac{2}{3} & 4 & 1 \\ 6 & 12 & 1 \end{vmatrix} = \begin{vmatrix} \frac{2}{3} & 4 \\ 6 & 12 \end{vmatrix} - \begin{vmatrix} x & y \\ 6 & 12 \end{vmatrix} + \begin{vmatrix} x & y \\ \frac{2}{3} & 4 \end{vmatrix} = -16 - (12x - 6y) + 4x - \frac{2}{3}y = 0$ or $3x - 2y + 6 = 0$

37. A horizontal stretch, $k = 2$, of the square with vertices $(0, 0), (0, 3), (3, 0)$ and $(3, 3)$.

$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}.$

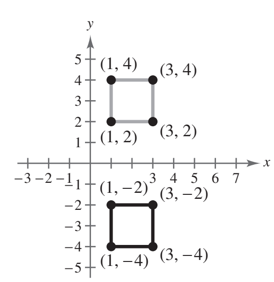
New Vertices: $(0, 0), (0, 3), (6, 0)$ and $(6, 3)$



38. A reflection in the x-axis of the square with vertices $(1, 2), (3, 2), (1, 4)$ and $(3, 4)$.

$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}.$

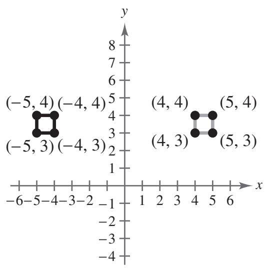
New Vertices: $(1, -2), (3, -2), (1, -4)$ and $(3, -4)$



39. A reflection in the y-axis of the square with vertices $(4, 3), (5, 3), (4, 4)$ and $(5, 4)$.

$$\begin{bmatrix}-1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}4\\3\end{bmatrix} = \begin{bmatrix}-4\\3\end{bmatrix}, \begin{bmatrix}-1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}5\\3\end{bmatrix} = \begin{bmatrix}-5\\3\end{bmatrix}, \begin{bmatrix}-1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}4\\4\end{bmatrix} = \begin{bmatrix}-4\\4\end{bmatrix} \text{ and } \begin{bmatrix}-1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}5\\4\end{bmatrix} = \begin{bmatrix}-5\\4\end{bmatrix}.$$

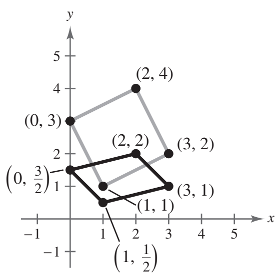
New Vertices: $(-4, 3), (-5, 3), (-4, 4)$ and $(-5, 4)$



40. A vertical shrink, $k = \frac{1}{2}$ of the square with vertices $(1, 1), (3, 2), (0, 3)$ and $(2, 4)$.

$$\begin{bmatrix}1 & 0\\0 & \frac{1}{2}\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}1\\\frac{1}{2}\end{bmatrix}, \begin{bmatrix}1 & 0\\0 & \frac{1}{2}\end{bmatrix}\begin{bmatrix}3\\2\end{bmatrix} = \begin{bmatrix}3\\1\end{bmatrix}, \begin{bmatrix}1 & 0\\0 & \frac{1}{2}\end{bmatrix}\begin{bmatrix}0\\3\end{bmatrix} = \begin{bmatrix}1\\\frac{3}{2}\end{bmatrix} \text{ and } \begin{bmatrix}1 & 0\\0 & \frac{1}{2}\end{bmatrix}\begin{bmatrix}2\\4\end{bmatrix} = \begin{bmatrix}2\\2\end{bmatrix}.$$

New Vertices: $\left(1, \frac{1}{2}\right), (3, 1), \left(0, \frac{3}{2}\right)$ and $(2, 2)$



41. The area of the parallelogram with vertices: $(0, 0), (1, 0), (2, 2)$ and $(3, 2) \Rightarrow a = 1, b = 2, c = 2$ and $d = 2$.

$$A = \begin{bmatrix}1 & 0\\2 & 2\end{bmatrix}$$

Area $= |\det(A)| = |2 - 0| = 2$ square units.

42. The area of the parallelogram with vertices: $(0, 0), (3, 0), (4, 1)$ and $(7, 1) \Rightarrow a = 3, b = 0, c = 4$ and $d = 1$.

$$A = \begin{bmatrix}3 & 0\\4 & 1\end{bmatrix}$$

Area $= |\det(A)| = |3 - 0| = 3$ square units.

43. The area of the parallelogram with vertices: $(0, 0), (-2, 0), (3, 5)$ and $(1, 5) \Rightarrow a = -2, b = 0, c = 3$ and $d = 5$.

$$A = \begin{bmatrix}-2 & 0\\3 & 5\end{bmatrix}$$

Area $= |\det(A)| = |-10 - 0| = 10$ square units.

44. The area of the parallelogram with vertices: $(0, 0), (0, 8), (8, -6)$ and $(8, 2) \Rightarrow a = 0, b = 8, c = 8$ and $d = -6$.

$$A = \begin{bmatrix}0 & 8\\8 & -6\end{bmatrix}$$

Area $= |\det(A)| = |-64| = 64$ square units.

45. (a) Uncoded: C O M E _ H O M E _ S O O N

$[3\ \ 15]\,[13\ \ 5]\,[0\ \ 8]\,[15\ \ 13]\,[5\ \ 0]\,[19\ \ 15]\,[15\ \ 14]$

(b) $[3\ \ 15]\begin{bmatrix}1 & 2\\3 & 5\end{bmatrix} = [48\ \ 81]$

$[13\ \ 5]\begin{bmatrix}1 & 2\\3 & 5\end{bmatrix} = [28\ \ 51]$

$[0\ \ 8]\begin{bmatrix}1 & 2\\3 & 5\end{bmatrix} = [24\ \ 40]$

$[15\ \ 13]\begin{bmatrix}1 & 2\\3 & 5\end{bmatrix} = [54\ \ 95]$

$[5\ \ 0]\begin{bmatrix}1 & 2\\3 & 5\end{bmatrix} = [5\ \ 10]$

$[19\ \ 15]\begin{bmatrix}1 & 2\\3 & 5\end{bmatrix} = [64\ \ 113]$

$[15\ \ 14]\begin{bmatrix}1 & 2\\3 & 5\end{bmatrix} = [57\ \ 100]$

Encoded: 48 81 28 51 24 40 54 95 5 10 64 113 57 100

46. (a) Uncoded: H E L P _ I S _ O N _ T H E _ W A Y

$[8\ \ 5]\,[12\ \ 16]\,[0\ \ 9]\,[19\ \ 0]\,[15\ \ 14]\,[0\ \ 20]\,[8\ \ 5]\,[0\ \ 23]\,[1\ \ 25]$

(b) $[8\ \ 5]\begin{bmatrix}-2 & 3\\-1 & 1\end{bmatrix} = [-21\ \ 29]$

$[12\ \ 16]\begin{bmatrix}-2 & 3\\-1 & 1\end{bmatrix} = [-40\ \ 52]$

$[0\ \ 9]\begin{bmatrix}-2 & 3\\-1 & 1\end{bmatrix} = [-9\ \ 9]$

$[19\ \ 0]\begin{bmatrix}-2 & 3\\-1 & 1\end{bmatrix} = [-38\ \ 57]$

$[15\ \ 14]\begin{bmatrix}-2 & 3\\-1 & 1\end{bmatrix} = [-44\ \ 59]$

$[0\ \ 20]\begin{bmatrix}-2 & 3\\-1 & 1\end{bmatrix} = [-20\ \ 20]$

$[8\ \ 5]\begin{bmatrix}-2 & 3\\-1 & 1\end{bmatrix} = [-21\ \ 29]$

$[0\ \ 23]\begin{bmatrix}-2 & 3\\-1 & 1\end{bmatrix} = [-23\ \ 23]$

$[1\ \ 25]\begin{bmatrix}-2 & 3\\-1 & 1\end{bmatrix} = [-27\ \ 28]$

Encoded:
–21 29 –40 52 –9 9 –38 57 –44 59 –20 20 –21 29 –23 23 –27 28

47. (a) Uncoded:

C A L L _ M E _ T O M O

[3 1 12] [12 0 13] [5 0 20] [15 13 15]

R R O W _ _

[18 18 15] [23 0 0]

(b) $$\begin{bmatrix}3 & 1 & 12\end{bmatrix}\begin{bmatrix}1 & -1 & 0\\ 1 & 0 & -1\\ -6 & 2 & 3\end{bmatrix} = \begin{bmatrix}-68 & 21 & 35\end{bmatrix}$$

$$\begin{bmatrix}12 & 0 & 13\end{bmatrix}\begin{bmatrix}1 & -1 & 0\\ 1 & 0 & -1\\ -6 & 2 & 3\end{bmatrix} = \begin{bmatrix}-66 & 14 & 39\end{bmatrix}$$

$$\begin{bmatrix}5 & 0 & 20\end{bmatrix}\begin{bmatrix}1 & -1 & 0\\ 1 & 0 & -1\\ -6 & 2 & 3\end{bmatrix} = \begin{bmatrix}-115 & 35 & 60\end{bmatrix}$$

$$\begin{bmatrix}15 & 13 & 15\end{bmatrix}\begin{bmatrix}1 & -1 & 0\\ 1 & 0 & -1\\ -6 & 2 & 3\end{bmatrix} = \begin{bmatrix}-62 & 15 & 32\end{bmatrix}$$

$$\begin{bmatrix}18 & 18 & 15\end{bmatrix}\begin{bmatrix}1 & -1 & 0\\ 1 & 0 & -1\\ -6 & 2 & 3\end{bmatrix} = \begin{bmatrix}-54 & 12 & 27\end{bmatrix}$$

$$\begin{bmatrix}23 & 0 & 0\end{bmatrix}\begin{bmatrix}1 & -1 & 0\\ 1 & 0 & -1\\ -6 & 2 & 3\end{bmatrix} = \begin{bmatrix}23 & -23 & 0\end{bmatrix}$$

Encoded:

−68 21 35 −66 14 39 −115 35 60

−62 15 32 −54 12 27 23 −23 0

48. (a) Uncoded:

P L E A S E _ S E N D _

[16 12 5] [1 19 5] [0 19 5] [14 4 0]

M O N E Y _

[13 15 14] [5 25 0]

(b) $$\begin{bmatrix}16 & 12 & 5\end{bmatrix}\begin{bmatrix}4 & 2 & 1\\ -3 & -3 & -1\\ 3 & 2 & 1\end{bmatrix} = \begin{bmatrix}43 & 6 & 9\end{bmatrix}$$

$$\begin{bmatrix}1 & 19 & 5\end{bmatrix}\begin{bmatrix}4 & 2 & 1\\ -3 & -3 & -1\\ 3 & 2 & 1\end{bmatrix} = \begin{bmatrix}-38 & -45 & -13\end{bmatrix}$$

$$\begin{bmatrix}0 & 19 & 5\end{bmatrix}\begin{bmatrix}4 & 2 & 1\\ -3 & -3 & -1\\ 3 & 2 & 1\end{bmatrix} = \begin{bmatrix}-42 & -47 & -14\end{bmatrix}$$

$$\begin{bmatrix}14 & 4 & 0\end{bmatrix}\begin{bmatrix}4 & 2 & 1\\ -3 & -3 & -1\\ 3 & 2 & 1\end{bmatrix} = \begin{bmatrix}44 & 16 & 10\end{bmatrix}$$

$$\begin{bmatrix}13 & 15 & 14\end{bmatrix}\begin{bmatrix}4 & 2 & 1\\ -3 & -3 & -1\\ 3 & 2 & 1\end{bmatrix} = \begin{bmatrix}49 & 9 & 12\end{bmatrix}$$

$$\begin{bmatrix}5 & 25 & 0\end{bmatrix}\begin{bmatrix}4 & 2 & 1\\ -3 & -3 & -1\\ 3 & 2 & 1\end{bmatrix} = \begin{bmatrix}-55 & -65 & -20\end{bmatrix}$$

Encoded:

43 6 9 −38 −45 −13 −42 −47 −14

44 16 10 49 9 12 −55 −65 −20

In Exercises 49–52, use the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix}$.

49. L A N D I N G _ S U C C E S S F U L

$[12 \quad 1 \quad 14][4 \quad 9 \quad 14][7 \quad 0 \quad 19][21 \quad 3 \quad 3][5 \quad 19 \quad 19][6 \quad 21 \quad 12]$

$$[12 \quad 1 \quad 14]\begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix} = [1 \quad -25 \quad -65]$$

$$[4 \quad 9 \quad 14]\begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix} = [17 \quad 15 \quad -9]$$

$$[7 \quad 0 \quad 19]\begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix} = [-12 \quad -62 \quad -119]$$

$$[21 \quad 3 \quad 3]\begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix} = [27 \quad 51 \quad 48]$$

$$[5 \quad 19 \quad 19]\begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix} = [43 \quad 67 \quad 48]$$

$$[6 \quad 21 \quad 12]\begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix} = [57 \quad 111 \quad 117]$$

Cryptogram: 1 –25 –65 17 15 –9 –12 –62 –119 27 51 48 43 67 48 57 111 117

50. I C E B E R G _ D E A D _ A H E A D

$[9 \quad 3 \quad 5][2 \quad 5 \quad 18][7 \quad 0 \quad 4][5 \quad 1 \quad 4][0 \quad 1 \quad 8][5 \quad 1 \quad 4]$

$$[9 \quad 3 \quad 5]\begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix} = [13 \quad 19 \quad 10]$$

$$[2 \quad 5 \quad 18]\begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix} = [-1 \quad -33 \quad -77]$$

$$[7 \quad 0 \quad 4]\begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix} = [3 \quad -2 \quad -14]$$

$$[5 \quad 1 \quad 4]\begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix} = [4 \quad 1 \quad -9]$$

$$[0 \quad 1 \quad 8]\begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix} = [-5 \quad -25 \quad -47]$$

$$[5 \quad 1 \quad 4]\begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix} = [4 \quad 1 \quad -9]$$

Cryptogram: 13 19 10 –1 –33 –77 3 –2 –14 4 1 –9 –5 –25 –47 4 1 –9

51. H A P P Y _ B I R T H D A Y _

$[8\ \ 1\ \ 16]\,[16\ \ 25\ \ 0]\,[2\ \ 9\ \ 18]\,[20\ \ 8\ \ 4]\,[1\ \ 25\ \ 0]$

$$[8\ \ 1\ \ 16]\begin{bmatrix}1 & 2 & 2\\ 3 & 7 & 9\\ -1 & -4 & -7\end{bmatrix} = [-5\ \ -41\ \ -87]$$

$$[16\ \ 25\ \ 0]\begin{bmatrix}1 & 2 & 2\\ 3 & 7 & 9\\ -1 & -4 & -7\end{bmatrix} = [91\ \ 207\ \ 257]$$

$$[2\ \ 9\ \ 18]\begin{bmatrix}1 & 2 & 2\\ 3 & 7 & 9\\ -1 & -4 & -7\end{bmatrix} = [11\ \ -5\ \ -41]$$

$$[20\ \ 8\ \ 4]\begin{bmatrix}1 & 2 & 2\\ 3 & 7 & 9\\ -1 & -4 & -7\end{bmatrix} = [40\ \ 80\ \ 84]$$

$$[1\ \ 25\ \ 0]\begin{bmatrix}1 & 2 & 2\\ 3 & 7 & 9\\ -1 & -4 & -7\end{bmatrix} = [76\ \ 177\ \ 227]$$

Cryptogram: −5 −41 −87 91 207 257 11 −5 −41 40 80 84 76 177 227

52. O P E R A T I O N _ O V E R L O A D

$[15\ \ 16\ \ 5]\,[18\ \ 1\ \ 20]\,[9\ \ 15\ \ 14]\,[0\ \ 15\ \ 22]\,[5\ \ 18\ \ 12]\,[15\ \ 1\ \ 4]$

$$[15\ \ 16\ \ 5]\begin{bmatrix}1 & 2 & 2\\ 3 & 7 & 9\\ -1 & -4 & -7\end{bmatrix} = [58\ \ 122\ \ 139]$$

$$[18\ \ 1\ \ 20]\begin{bmatrix}1 & 2 & 2\\ 3 & 7 & 9\\ -1 & -4 & -7\end{bmatrix} = [1\ \ -37\ \ -95]$$

$$[9\ \ 15\ \ 14]\begin{bmatrix}1 & 2 & 2\\ 3 & 7 & 9\\ -1 & -4 & -7\end{bmatrix} = [40\ \ 67\ \ 55]$$

$$[0\ \ 15\ \ 22]\begin{bmatrix}1 & 2 & 2\\ 3 & 7 & 9\\ -1 & -4 & -7\end{bmatrix} = [23\ \ 17\ \ -19]$$

$$[5\ \ 18\ \ 12]\begin{bmatrix}1 & 2 & 2\\ 3 & 7 & 9\\ -1 & -4 & -7\end{bmatrix} = [47\ \ 88\ \ 88]$$

$$[15\ \ 1\ \ 4]\begin{bmatrix}1 & 2 & 2\\ 3 & 7 & 9\\ -1 & -4 & -7\end{bmatrix} = [14\ \ 21\ \ 11]$$

Cryptogram: 58 122 139 1 −37 −95 40 67 55 23 17 −19 47 88 88 14 21 11

53. $A^{-1} = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}^{-1} = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$

$$\begin{bmatrix} 11 & 21 \\ 64 & 112 \\ 25 & 50 \\ 29 & 53 \\ 23 & 46 \\ 40 & 75 \\ 55 & 92 \end{bmatrix} \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 8 & 1 \\ 16 & 16 \\ 25 & 0 \\ 14 & 5 \\ 23 & 0 \\ 25 & 5 \\ 1 & 18 \end{bmatrix} \begin{matrix} H & A \\ P & P \\ Y & \_ \\ N & E \\ W & \_ \\ Y & E \\ A & R \end{matrix}$$

Message: HAPPY NEW YEAR

54. $A^{-1} = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}$

$[85 \quad 120]\begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix} = [20 \quad 15]$ T O

$[6 \quad 8]\begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix} = [0 \quad 2]$ _ B

$[10 \quad 15]\begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix} = [5 \quad 0]$ E _

$[84 \quad 117]\begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix} = [15 \quad 18]$ O R

$[42 \quad 56]\begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix} = [0 \quad 14]$ _ N

$[90 \quad 125]\begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix} = [15 \quad 20]$ O T

$[60 \quad 80]\begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix} = [0 \quad 20]$ _ T

$[30 \quad 45]\begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix} = [15 \quad 0]$ O _

$[19 \quad 26]\begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix} = [2 \quad 5]$ B E

Message: TO BE OR NOT TO BE

55. $A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -6 & 2 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} -2 & -3 & -1 \\ -3 & -3 & -1 \\ -2 & -4 & -1 \end{bmatrix}$

$$\begin{bmatrix} 9 & -1 & -9 \\ 38 & -19 & -19 \\ 28 & -9 & -19 \\ -80 & 25 & 41 \\ -64 & 21 & 31 \\ 9 & -5 & -4 \end{bmatrix} \begin{bmatrix} -2 & -3 & -1 \\ -3 & -3 & -1 \\ -2 & -4 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 12 & 1 \\ 19 & 19 & 0 \\ 9 & 19 & 0 \\ 3 & 1 & 14 \\ 3 & 5 & 12 \\ 5 & 4 & 0 \end{bmatrix} \begin{matrix} C & L & A \\ S & S & \_ \\ I & S & \_ \\ C & A & N \\ C & E & L \\ E & D & \_ \end{matrix}$$

Message: CLASS IS CANCELED

56. $A^{-1} = \begin{bmatrix} 11 & 2 & -8 \\ 4 & 1 & -3 \\ -8 & -1 & 6 \end{bmatrix}$

$[112 \quad -140 \quad 83]\begin{bmatrix} 11 & 2 & -8 \\ 4 & 1 & -3 \\ -8 & -1 & 6 \end{bmatrix} = [8 \quad 1 \quad 22]$ H A V

$[19 \quad -25 \quad 13]\begin{bmatrix} 11 & 2 & -8 \\ 4 & 1 & -3 \\ -8 & -1 & 6 \end{bmatrix} = [5 \quad 0 \quad 1]$ E _ A

$[72 \quad -76 \quad 61]\begin{bmatrix} 11 & 2 & -8 \\ 4 & 1 & -3 \\ -8 & -1 & 6 \end{bmatrix} = [0 \quad 7 \quad 18]$ _ G R

$[95 \quad -118 \quad 71]\begin{bmatrix} 11 & 2 & -8 \\ 4 & 1 & -3 \\ -8 & -1 & 6 \end{bmatrix} = [5 \quad 1 \quad 20]$ E A T

$[20 \quad 21 \quad 38]\begin{bmatrix} 11 & 2 & -8 \\ 4 & 1 & -3 \\ -8 & -1 & 6 \end{bmatrix} = [0 \quad 23 \quad 5]$ _ W E

$[35 \quad -23 \quad 36]\begin{bmatrix} 11 & 2 & -8 \\ 4 & 1 & -3 \\ -8 & -1 & 6 \end{bmatrix} = [5 \quad 11 \quad 5]$ E K E

$[42 \quad -48 \quad 32]\begin{bmatrix} 11 & 2 & -8 \\ 4 & 1 & -3 \\ -8 & -1 & 6 \end{bmatrix} = [14 \quad 4 \quad 0]$ N D _

Message: HAVE A GREAT WEEKEND

57. $A^{-1} = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix}^{-1} = \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix}$

$\begin{bmatrix} 20 & 17 & -15 \\ -12 & -56 & -104 \\ 1 & -25 & -65 \\ 62 & 143 & 181 \end{bmatrix}\begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 19 & 5 & 14 \\ 4 & 0 & 16 \\ 12 & 1 & 14 \\ 5 & 19 & 0 \end{bmatrix}$
S E N
D _ P
L A N
E S _

Message: SEND PLANES

58. $\begin{bmatrix}13 & -9 & -59\end{bmatrix}\begin{bmatrix}-13 & 6 & 4\\ 12 & -5 & -3\\ -5 & 2 & 1\end{bmatrix} = \begin{bmatrix}18 & 5 & 20\end{bmatrix}$ R E T

$\begin{bmatrix}61 & 112 & 106\end{bmatrix}\begin{bmatrix}-13 & 6 & 4\\ 12 & -5 & -3\\ -5 & 2 & 1\end{bmatrix} = \begin{bmatrix}21 & 18 & 14\end{bmatrix}$ U R N

$\begin{bmatrix}-17 & -73 & -131\end{bmatrix}\begin{bmatrix}-13 & 6 & 4\\ 12 & -5 & -3\\ -5 & 2 & 1\end{bmatrix} = \begin{bmatrix}0 & 1 & 20\end{bmatrix}$ _ A T Message: RETURN AT DAWN

$\begin{bmatrix}11 & 24 & 29\end{bmatrix}\begin{bmatrix}-13 & 6 & 4\\ 12 & -5 & -3\\ -5 & 2 & 1\end{bmatrix} = \begin{bmatrix}0 & 4 & 1\end{bmatrix}$ _ D A

$\begin{bmatrix}65 & 144 & 172\end{bmatrix}\begin{bmatrix}-13 & 6 & 4\\ 12 & -5 & -3\\ -5 & 2 & 1\end{bmatrix} = \begin{bmatrix}23 & 14 & 0\end{bmatrix}$ W N _

59. Let A be the 2×2 matrix needed to decode the message.

$\begin{bmatrix}-18 & -18\\ 1 & 16\end{bmatrix}A = \begin{bmatrix}0 & 18\\ 15 & 14\end{bmatrix}$ $\begin{matrix}\_ & R\\ O & N\end{matrix}$

$$A = \begin{bmatrix}-18 & -18\\ 1 & 16\end{bmatrix}^{-1}\begin{bmatrix}0 & 18\\ 15 & 14\end{bmatrix} = \begin{bmatrix}-\frac{8}{135} & -\frac{1}{15}\\ \frac{1}{270} & \frac{1}{15}\end{bmatrix}\begin{bmatrix}0 & 18\\ 15 & 14\end{bmatrix} = \begin{bmatrix}-1 & -2\\ 1 & 1\end{bmatrix}$$

$$\begin{bmatrix}8 & 21\\ -15 & -10\\ -13 & -13\\ 5 & 10\\ 5 & 25\\ 5 & 19\\ -1 & 6\\ 20 & 40\\ -18 & -18\\ 1 & 16\end{bmatrix}\begin{bmatrix}-1 & -2\\ 1 & 1\end{bmatrix} = \begin{bmatrix}13 & 5\\ 5 & 20\\ 0 & 13\\ 5 & 0\\ 20 & 15\\ 14 & 9\\ 7 & 8\\ 20 & 0\\ 0 & 18\\ 15 & 14\end{bmatrix}\begin{matrix}M & E\\ E & T\\ \_ & M\\ E & \_\\ T & O\\ N & I\\ G & H\\ T & \_\\ \_ & R\\ O & N\end{matrix}$$

Message: MEET ME TONIGHT RON

60. Let A be the 2×2 matrix needed to decode the message.

$\begin{bmatrix}-19 & -19\\ 37 & 16\end{bmatrix}A = \begin{bmatrix}0 & 19\\ 21 & 5\end{bmatrix}$ $\begin{matrix}\_ & S\\ U & E\end{matrix}$

$$A^{-1} = \begin{bmatrix}-19 & -19\\ 37 & 16\end{bmatrix}^{-1}\begin{bmatrix}0 & 19\\ 21 & 5\end{bmatrix} = \begin{bmatrix}\frac{16}{399} & \frac{1}{21}\\ -\frac{37}{399} & -\frac{1}{21}\end{bmatrix}\begin{bmatrix}0 & 19\\ 21 & 5\end{bmatrix} = \begin{bmatrix}1 & 1\\ -1 & -2\end{bmatrix}$$

$$\begin{bmatrix}5 & 2\\ 25 & 11\\ -2 & -7\\ -15 & -15\\ 32 & 14\\ -8 & -13\\ 38 & 19\\ -19 & -19\\ 37 & 16\end{bmatrix}\begin{bmatrix}1 & 1\\ -1 & -2\end{bmatrix} = \begin{bmatrix}3 & 1\\ 14 & 3\\ 5 & 12\\ 0 & 15\\ 18 & 4\\ 5 & 18\\ 19 & 0\\ 0 & 19\\ 21 & 5\end{bmatrix}\begin{matrix}C & A\\ N & C\\ E & L\\ \_ & O\\ R & D\\ E & R\\ S & \_\\ \_ & S\\ U & E\end{matrix}$$

Message: CANCEL ORDERS SUE

61. $D = \begin{vmatrix} 4 & 0 & 8 \\ 0 & 2 & 8 \\ 1 & 1 & -1 \end{vmatrix} = -56$

$I_1 = \dfrac{\begin{vmatrix} 2 & 0 & 8 \\ 6 & 2 & 8 \\ 0 & 1 & -1 \end{vmatrix}}{-56} = -\dfrac{28}{56} = -\dfrac{1}{2}$

$I_2 = \dfrac{\begin{vmatrix} 4 & 2 & 8 \\ 0 & 6 & 8 \\ 1 & 0 & -1 \end{vmatrix}}{-56} = \dfrac{-56}{-56} = 1$

$I_3 = \dfrac{\begin{vmatrix} 4 & 0 & 2 \\ 0 & 2 & 6 \\ 1 & 1 & 0 \end{vmatrix}}{-56} = \dfrac{-28}{-56} = \dfrac{1}{2}$

So, the solution is $I_1 = -0.5$ ampere, $I_2 = 1$ ampere, and $I_3 = 0.5$ ampere.

62. $D = \begin{vmatrix} 1 & -2 & 0 \\ 1 & 0 & -3 \\ 0 & 1 & 2 \end{vmatrix} = 7$

$t_1 = \dfrac{\begin{vmatrix} 0 & -2 & 0 \\ 192 & 0 & -3 \\ 64 & 1 & 2 \end{vmatrix}}{7} = \dfrac{1152}{7} \approx 164.6 \text{ lb}$

$t_2 = \dfrac{\begin{vmatrix} 1 & 0 & 0 \\ 1 & 192 & -3 \\ 0 & 64 & 2 \end{vmatrix}}{7} = \dfrac{576}{7} \approx 82.3 \text{ lb}$

$a = \dfrac{\begin{vmatrix} 1 & -2 & 0 \\ 1 & 0 & 192 \\ 0 & 1 & 64 \end{vmatrix}}{7} = -\dfrac{64}{7} \approx -9.1 \text{ ft/sq}^2$

The solution is $t_1 \approx 164.6$ lb, $t_2 \approx 82.3$ lb, and $a \approx -9.1 \text{ ft/sec}^2$.

63. False. In Cramer's Rule, the denominator is the determinant of the coefficient matrix.

64. True. If the determinant of the coefficient matrix is zero, the solution of the system would result in division by zero, which is undefined.

65. If the determinant of the coefficient matrix is zero, the system has either no solution or infinitely many solutions.

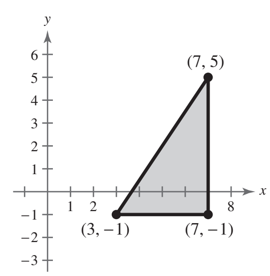
66. Answers will vary. *Sample answer:* To find the equation of a line through two points, you could find the slope and then use the point-slope form of an equation, or use a matrix and evaluate the determinant. Using the point-slope form of an equation may be easier because the work is straightforward. However, it can be easy to make a mistake when working with fractions. Using the determinant of a matrix may be easier because the determinant of a 2×2 matrix is $ad - bc$. However, it may be difficult to remember how the determinant can be used to find the equation of a line.

67. $\text{Area} = \frac{1}{2}\begin{vmatrix} 3 & -1 & 1 \\ 7 & -1 & 1 \\ 7 & 5 & 1 \end{vmatrix}$

$= \frac{1}{2}\left(3\begin{vmatrix} -1 & 1 \\ 5 & 1 \end{vmatrix} + 1\begin{vmatrix} 7 & 1 \\ 7 & 1 \end{vmatrix} + 1\begin{vmatrix} 7 & -1 \\ 7 & 5 \end{vmatrix}\right)$

$= \frac{1}{2}(-18 + 0 + 42)$

$= 12$ square units



$\text{Area} = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(7-3)(5-(-1)) = \frac{1}{2}(4)(6) = 12$ square units

68. Answers will vary.

Review Exercises for Chapter 8

1. $\begin{bmatrix} -1 & 3 \end{bmatrix}$

Order: 1×2

2. $\begin{bmatrix} 3 & 1 \\ 5 & -2 \end{bmatrix}$

Order: 2×2

3. $\begin{bmatrix} 2 & 1 & 0 & 4 & -1 \\ 6 & 2 & 1 & 8 & 0 \end{bmatrix}$

Order: 2×5

4. $[5]$

Order: 1×1

5. $\begin{cases} 3x - 10y = 15 \\ 5x + 4y = 22 \end{cases}$

$$\begin{bmatrix} 3 & -10 & \vdots & 15 \\ 5 & 4 & \vdots & 22 \end{bmatrix}$$

6. $\begin{cases} 8x - 7y + 4z = 12 \\ 3x - 5y + 2z = 20 \end{cases}$

$$\begin{bmatrix} 8 & -7 & 4 & \vdots & 12 \\ 3 & -5 & 2 & \vdots & 20 \end{bmatrix}$$

7. $\begin{bmatrix} 1 & 0 & 2 & \vdots & -8 \\ 2 & -2 & 3 & \vdots & 12 \\ 4 & 7 & 1 & \vdots & 3 \end{bmatrix}$

$$\begin{cases} x \qquad + 2z = -8 \\ 2x - 2y + 3z = 12 \\ 4x + 7y + z = 3 \end{cases}$$

8. $\begin{bmatrix} 2 & 10 & 8 & 5 & \vdots & -1 \\ -3 & 4 & 0 & 9 & \vdots & 2 \end{bmatrix}$

$$\begin{cases} 2x + 10y + 8z + 5w = -1 \\ -3x + 4y \qquad + 9w = 2 \end{cases}$$

9. $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 2 & 2 \end{bmatrix}$

$$\begin{matrix} R_1 \\ R_2 \\ \\ \end{matrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}$$

$$\begin{matrix} \\ \\ -2R_1 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & -2 & -4 \end{bmatrix}$$

$$\begin{matrix} \\ \\ 2R_2 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{bmatrix}$$

$$\begin{matrix} \\ \\ -\frac{1}{2}R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

10. $\begin{bmatrix} 4 & 8 & 16 \\ 3 & -1 & 2 \\ -2 & 10 & 12 \end{bmatrix}$

$$\begin{matrix} \frac{1}{4}R_1 \rightarrow \\ \\ -\frac{1}{2}R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 2 & 4 \\ 3 & -1 & 2 \\ 1 & -5 & -6 \end{bmatrix}$$

$$\begin{matrix} \\ -3R_1 + R_2 \rightarrow \\ -R_1 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 2 & 4 \\ 0 & -7 & -10 \\ 0 & -7 & -10 \end{bmatrix}$$

$$\begin{matrix} \\ \\ -R_2 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 2 & 4 \\ 0 & -7 & -10 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{matrix} \\ -\frac{1}{7}R_2 \rightarrow \\ \\ \end{matrix} \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & \frac{10}{7} \\ 0 & 0 & 0 \end{bmatrix}$$

11. $\begin{bmatrix} 1 & 2 & 3 & \vdots & 9 \\ 0 & 1 & -2 & \vdots & 2 \\ 0 & 0 & 1 & \vdots & -1 \end{bmatrix} \begin{matrix} \Rightarrow \\ \Rightarrow \\ \Rightarrow \end{matrix} \begin{cases} x + 2y + 3z = 9 \\ y - 2z = 2 \\ z = -1 \end{cases}$

$$y - 2(-1) = 2 \Rightarrow y = 0$$

$$x + 2(0) + 3(-1) = 9 \Rightarrow x = 12$$

Solution: $(12, 0, -1)$

12. $\begin{cases} x + 3y - 9z = 4 \\ y - z = 10 \\ z = -2 \end{cases}$

$$y - (-2) = 10$$

$$y = 8$$

$$x + 3(8) - 9(-2) = 4$$

$$x = -38$$

Solution: $(-38, 8, -2)$

13. $\begin{bmatrix} 1 & 3 & 4 & \vdots & 1 \\ 0 & 1 & 2 & \vdots & 3 \\ 0 & 0 & 1 & \vdots & 4 \end{bmatrix} \begin{matrix} \Rightarrow \\ \Rightarrow \\ \Rightarrow \end{matrix} \begin{cases} x + 3y + 4z = 1 \\ y + 2z = 3 \\ z = 4 \end{cases}$

$$y + 2(4) = 3 \Rightarrow y = -5$$

$$x + 3(-5) + 4(4) = 1 \Rightarrow x = 0$$

Solution: $(0, -5, 4)$

14. $\begin{cases} x - 8y = -2 \\ y - z = -7 \\ z = 1 \end{cases}$

$y - 1 = -7$

$y = -6$

$x - 8(-6) = -2$

$x = -50$

Solution: $(-50, -6, 1)$

15. $\begin{bmatrix} 5 & 4 & \vdots & 2 \\ -1 & 1 & \vdots & -22 \end{bmatrix}$

$4R_2 + R_1 \rightarrow \begin{bmatrix} 1 & 8 & \vdots & -86 \\ -1 & 1 & \vdots & -22 \end{bmatrix}$

$R_1 + R_2 \rightarrow \begin{bmatrix} 1 & 8 & \vdots & -86 \\ 0 & 9 & \vdots & -108 \end{bmatrix}$

$\frac{1}{9}R_2 \rightarrow \begin{bmatrix} 1 & 8 & \vdots & -86 \\ 0 & 1 & \vdots & -12 \end{bmatrix}$

$\begin{cases} x + 8y = -86 \\ y = -12 \end{cases}$

$y = -12$

$x + 8(-12) = -86 \Rightarrow x = 10$

Solution: $(10, -12)$

16. $\begin{bmatrix} 2 & -5 & \vdots & 2 \\ 3 & -7 & \vdots & 1 \end{bmatrix}$

$\frac{1}{2}R_1 \rightarrow \begin{bmatrix} 1 & -\frac{5}{2} & \vdots & 1 \\ 3 & -7 & \vdots & 1 \end{bmatrix}$

$-3R_1 + R_2 \rightarrow \begin{bmatrix} 1 & -\frac{5}{2} & \vdots & 1 \\ 0 & \frac{1}{2} & \vdots & -2 \end{bmatrix}$

$2R_3 \rightarrow \begin{bmatrix} 1 & -\frac{5}{2} & \vdots & 1 \\ 0 & 1 & \vdots & -4 \end{bmatrix}$

$\begin{cases} x - \frac{5}{2}y = 1 \\ y = -4 \end{cases}$

$y = -4$

$x - \frac{5}{2}(-4) = 1 \Rightarrow x = -9$

Solution: $(-9, -4)$

17. $\begin{bmatrix} 0.3 & -0.1 & \vdots & -0.13 \\ 0.2 & -0.3 & \vdots & -0.25 \end{bmatrix}$

$\begin{matrix} 10R_1 \rightarrow \\ 10R_2 \rightarrow \end{matrix} \begin{bmatrix} 3 & -1 & \vdots & -1.3 \\ 2 & -3 & \vdots & -2.5 \end{bmatrix}$

$-R_2 + R_1 \rightarrow \begin{bmatrix} 1 & 2 & \vdots & 1.2 \\ 2 & -3 & \vdots & -2.5 \end{bmatrix}$

$-2R_1 + R_2 \rightarrow \begin{bmatrix} 1 & 2 & \vdots & 1.2 \\ 0 & -7 & \vdots & -4.9 \end{bmatrix}$

$-\frac{1}{7}R_2 \rightarrow \begin{bmatrix} 1 & 2 & \vdots & 1.2 \\ 0 & 1 & \vdots & 0.7 \end{bmatrix}$

$\begin{cases} x + 2y = 1.2 \\ y = 0.7 \end{cases}$

$y = 0.7$

$x + 2(0.7) = 1.2 \Rightarrow x = -0.2$

Solution: $(-0.2, 0.7) = \left(-\frac{1}{5}, \frac{7}{10}\right)$

18. $\begin{bmatrix} 0.2 & -0.1 & \vdots & 0.07 \\ 0.4 & -0.5 & \vdots & -0.01 \end{bmatrix}$

$\begin{matrix} 5R_1 \rightarrow \\ -2R_1 + R_2 \rightarrow \end{matrix} \begin{bmatrix} 1 & -0.5 & \vdots & 0.35 \\ 0 & -0.3 & \vdots & -0.15 \end{bmatrix}$

$-\frac{1}{0.3}R_2 \rightarrow \begin{bmatrix} 1 & -0.5 & \vdots & 0.35 \\ 0 & 1 & \vdots & 0.5 \end{bmatrix}$

$\begin{cases} x - 0.5y = 0.35 \\ y = 0.5 \end{cases}$

$y = 0.5$

$x - 0.5(0.5) = 0.35 \Rightarrow x = 0.6$

Solution: $(0.6, 0.5) = \left(\frac{3}{5}, \frac{1}{2}\right)$

19. $\begin{cases} -x + 2y = 3 \\ 2x - 4y = 6 \end{cases}$

$\begin{bmatrix} -1 & 2 & \vdots & 3 \\ 2 & -4 & \vdots & 6 \end{bmatrix}$

$2R_1 + R_2 \rightarrow \begin{bmatrix} -1 & 2 & \vdots & 3 \\ 0 & 0 & \vdots & 12 \end{bmatrix}$

Because the last row consists of all zeros except for the last entry, the system is inconsistent and there is no solution.

20. $\begin{cases} -x + 2y = 3 \\ 2x - 4y = -6 \end{cases}$

$$\begin{bmatrix} -1 & 2 & \vdots & 3 \\ 2 & -4 & \vdots & -6 \end{bmatrix}$$

$$2R_1 + R_2 \rightarrow \begin{bmatrix} -1 & 2 & \vdots & 3 \\ 0 & 0 & \vdots & 0 \end{bmatrix}$$

$-x + 2y = 3$

Let $y = a$, then:

$-x + 2a = 3 \Rightarrow x = 2a - 3$

Solution: $(2a - 3, a)$ where a is any real number

21. $\begin{cases} x - 2y + z = 7 \\ 2x + y - 2z = -4 \\ -x + 3y + 2z = -3 \end{cases}$

$$\begin{bmatrix} 1 & -2 & 1 & \vdots & 7 \\ 2 & 1 & -2 & \vdots & -4 \\ -1 & 3 & 2 & \vdots & -3 \end{bmatrix}$$

$$\begin{matrix} \\ -2R_1 + R_2 \rightarrow \\ R_1 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & -2 & 1 & \vdots & 7 \\ 0 & 5 & -4 & \vdots & -18 \\ 0 & 1 & 3 & \vdots & 4 \end{bmatrix}$$

$$\begin{matrix} \\ R_2 + (-5)R_3 \rightarrow \\ \\ \end{matrix} \begin{bmatrix} 1 & -2 & 1 & \vdots & 7 \\ 0 & 0 & -19 & \vdots & -38 \\ 0 & 1 & 3 & \vdots & 4 \end{bmatrix}$$

$-19z = -38$

$z = 2$

$y + 3(2) = 4 \Rightarrow y = -2$

$x - 2(-2) + 2 = 7 \Rightarrow x = 1$

Solution: $(1, -2, 2)$

22. $\begin{cases} x - 2y + z = 4 \\ 2x + y - 2z = -24 \\ -x + 3y + 2z = 20 \end{cases}$

$$\begin{bmatrix} 1 & -2 & 1 & \vdots & 4 \\ 2 & 1 & -2 & \vdots & -24 \\ -1 & 3 & 2 & \vdots & 20 \end{bmatrix}$$

$$\begin{matrix} \\ -2R_1 + R_2 \rightarrow \\ R_1 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & -2 & 1 & \vdots & 4 \\ 0 & 5 & -4 & \vdots & -32 \\ 0 & 1 & 3 & \vdots & 24 \end{bmatrix}$$

$$\begin{matrix} \\ \\ R_2 + (-5)R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & -2 & 1 & \vdots & 4 \\ 0 & 5 & -4 & \vdots & -32 \\ 0 & 0 & -19 & \vdots & -152 \end{bmatrix}$$

$-19z = -152$

$z = 8$

$5y - 4(8) = -32 \Rightarrow y = 0$

$x - 2(0) + 8 = 4 \Rightarrow x = -4$

Solution: $(-4, 0, 8)$

23. $$\begin{bmatrix} 2 & 1 & 2 & \vdots & 4 \\ 2 & 2 & 0 & \vdots & 5 \\ 2 & -1 & 6 & \vdots & 2 \end{bmatrix}$$

$$\begin{matrix} \\ -R_1 + R_2 \rightarrow \\ -R_1 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 2 & 1 & 2 & \vdots & 4 \\ 0 & 1 & -2 & \vdots & 1 \\ 0 & -2 & 4 & \vdots & -2 \end{bmatrix}$$

$$\begin{matrix} -R_2 + R_1 \rightarrow \\ \\ 2R_2 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 2 & 0 & 4 & \vdots & 3 \\ 0 & 1 & -2 & \vdots & 1 \\ 0 & 0 & 0 & \vdots & 0 \end{bmatrix}$$

$$\begin{matrix} \frac{1}{2}R_1 \rightarrow \\ \\ \\ \end{matrix} \begin{bmatrix} 1 & 0 & 2 & \vdots & \frac{3}{2} \\ 0 & 1 & -2 & \vdots & 1 \\ 0 & 0 & 0 & \vdots & 0 \end{bmatrix}$$

Let $z = a$, then:

$y - 2a = 1 \Rightarrow y = 2a + 1$

$x + 2a = \frac{3}{2} \Rightarrow x = -2a + \frac{3}{2}$

Solution: $\left(-2a + \frac{3}{2}, 2a + 1, a\right)$ where a is any real number

24. $$\begin{bmatrix} 1 & 2 & 6 & \vdots & 1 \\ 2 & 5 & 15 & \vdots & 4 \\ 3 & 1 & 3 & \vdots & -6 \end{bmatrix}$$

$$\begin{matrix} \\ -2R_1 + R_2 \rightarrow \\ -3R_1 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 2 & 6 & \vdots & 1 \\ 0 & 1 & 3 & \vdots & 2 \\ 0 & -5 & -15 & \vdots & -9 \end{bmatrix}$$

$$\begin{matrix} \\ \\ 5R_2 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 2 & 6 & \vdots & 1 \\ 0 & 1 & 3 & \vdots & 2 \\ 0 & 0 & 0 & \vdots & 1 \end{bmatrix}$$

Because the last row consists of all zeros except for the last entry, the system is inconsistent and there is no solution.

25.
$$\begin{bmatrix} 2 & 3 & 1 & \vdots & 10 \\ 2 & -3 & -3 & \vdots & 22 \\ 4 & -2 & 3 & \vdots & -2 \end{bmatrix}$$

$$\begin{matrix} \\ -R_1 + R_2 \rightarrow \\ -2R_1 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 2 & 3 & 1 & \vdots & 10 \\ 0 & -6 & -4 & \vdots & 12 \\ 0 & -8 & 1 & \vdots & -22 \end{bmatrix}$$

$$\begin{matrix} \frac{1}{2}R_1 \rightarrow \\ -\frac{1}{6}R_2 \rightarrow \\ \\ \end{matrix} \begin{bmatrix} 1 & \frac{3}{2} & \frac{1}{2} & \vdots & 5 \\ 0 & 1 & \frac{2}{3} & \vdots & -2 \\ 0 & -8 & 1 & \vdots & -22 \end{bmatrix}$$

$$\begin{matrix} \\ \\ 8R_2 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & \frac{3}{2} & \frac{1}{2} & \vdots & 5 \\ 0 & 1 & \frac{2}{3} & \vdots & -2 \\ 0 & 0 & \frac{19}{3} & \vdots & -38 \end{bmatrix}$$

$$\begin{matrix} \\ \\ \frac{3}{19}R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & \frac{3}{2} & \frac{1}{2} & \vdots & 5 \\ 0 & 1 & \frac{2}{3} & \vdots & -2 \\ 0 & 0 & 1 & \vdots & -6 \end{bmatrix}$$

$z = -6$

$y + \frac{2}{3}(-6) = -2 \Rightarrow y = 2$

$x + \frac{3}{2}(2) + \frac{1}{2}(-6) = 5 \Rightarrow x = 5$

Solution: $(5, 2, -6)$

26.
$$\begin{bmatrix} 2 & 3 & 3 & \vdots & 3 \\ 6 & 6 & 12 & \vdots & 13 \\ 12 & 9 & -1 & \vdots & 2 \end{bmatrix}$$

$$\begin{matrix} \\ -3R_1 + R_2 \rightarrow \\ -6R_1 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 2 & 3 & 3 & \vdots & 3 \\ 0 & -3 & 3 & \vdots & 4 \\ 0 & -9 & -19 & \vdots & -16 \end{bmatrix}$$

$$\begin{matrix} R_2 + R_1 \rightarrow \\ \\ -3R_2 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 2 & 0 & 6 & \vdots & 7 \\ 0 & -3 & 3 & \vdots & 4 \\ 0 & 0 & -28 & \vdots & -28 \end{bmatrix}$$

$$\begin{matrix} \frac{1}{2}R_1 \rightarrow \\ -\frac{1}{3}R_2 \rightarrow \\ -\frac{1}{28}R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 0 & 3 & \vdots & \frac{7}{2} \\ 0 & 1 & -1 & \vdots & -\frac{4}{3} \\ 0 & 0 & 1 & \vdots & 1 \end{bmatrix}$$

$$\begin{cases} x + 3z = \frac{7}{2} \\ y - z = -\frac{4}{3} \\ z = 1 \end{cases}$$

$z = 1$

$y - 1 = -\frac{4}{3} \Rightarrow y = -\frac{1}{3}$

$x + 3(1) = \frac{7}{2} \Rightarrow x = \frac{1}{2}$

Solution: $\left(\frac{1}{2}, -\frac{1}{3}, 1\right)$

27.
$$\begin{cases} x + 2y - z = 3 \\ x - y - z = -3 \\ 2x + y + 3z = 10 \end{cases}$$

$$\begin{bmatrix} 1 & 2 & -1 & \vdots & 3 \\ 1 & -1 & -1 & \vdots & -3 \\ 2 & 1 & 3 & \vdots & 10 \end{bmatrix}$$

$$\begin{matrix} \\ -R_1 + R_2 \rightarrow \\ -2R_2 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 2 & -1 & \vdots & 3 \\ 0 & -3 & 0 & \vdots & -6 \\ 0 & 3 & 5 & \vdots & 16 \end{bmatrix}$$

$$\begin{matrix} \\ \\ R_2 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 2 & -1 & \vdots & 3 \\ 0 & -3 & 0 & \vdots & -6 \\ 0 & 0 & 5 & \vdots & 10 \end{bmatrix}$$

$$\begin{matrix} 3R_1 + 2R_2 \rightarrow \\ \\ \\ \end{matrix} \begin{bmatrix} 3 & 0 & -3 & \vdots & -3 \\ 0 & -3 & 0 & \vdots & -6 \\ 0 & 0 & 5 & \vdots & 10 \end{bmatrix}$$

$$\begin{matrix} 5R_1 + 3R_3 \rightarrow \\ \\ \\ \end{matrix} \begin{bmatrix} 15 & 0 & 0 & \vdots & 15 \\ 0 & -3 & 0 & \vdots & -6 \\ 0 & 0 & 5 & \vdots & 10 \end{bmatrix}$$

$$\begin{matrix} \frac{1}{15}R_1 \rightarrow \\ \frac{1}{3}R_2 \rightarrow \\ \frac{1}{5}R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 0 & 0 & \vdots & 1 \\ 0 & 1 & 0 & \vdots & 2 \\ 0 & 0 & 1 & \vdots & 2 \end{bmatrix}$$

$x = 1$

$y = 2$

$z = 2$

Solution: $(1, 2, 2)$

28. $\begin{cases} x - 3y + z = 2 \\ 3x - y - z = -6 \\ -x + y - 3z = -2 \end{cases}$

$$\begin{bmatrix} 1 & -3 & 1 & \vdots & 2 \\ 3 & -1 & -1 & \vdots & -6 \\ -1 & 1 & -3 & \vdots & -2 \end{bmatrix}$$

$$\begin{array}{r} \\ 3R_3 + R_2 \rightarrow \\ R_1 + R_3 \rightarrow \end{array} \begin{bmatrix} 1 & -3 & 1 & \vdots & 2 \\ 0 & 2 & -10 & \vdots & -12 \\ 0 & -2 & -2 & \vdots & 0 \end{bmatrix}$$

$$\begin{array}{r} \\ \frac{1}{2}R_2 \rightarrow \\ R_2 + R_3 \rightarrow \end{array} \begin{bmatrix} 1 & -3 & 1 & \vdots & 2 \\ 0 & 1 & -5 & \vdots & -6 \\ 0 & 0 & -12 & \vdots & -12 \end{bmatrix}$$

$$\begin{array}{r} 3R_2 + R_1 \rightarrow \\ \\ -\frac{1}{12}R_3 \rightarrow \end{array} \begin{bmatrix} 1 & 0 & -14 & \vdots & -16 \\ 0 & 1 & -5 & \vdots & -6 \\ 0 & 0 & 1 & \vdots & 1 \end{bmatrix}$$

$$\begin{array}{r} 14R_3 + R_1 \rightarrow \\ 5R_3 + R_2 \rightarrow \\ \\ \end{array} \begin{bmatrix} 1 & 0 & 0 & \vdots & -2 \\ 0 & 1 & 0 & \vdots & -1 \\ 0 & 0 & 1 & \vdots & 1 \end{bmatrix}$$

$x = -2$

$y = -1$

$z = 1$

Solution: $(-2, -1, 1)$

29. $$\begin{bmatrix} -1 & 1 & 2 & \vdots & 1 \\ 2 & 3 & 1 & \vdots & -2 \\ 5 & 4 & 2 & \vdots & 4 \end{bmatrix}$$

$$\begin{array}{r} -R_1 \rightarrow \\ \\ \\ \end{array} \begin{bmatrix} 1 & -1 & -2 & \vdots & -1 \\ 2 & 3 & 1 & \vdots & -2 \\ 5 & 4 & 2 & \vdots & 4 \end{bmatrix}$$

$$\begin{array}{r} \\ -2R_1 + R_2 \rightarrow \\ -5R_1 + R_3 \rightarrow \end{array} \begin{bmatrix} 1 & -1 & -2 & \vdots & -1 \\ 0 & 5 & 5 & \vdots & 0 \\ 0 & 9 & 12 & \vdots & 9 \end{bmatrix}$$

$$\begin{array}{r} \\ \frac{1}{5}R_2 \rightarrow \\ \\ \end{array} \begin{bmatrix} 1 & -1 & -2 & \vdots & -1 \\ 0 & 1 & 1 & \vdots & 0 \\ 0 & 9 & 12 & \vdots & 9 \end{bmatrix}$$

$$\begin{array}{r} R_2 + R_1 \rightarrow \\ \\ -9R_2 + R_3 \rightarrow \end{array} \begin{bmatrix} 1 & 0 & -1 & \vdots & -1 \\ 0 & 1 & 1 & \vdots & 0 \\ 0 & 0 & 3 & \vdots & 9 \end{bmatrix}$$

$$\begin{array}{r} \\ \\ \frac{1}{3}R_3 \rightarrow \end{array} \begin{bmatrix} 1 & 0 & -1 & \vdots & -1 \\ 0 & 1 & 1 & \vdots & 0 \\ 0 & 0 & 1 & \vdots & 3 \end{bmatrix}$$

$$\begin{array}{r} R_3 + R_1 \rightarrow \\ -R_3 + R_2 \rightarrow \\ \\ \end{array} \begin{bmatrix} 1 & 0 & 0 & \vdots & 2 \\ 0 & 1 & 0 & \vdots & -3 \\ 0 & 0 & 1 & \vdots & 3 \end{bmatrix}$$

$x = 2, y = -3, z = 3$

Solution: $(2, -3, 3)$

30. $\begin{cases} 4x + 4y + 4z = 5 \\ 4x - 2y - 8z = 1 \\ 5x + 3y + 8z = 6 \end{cases}$

$$\begin{bmatrix} 4 & 4 & 4 & \vdots & 5 \\ 4 & -2 & -8 & \vdots & 1 \\ 5 & 3 & 8 & \vdots & 6 \end{bmatrix}$$

$$\begin{array}{r} \frac{1}{4}R_1 \rightarrow \\ \\ \\ \end{array} \begin{bmatrix} 1 & 1 & 1 & \vdots & \frac{5}{4} \\ 4 & -2 & -8 & \vdots & 1 \\ 5 & 3 & 8 & \vdots & 6 \end{bmatrix}$$

$$\begin{array}{r} \\ -4R_1 + R_2 \rightarrow \\ -5R_1 + R_3 \rightarrow \end{array} \begin{bmatrix} 1 & 1 & 1 & \vdots & \frac{5}{4} \\ 0 & -6 & -12 & \vdots & -4 \\ 0 & -2 & 3 & \vdots & -\frac{1}{4} \end{bmatrix}$$

$$\begin{array}{r} \\ -\frac{1}{6}R_2 \rightarrow \\ \\ \end{array} \begin{bmatrix} 1 & 1 & 1 & \vdots & \frac{5}{4} \\ 0 & 1 & 2 & \vdots & \frac{2}{3} \\ 0 & -2 & 3 & \vdots & -\frac{1}{4} \end{bmatrix}$$

$$\begin{array}{r} -R_2 + R_1 \rightarrow \\ \\ 2R_2 + R_3 \rightarrow \end{array} \begin{bmatrix} 1 & 0 & -1 & \vdots & \frac{7}{12} \\ 0 & 1 & 2 & \vdots & \frac{2}{3} \\ 0 & 0 & 7 & \vdots & \frac{13}{12} \end{bmatrix}$$

$$\begin{array}{r} \\ \\ \frac{1}{7}R_3 \rightarrow \end{array} \begin{bmatrix} 1 & 0 & -1 & \vdots & \frac{7}{12} \\ 0 & 1 & 2 & \vdots & \frac{2}{3} \\ 0 & 0 & 1 & \vdots & \frac{13}{84} \end{bmatrix}$$

$$\begin{array}{r} R_3 + R_1 \rightarrow \\ -2R_3 + R_2 \rightarrow \\ \\ \end{array} \begin{bmatrix} 1 & 0 & 0 & \vdots & \frac{31}{42} \\ 0 & 1 & 0 & \vdots & \frac{5}{14} \\ 0 & 0 & 1 & \vdots & \frac{13}{84} \end{bmatrix}$$

$x = \frac{31}{42}$

$y = \frac{5}{14}$

$z = \frac{13}{84}$

Solution: $\left(\frac{31}{42}, \frac{5}{14}, \frac{13}{84}\right)$

31. Use the reduced row-echelon form feature of a graphing utility.

$$\begin{bmatrix} 3 & -1 & 5 & -2 & \vdots & -44 \\ 1 & 6 & 4 & -1 & \vdots & 1 \\ 5 & -1 & 1 & 3 & \vdots & -15 \\ 0 & 4 & -1 & -8 & \vdots & 58 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & \vdots & 2 \\ 0 & 1 & 0 & 0 & \vdots & 6 \\ 0 & 0 & 1 & 0 & \vdots & -10 \\ 0 & 0 & 0 & 1 & \vdots & -3 \end{bmatrix}$$

$x = 2, y = 6, z = -10, w = -3$

Solution: $(2, 6, -10, -3)$

32. Use the reduced row-echelon form feature of the graphing utility.

$$\begin{bmatrix} 4 & 12 & 2 & \vdots & 20 \\ 1 & 6 & 4 & \vdots & 12 \\ 1 & 6 & 1 & \vdots & 8 \\ -2 & -10 & -2 & \vdots & -10 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & \vdots & 0 \\ 0 & 1 & 0 & \vdots & 0 \\ 0 & 0 & 1 & \vdots & 0 \\ 0 & 0 & 0 & \vdots & 1 \end{bmatrix}$$

The system is inconsistent and there is no solution.

33. $\begin{bmatrix} -1 & x \\ y & 9 \end{bmatrix} = \begin{bmatrix} -1 & 12 \\ 11 & 9 \end{bmatrix} \Rightarrow x = 12$ and $y = 11$

34. $\begin{bmatrix} -1 & 0 \\ x & 5 \\ -4 & -3 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 8 & 5 \\ -4 & y \end{bmatrix} \Rightarrow x = 8, y = -3$

35. $\begin{bmatrix} x+3 & -4 & 44 \\ 0 & -3 & 2 \\ -2 & y+5 & 6 \end{bmatrix} = \begin{bmatrix} 5x-1 & -4 & 44 \\ 0 & -3 & 2 \\ -2 & 16 & 6 \end{bmatrix}$

$$\left.\begin{aligned} x + 3 &= 5x - 1 \\ 4 &= 4x \\ y + 5 &= 16 \\ y &= 11 \end{aligned}\right\} x = 1 \text{ and } y = 11$$

36. $\begin{bmatrix} -9 & 4 & 2 & -5 \\ 0 & -3 & 7 & 2y \\ 6 & -1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -9 & 4 & x-10 & -5 \\ 0 & -3 & 7 & -6 \\ 6 & -1 & 1 & 0 \end{bmatrix}$

$$\left.\begin{aligned} 2 &= x - 10 \\ 12 &= x \\ 2y &= -6 \\ y &= -3 \end{aligned}\right\} x = 12, y = -3$$

37. (a) $A + B = \begin{bmatrix} 2 & -2 \\ 3 & 5 \end{bmatrix} + \begin{bmatrix} -3 & 10 \\ 12 & 8 \end{bmatrix} = \begin{bmatrix} -1 & 8 \\ 15 & 13 \end{bmatrix}$

(b) $A - B = \begin{bmatrix} 2 & -2 \\ 3 & 5 \end{bmatrix} - \begin{bmatrix} -3 & 10 \\ 12 & 8 \end{bmatrix} = \begin{bmatrix} 5 & -12 \\ -9 & -3 \end{bmatrix}$

(c) $4A = 4\begin{bmatrix} 2 & -2 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ 12 & 20 \end{bmatrix}$

(d) $2A + 2B = 2\begin{bmatrix} 2 & -2 \\ 3 & 5 \end{bmatrix} + 2\begin{bmatrix} -3 & 10 \\ 12 & 8 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ 6 & 10 \end{bmatrix} + \begin{bmatrix} -6 & 20 \\ 24 & 16 \end{bmatrix} = \begin{bmatrix} -2 & 16 \\ 30 & 26 \end{bmatrix}$

38. $A = \begin{bmatrix} 4 & 3 \\ -6 & 1 \\ 10 & 1 \end{bmatrix}, B = \begin{bmatrix} 3 & 11 \\ 15 & 25 \\ 20 & 29 \end{bmatrix}$

(a) $A + B = \begin{bmatrix} 4 & 3 \\ -6 & 1 \\ 10 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 11 \\ 15 & 25 \\ 20 & 29 \end{bmatrix} = \begin{bmatrix} 4+3 & 3+11 \\ -6+15 & 1+25 \\ 10+20 & 1+29 \end{bmatrix} = \begin{bmatrix} 7 & 14 \\ 9 & 26 \\ 30 & 30 \end{bmatrix}$

(b) $A - B = \begin{bmatrix} 4 & 3 \\ -6 & 1 \\ 10 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 11 \\ 15 & 25 \\ 20 & 29 \end{bmatrix} = \begin{bmatrix} 4-3 & 3-11 \\ -6-15 & 1-25 \\ 10-20 & 1-29 \end{bmatrix} = \begin{bmatrix} 1 & -8 \\ -21 & -24 \\ -10 & -28 \end{bmatrix}$

(c) $4A = 4\begin{bmatrix} 4 & 3 \\ -6 & 1 \\ 10 & 1 \end{bmatrix} = \begin{bmatrix} 4(4) & 4(3) \\ 4(-6) & 4(1) \\ 4(10) & 4(1) \end{bmatrix} = \begin{bmatrix} 16 & 12 \\ -24 & 4 \\ 40 & 4 \end{bmatrix}$

(d) $2A + 2B = 2\begin{bmatrix} 4 & 3 \\ -6 & 1 \\ 10 & 1 \end{bmatrix} + 2\begin{bmatrix} 3 & 11 \\ 15 & 25 \\ 20 & 29 \end{bmatrix} = \begin{bmatrix} 8 & 6 \\ -12 & 2 \\ 20 & 2 \end{bmatrix} + \begin{bmatrix} 6 & 22 \\ 30 & 50 \\ 40 & 58 \end{bmatrix} = \begin{bmatrix} 14 & 28 \\ 18 & 52 \\ 60 & 60 \end{bmatrix}$

39. (a) $A + B = \begin{bmatrix} 5 & 4 \\ -7 & 2 \\ 11 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ 4 & 12 \\ 20 & 40 \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ -3 & 14 \\ 31 & 42 \end{bmatrix}$

(b) $A - B = \begin{bmatrix} 5 & 4 \\ -7 & 2 \\ 11 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ 4 & 12 \\ 20 & 40 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -11 & -10 \\ -9 & -38 \end{bmatrix}$

(c) $4A = 4\begin{bmatrix} 5 & 4 \\ -7 & 2 \\ 11 & 2 \end{bmatrix} = \begin{bmatrix} 20 & 16 \\ -28 & 8 \\ 44 & 8 \end{bmatrix}$

(d) $2A + 2B = 2\begin{bmatrix} 5 & 4 \\ -7 & 2 \\ 11 & 2 \end{bmatrix} + 2\begin{bmatrix} 0 & 3 \\ 4 & 12 \\ 20 & 40 \end{bmatrix} = \begin{bmatrix} 10 & 8 \\ -14 & 4 \\ 22 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 6 \\ 8 & 24 \\ 40 & 80 \end{bmatrix} = \begin{bmatrix} 10 & 14 \\ -6 & 28 \\ 62 & 84 \end{bmatrix}$

40. (a) $A + B$ is not possible, A and B do not have the same order. A is 1×3 and B is 3×1.

(b) $A - B$ is not possible. A and B do not have the same order. A is 1×3 and B is 3×1.

(c) $4A = 4\begin{bmatrix} 6 & -5 & 7 \end{bmatrix} = \begin{bmatrix} 24 & -20 & 28 \end{bmatrix}$

(d) $2A + 2B$ is not possible. A and B do not have the same order. A is 1×3 and B is 3×1.

41. $\begin{bmatrix} 7 & 3 \\ -1 & 5 \end{bmatrix} + \begin{bmatrix} 10 & -20 \\ 14 & -3 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 1 & 9 \end{bmatrix} = \begin{bmatrix} 7 + 10 + 5 & 3 - 20 + 0 \\ -1 + 14 + 1 & 5 - 3 + 9 \end{bmatrix} = \begin{bmatrix} 22 & -17 \\ 14 & 11 \end{bmatrix}$

42. $\begin{bmatrix} -11 & -7 \\ 16 & -2 \\ 19 & 1 \end{bmatrix} - \begin{bmatrix} 6 & 0 \\ 8 & -4 \\ -2 & 10 \end{bmatrix} + \begin{bmatrix} -3 & 1 \\ 2 & 28 \\ 12 & -2 \end{bmatrix} = \begin{bmatrix} -20 & -6 \\ 10 & 30 \\ 33 & -11 \end{bmatrix}$

43. $-2\left(\begin{bmatrix} 1 & 2 \\ 5 & -4 \\ 6 & 0 \end{bmatrix} + \begin{bmatrix} 7 & 1 \\ 1 & 2 \\ 1 & 4 \end{bmatrix}\right) = -2\begin{bmatrix} 8 & 3 \\ 6 & -2 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ -12 & 4 \\ -14 & -8 \end{bmatrix}$

44. $5\left(\begin{bmatrix} 8 & -1 & 8 \\ -2 & 4 & 12 \\ 0 & -6 & 0 \end{bmatrix} - \begin{bmatrix} -2 & 0 & -4 \\ 3 & -1 & 1 \\ 6 & 12 & -8 \end{bmatrix}\right) = 5\begin{bmatrix} 10 & -1 & 12 \\ -5 & 5 & 11 \\ -6 & -18 & 8 \end{bmatrix}$

$= \begin{bmatrix} 50 & -5 & 60 \\ -25 & 25 & 55 \\ -30 & -90 & 40 \end{bmatrix}$

45. $X = 2A - 3B = 2\begin{bmatrix} -4 & 0 \\ 1 & -5 \\ -3 & 2 \end{bmatrix} - 3\begin{bmatrix} 1 & 2 \\ -2 & 1 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} -8 & 0 \\ 2 & -10 \\ -6 & 4 \end{bmatrix} + \begin{bmatrix} -3 & -6 \\ 6 & -3 \\ -12 & -12 \end{bmatrix} = \begin{bmatrix} -11 & -6 \\ 8 & -13 \\ -18 & -8 \end{bmatrix}$

46. $X = \frac{1}{6}(4A + 3B) = \frac{1}{6}\left(4\begin{bmatrix} -4 & 0 \\ 1 & -5 \\ -3 & 2 \end{bmatrix} + 3\begin{bmatrix} 1 & 2 \\ -2 & 1 \\ 4 & 4 \end{bmatrix}\right) = \frac{1}{6}\left(\begin{bmatrix} -16 & 0 \\ 4 & -20 \\ -12 & 8 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ -6 & 3 \\ 12 & 12 \end{bmatrix}\right) = \frac{1}{6}\begin{bmatrix} -16 + 3 & 0 + 6 \\ 4 - 6 & -20 + 3 \\ -12 + 12 & 8 + 12 \end{bmatrix}$

$= \frac{1}{6}\begin{bmatrix} -13 & 6 \\ -2 & -17 \\ 0 & 20 \end{bmatrix} = \begin{bmatrix} -\frac{13}{6} & 1 \\ -\frac{1}{3} & -\frac{17}{6} \\ 0 & \frac{10}{3} \end{bmatrix}$

47. $X = \frac{1}{3}[B - 2A] = \frac{1}{3}\left(\begin{bmatrix} 1 & 2 \\ -2 & 1 \\ 4 & 4 \end{bmatrix} - 2\begin{bmatrix} -4 & 0 \\ 1 & -5 \\ -3 & 2 \end{bmatrix}\right) = \frac{1}{3}\begin{bmatrix} 9 & 2 \\ -4 & 11 \\ 10 & 0 \end{bmatrix} = \begin{bmatrix} 3 & \frac{2}{3} \\ -\frac{4}{3} & \frac{11}{3} \\ \frac{10}{3} & 0 \end{bmatrix}$

48. $X = \frac{1}{3}(2A - 5B) = \frac{1}{3}\left(2\begin{bmatrix} -4 & 0 \\ 1 & -5 \\ -3 & 2 \end{bmatrix} - 5\begin{bmatrix} 1 & 2 \\ -2 & 1 \\ 4 & 4 \end{bmatrix}\right) = \frac{1}{3}\left(\begin{bmatrix} -8 & 0 \\ 2 & -10 \\ -6 & 4 \end{bmatrix} + \begin{bmatrix} -5 & -10 \\ 10 & -5 \\ -20 & -20 \end{bmatrix}\right) = \frac{1}{3}\begin{bmatrix} -8-5 & 0-10 \\ 2+10 & -10-5 \\ -6-20 & 4-20 \end{bmatrix}$

$= \frac{1}{3}\begin{bmatrix} -13 & -10 \\ 12 & -15 \\ -26 & -16 \end{bmatrix} = \begin{bmatrix} -\frac{13}{3} & -\frac{10}{3} \\ 4 & -5 \\ -\frac{26}{3} & -\frac{16}{3} \end{bmatrix}$

49. A and B are both 2×2, so AB exists and has dimensions 2×2.

$AB = \begin{bmatrix} 2 & -2 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} -3 & 10 \\ 12 & 8 \end{bmatrix} = \begin{bmatrix} 2(-3) + (-2)(12) & 2(10) + (-2)(8) \\ 3(-3) + 5(12) & 3(10) + 5(8) \end{bmatrix} = \begin{bmatrix} -30 & 4 \\ 51 & 70 \end{bmatrix}$

50. Not possible because the number of columns of A does not equal the number of rows of B.

51. Because A is 3×2 and B is 2×2, AB exists and has dimensions 3×2.

$AB = \begin{bmatrix} 5 & 4 \\ -7 & 2 \\ 11 & 2 \end{bmatrix}\begin{bmatrix} 4 & 12 \\ 20 & 40 \end{bmatrix} = \begin{bmatrix} 5(4) + 4(20) & 5(12) + 4(40) \\ -7(4) + 2(20) & -7(12) + 2(40) \\ 11(4) + 2(20) & 11(12) + 2(40) \end{bmatrix} = \begin{bmatrix} 100 & 220 \\ 12 & -4 \\ 84 & 212 \end{bmatrix}$

52. Because A is a 1×3 and B is 3×1, AB exists and has dimensions 1×1.

$AB = \begin{bmatrix} 6 & -5 & 7 \end{bmatrix}\begin{bmatrix} -1 \\ 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 6(-1) - 5(4) + 7(8) \end{bmatrix} = \begin{bmatrix} 30 \end{bmatrix}$

53. $\begin{bmatrix} 4 & 1 \\ 11 & -7 \\ 12 & 3 \end{bmatrix}\begin{bmatrix} 3 & -5 & 6 \\ 2 & -2 & -2 \end{bmatrix} = \begin{bmatrix} 14 & -22 & 22 \\ 19 & -41 & 80 \\ 42 & -66 & 66 \end{bmatrix}$

54. $\begin{bmatrix} -2 & 3 & 10 \\ 4 & -2 & 2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -5 & 2 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 13 & 24 \\ 20 & 4 \end{bmatrix}$

55. Not possible. The number of columns of the first matrix does not equal the number of rows of the second matrix.

56. $\begin{bmatrix} 4 & -2 & 6 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ 0 & -3 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 4(-2) - 2(0) + 6(2) & 4(1) - 2(-3) + 6(0) \end{bmatrix} = \begin{bmatrix} 4 & 10 \end{bmatrix}$

57. (a) $AB = \begin{bmatrix} 1 & 3 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} (1)(5) + (3)(-2) & (1)(-1) + (3)(0) \\ (4)(5) + (1)(-2) & (4)(-1) + (1)(0) \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 18 & -4 \end{bmatrix}$

(b) $BA = \begin{bmatrix} 5 & -1 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} (5)(1) + (-1)(4) & (5)(3) + (-1)(1) \\ (-2)(1) + (0)(4) & (-2)(3) + (0)(1) \end{bmatrix} = \begin{bmatrix} 1 & 14 \\ -2 & -6 \end{bmatrix}$

(c) $A^2 = \begin{bmatrix} 1 & 3 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} (1)(1) + (3)(4) & (1)(3) + (3)(1) \\ (4)(1) + (1)(4) & (4)(3) + (1)(1) \end{bmatrix} = \begin{bmatrix} 13 & 6 \\ 8 & 13 \end{bmatrix}$

58. (a) $AB = \begin{bmatrix} 2 & 3 \\ 8 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \end{bmatrix} = \begin{bmatrix} (2)(4) + (3)(1) \\ (8)(4) + (-1)(1) \\ (0)(4) + (2)(1) \end{bmatrix} = \begin{bmatrix} 11 \\ 31 \\ 2 \end{bmatrix}$

(b) BA is not possible. The number of columns of the first matrix does not equal the number of rows of the second matrix,
B is 2×1 and A is 3×2.

(c) A^2 is not possible. The number of columns of the first matrix does not equal the number of rows of the second matrix,
A is 3×2.

In Exercises 59-62, $\mathbf{v} = \langle 2, 5 \rangle = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$

59. $A\mathbf{v} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \end{bmatrix} = \langle 2, -5 \rangle$ is a reflection in the x-axis.

60. $A\mathbf{v} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} -5 \\ -2 \end{bmatrix} = \langle -5, -2 \rangle$ is a reflection in the line $y = x$.

61. $A\mathbf{v} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \end{bmatrix} = \langle 1, 5 \rangle$ is a horizontal shrink.

62. $A\mathbf{v} = \begin{bmatrix} 1 & 0 \\ 0 & 6 \end{bmatrix}\begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ 30 \end{bmatrix} = \langle 2, 30 \rangle$ is a vertical stretch.

63. $0.95A = 0.95\begin{bmatrix} 80 & 120 & 140 \\ 40 & 100 & 80 \end{bmatrix} = \begin{bmatrix} 76 & 114 & 133 \\ 38 & 95 & 76 \end{bmatrix}$

64. $T = \begin{bmatrix} 120 & 80 & 20 \end{bmatrix}$

$$TC = \begin{bmatrix} 120 & 80 & 20 \end{bmatrix}\begin{bmatrix} 0.07 & 0.095 \\ 0.10 & 0.08 \\ 0.28 & 0.25 \end{bmatrix} = \begin{bmatrix} \$22 & \$22.8 \end{bmatrix}$$

Your cost with company A is \$22.00. Your cost with company B is \$22.80.

65. $$AB = \begin{bmatrix} -4 & -1 \\ 7 & 2 \end{bmatrix}\begin{bmatrix} -2 & -1 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} -4(-2) + (-1)(7) & -4(-1) + (-1)(4) \\ 7(-2) + 2(7) & 7(-1) + 2(4) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$BA = \begin{bmatrix} -2 & -1 \\ 7 & 4 \end{bmatrix}\begin{bmatrix} -4 & -1 \\ 7 & 2 \end{bmatrix} = \begin{bmatrix} -2(-4) + (-1)(7) & -2(-1) + (-1)(2) \\ 7(-4) + 4(7) & 7(-1) + 4(2) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

66. $$AB = \begin{bmatrix} 5 & -1 \\ 11 & -2 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ -11 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$BA = \begin{bmatrix} -2 & 1 \\ -11 & 5 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 11 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

67. $$AB = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 6 & 2 & 3 \end{bmatrix}\begin{bmatrix} -2 & -3 & 1 \\ 3 & 3 & -1 \\ 2 & 4 & -1 \end{bmatrix} = \begin{bmatrix} 1(-2) + 1(3) + 0(2) & 1(-3) + 1(3) + 0(4) & 1(1) + 1(-1) + 0(-1) \\ 1(-2) + 0(3) + 1(2) & 1(-3) + 0(3) + 1(4) & 1(1) + 0(-1) + 1(-1) \\ 6(-2) + 2(3) + 3(2) & 6(-3) + 2(3) + 3(4) & 6(1) + 2(-1) + 3(-1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$$BA = \begin{bmatrix} -2 & -3 & 1 \\ 3 & 3 & -1 \\ 2 & 4 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 6 & 2 & 3 \end{bmatrix} = \begin{bmatrix} -2(1) + (-3)(1) + 1(6) & -2(1) + (-3)(0) + 1(2) & -2(0) + (-3)(1) + 1(3) \\ 3(1) + 3(1) + (-1)(6) & 3(1) + 3(0) + (-1)(2) & 3(0) + 3(1) + (-1)(3) \\ 2(1) + 4(1) + (-1)(6) & 2(1) + 4(0) + (-1)(2) & 2(0) + 4(1) + (-1)(3) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

68. $$AB = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 0 & -1 \\ 8 & -4 & 2 \end{bmatrix}\begin{bmatrix} -2 & 1 & \frac{1}{2} \\ -3 & 1 & \frac{1}{2} \\ 2 & -2 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$$BA = \begin{bmatrix} -2 & 1 & \frac{1}{2} \\ -3 & 1 & \frac{1}{2} \\ 2 & -2 & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 0 & -1 \\ 8 & -4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

69. $[A \; \vdots \; I] = \begin{bmatrix} -6 & 5 & \vdots & 1 & 0 \\ -5 & 4 & \vdots & 0 & 1 \end{bmatrix}$

$-\frac{1}{6}R_1 \rightarrow \begin{bmatrix} 1 & -\frac{5}{6} & \vdots & -\frac{1}{6} & 0 \\ -5 & 4 & \vdots & 0 & 1 \end{bmatrix}$

$5R_1 + R_2 \rightarrow \begin{bmatrix} 1 & -\frac{5}{6} & \vdots & -\frac{1}{6} & 0 \\ 0 & -\frac{1}{6} & \vdots & -\frac{5}{6} & 1 \end{bmatrix}$

$-6R_2 \rightarrow \begin{bmatrix} 1 & -\frac{5}{6} & \vdots & -\frac{1}{6} & 0 \\ 0 & 1 & \vdots & 5 & -6 \end{bmatrix}$

$\frac{5}{6}R_2 + R_1 \rightarrow \begin{bmatrix} 1 & 0 & \vdots & 4 & -5 \\ 0 & 1 & \vdots & 5 & -6 \end{bmatrix} = [I \; \vdots \; A^{-1}]$

$A^{-1} = \begin{bmatrix} 4 & -5 \\ 5 & -6 \end{bmatrix}$

70. $[A \; \vdots \; I] = \begin{bmatrix} 3 & 4 & \vdots & 1 & 0 \\ 6 & 8 & \vdots & 0 & 1 \end{bmatrix}$

$-R_2 + 2R_1 \rightarrow \begin{bmatrix} 3 & 4 & \vdots & 1 & 0 \\ 0 & 0 & \vdots & 2 & -1 \end{bmatrix}$

A^{-1} does not exist.

71. $[A \; \vdots \; I] = \begin{bmatrix} 2 & 0 & 3 & \vdots & 1 & 0 & 0 \\ -1 & 1 & 1 & \vdots & 0 & 1 & 0 \\ 2 & -2 & 1 & \vdots & 0 & 0 & 1 \end{bmatrix}$

$2R_2 + R_3 \rightarrow \begin{bmatrix} 2 & 0 & 3 & \vdots & 1 & 0 & 0 \\ -1 & 1 & 1 & \vdots & 0 & 1 & 0 \\ 0 & 0 & 3 & \vdots & 0 & 2 & 1 \end{bmatrix}$

$-R_3 + R_1 \rightarrow \begin{bmatrix} 2 & 0 & 0 & \vdots & 1 & -2 & -1 \\ -1 & 1 & 1 & \vdots & 0 & 1 & 0 \\ 0 & 0 & 3 & \vdots & 0 & 2 & 1 \end{bmatrix}$

$\begin{matrix} \frac{1}{2}R_1 \rightarrow \\ \\ \frac{1}{3}R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 0 & 0 & \vdots & \frac{1}{2} & -1 & -\frac{1}{2} \\ -1 & 1 & 1 & \vdots & 0 & 1 & 0 \\ 0 & 0 & 1 & \vdots & 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$

$R_1 + R_2 \rightarrow \begin{bmatrix} 1 & 0 & 0 & \vdots & \frac{1}{2} & -1 & -\frac{1}{2} \\ 0 & 1 & 1 & \vdots & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & \vdots & 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$

$-R_3 + R_2 \rightarrow \begin{bmatrix} 1 & 0 & 0 & \vdots & \frac{1}{2} & -1 & -\frac{1}{2} \\ 0 & 1 & 0 & \vdots & \frac{1}{2} & -\frac{2}{3} & -\frac{5}{6} \\ 0 & 0 & 1 & \vdots & 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix} = [I \; \vdots \; A^{-1}]$

$A^{-1} = \begin{bmatrix} \frac{1}{2} & -1 & -\frac{1}{2} \\ \frac{1}{2} & -\frac{2}{3} & -\frac{5}{6} \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$

72. $[A \;\vdots\; I] = \begin{bmatrix} 0 & -2 & 1 & \vdots & 1 & 0 & 0 \\ -5 & -2 & -3 & \vdots & 0 & 1 & 0 \\ 7 & 3 & 4 & \vdots & 0 & 0 & 1 \end{bmatrix}$

$$\begin{matrix} R_3 \\ \\ R_1 \end{matrix} \begin{bmatrix} 7 & 3 & 4 & \vdots & 0 & 0 & 1 \\ -5 & -2 & -3 & \vdots & 0 & 1 & 0 \\ 0 & -2 & 1 & \vdots & 1 & 0 & 0 \end{bmatrix}$$

$$R_2 + R_1 \rightarrow \begin{bmatrix} 2 & 1 & 1 & \vdots & 0 & 1 & 1 \\ -5 & -2 & -3 & \vdots & 0 & 1 & 0 \\ 0 & -2 & 1 & \vdots & 1 & 0 & 0 \end{bmatrix}$$

$$5R_1 + 2R_2 \rightarrow \begin{bmatrix} 2 & 1 & 1 & \vdots & 0 & 1 & 1 \\ 0 & 1 & -1 & \vdots & 0 & 7 & 5 \\ 0 & -2 & 1 & \vdots & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{matrix} -R_2 + R_1 \rightarrow \\ \\ 2R_2 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 2 & 0 & 2 & \vdots & 0 & -6 & -4 \\ 0 & 1 & -1 & \vdots & 0 & 7 & 5 \\ 0 & 0 & -1 & \vdots & 1 & 14 & 10 \end{bmatrix}$$

$$\begin{matrix} \frac{1}{2}R_1 \rightarrow \\ \\ -R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 0 & 1 & \vdots & 0 & -3 & -2 \\ 0 & 1 & -1 & \vdots & 0 & 7 & 5 \\ 0 & 0 & 1 & \vdots & -1 & -14 & -10 \end{bmatrix}$$

$$\begin{matrix} -R_3 + R_1 \rightarrow \\ R_3 + R_2 \rightarrow \\ \\ \end{matrix} \begin{bmatrix} 1 & 0 & 0 & \vdots & 1 & 11 & 8 \\ 0 & 1 & 0 & \vdots & -1 & -7 & -5 \\ 0 & 0 & 1 & \vdots & -1 & -14 & -10 \end{bmatrix} = [I \;\vdots\; A^{-1}]$$

$$A^{-1} = \begin{bmatrix} 1 & 11 & 8 \\ -1 & -7 & -5 \\ -1 & -14 & -10 \end{bmatrix}$$

73. $\begin{bmatrix} -1 & -2 & -2 \\ 3 & 7 & 9 \\ 1 & 4 & 7 \end{bmatrix}^{-1} = \begin{bmatrix} 13 & 6 & -4 \\ -12 & -5 & 3 \\ 5 & 2 & -1 \end{bmatrix}$

74. $A = \begin{bmatrix} 8 & 0 & 2 & 8 \\ 4 & -2 & 0 & -2 \\ 1 & 2 & 1 & 4 \\ -1 & 4 & 1 & 1 \end{bmatrix}$

$$A^{-1} = \begin{bmatrix} -2.5 & 3 & 7 & -2 \\ -4 & 4.5 & 11 & -3 \\ 14.5 & -16 & -40 & 12 \\ -1 & 1 & 3 & -1 \end{bmatrix}$$

75. $A = \begin{bmatrix} -7 & 2 \\ -8 & 2 \end{bmatrix}$

$$A^{-1} = \frac{1}{-7(2) - 2(-8)}\begin{bmatrix} 2 & -2 \\ 8 & -7 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 & -2 \\ 8 & -7 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 4 & -\frac{7}{2} \end{bmatrix}$$

76. $A = \begin{bmatrix} 10 & 4 \\ 7 & 3 \end{bmatrix}$

$$A^{-1} = \frac{1}{10(3) - 4(7)}\begin{bmatrix} 3 & -4 \\ -7 & 10 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 3 & -4 \\ -7 & 10 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & -2 \\ -\frac{7}{2} & 5 \end{bmatrix}$$

77. $A = \begin{bmatrix} -12 & 6 \\ 10 & -5 \end{bmatrix}$

$ad - bc = (-12)(-5) - (6)(10) = 0$

A^{-1} does not exist.

78. $A = \begin{bmatrix} -18 & -15 \\ -6 & -5 \end{bmatrix}$

$ad - bc = (-18)(-5) - (-15)(-6) = 0$

A^{-1} does not exist.

79. $\begin{cases} -x + 4y = 8 \\ 2x - 7y = -5 \end{cases}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 2 & -7 \end{bmatrix}^{-1} \begin{bmatrix} 8 \\ -5 \end{bmatrix} = \begin{bmatrix} 7 & 4 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 8 \\ -5 \end{bmatrix} = \begin{bmatrix} 7(8) + 4(-5) \\ 2(8) + 1(-5) \end{bmatrix} = \begin{bmatrix} 36 \\ 11 \end{bmatrix}$$

Solution: $(36, 11)$

80. $\begin{cases} 5x - y = 13 \\ -9x + 2y = -24 \end{cases}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 & -1 \\ -9 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 13 \\ -24 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 9 & 5 \end{bmatrix} \begin{bmatrix} 13 \\ -24 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

Solution: $(2, -3)$

81. $\begin{cases} -3x + 10y = 8 \\ 5x - 17y = -13 \end{cases}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 & 10 \\ 5 & -17 \end{bmatrix}^{-1} \begin{bmatrix} 8 \\ -13 \end{bmatrix} = \begin{bmatrix} -17 & -10 \\ -5 & -3 \end{bmatrix} \begin{bmatrix} 8 \\ -13 \end{bmatrix} = \begin{bmatrix} -17(8) + (-10)(-13) \\ -5(8) + (-3)(-13) \end{bmatrix} = \begin{bmatrix} -6 \\ -1 \end{bmatrix}$$

Solution: $(-6, -1)$

82. $\begin{cases} 4x - 2y = -10 \\ -19x + 9y = 47 \end{cases}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -19 & 9 \end{bmatrix}^{-1} \begin{bmatrix} -10 \\ 47 \end{bmatrix} = \begin{bmatrix} -\frac{9}{2} & -1 \\ -\frac{19}{2} & -2 \end{bmatrix} \begin{bmatrix} -10 \\ 47 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

Solution: $(-2, 1)$

83. $\begin{cases} \frac{1}{2}x + \frac{1}{3}y = 2 \\ -3x + 2y = 0 \end{cases}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{3} \\ -3 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{6} \\ \frac{3}{2} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Solution: $(2, 3)$

84. $\begin{cases} -\frac{5}{6}x + \frac{3}{8}y = -2 \\ 4x - 3y = 0 \end{cases}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{5}{6} & \frac{3}{8} \\ 4 & -3 \end{bmatrix}^{-1} \begin{bmatrix} -2 \\ 0 \end{bmatrix} = \begin{bmatrix} -3 & -\frac{3}{8} \\ -4 & -\frac{5}{6} \end{bmatrix} \begin{bmatrix} -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$$

Solution: $(6, 8)$

85. $\begin{cases} 0.3x + 0.7y = 10.2 \\ 0.4x + 0.6y = 7.6 \end{cases}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.3 & 0.7 \\ 0.4 & 0.6 \end{bmatrix}^{-1} \begin{bmatrix} 10.2 \\ 7.6 \end{bmatrix} = \begin{bmatrix} -6 & 7 \\ 4 & -3 \end{bmatrix} \begin{bmatrix} 10.2 \\ 7.6 \end{bmatrix} = \begin{bmatrix} -8 \\ 18 \end{bmatrix}$$

Solution: $(-8, 18)$

86. $\begin{cases} 3.5x - 4.5y = 8 \\ 2.5x - 7.5y = 25 \end{cases}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3.5 & -4.5 \\ 2.5 & -7.5 \end{bmatrix}^{-1} \begin{bmatrix} 8 \\ 25 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{3}{10} \\ \frac{1}{6} & -\frac{7}{30} \end{bmatrix} \begin{bmatrix} 8 \\ 25 \end{bmatrix} = \begin{bmatrix} -3.5 \\ -4.5 \end{bmatrix}$$

Solution: $(-3.5, -4.5)$

87. $\begin{cases} 3x + 2y - z = 6 \\ x - y + 2z = -1 \\ 5x + y + z = 7 \end{cases}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 & 2 & -1 \\ 1 & -1 & 2 \\ 5 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 6 \\ -1 \\ 7 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 1 \\ 3 & \frac{8}{3} & -\frac{7}{3} \\ 2 & \frac{7}{3} & -\frac{5}{3} \end{bmatrix} \begin{bmatrix} 6 \\ -1 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} -1(6) - 1(-1) + 1(7) \\ 3(6) + \frac{8}{3}(-1) - \frac{7}{3}(7) \\ 2(6) + \frac{7}{3}(-1) - \frac{5}{3}(7) \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}$$

Solution: $(2, -1, -2)$

88. $\begin{cases} -x + 4y - 2z = 12 \\ 2x - 9y + 5z = -25 \\ -x + 5y - 4z = 10 \end{cases}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & 4 & -2 \\ 2 & -9 & 5 \\ -1 & 5 & -4 \end{bmatrix}^{-1} \begin{bmatrix} 12 \\ -25 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} -11 & -6 & -2 \\ -3 & -2 & -1 \\ -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 12 \\ -25 \\ 10 \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \\ 3 \end{bmatrix}$$

Solution: $(-2, 4, 3)$

89. $\begin{cases} x + 2y = -1 \\ 3x + 4y = -5 \end{cases}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ -5 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} -1 \\ -5 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

Solution: $(-3, 1)$

90. $\begin{cases} x + 3y = 23 \\ -6x + 2y = -18 \end{cases}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ -6 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 23 \\ -18 \end{bmatrix} = \begin{bmatrix} 0.1 & -0.15 \\ 0.3 & 0.05 \end{bmatrix} \begin{bmatrix} 23 \\ -18 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

Solution: $(5, 6)$

91. $\begin{cases} \frac{6}{5}x - \frac{4}{7}y = \frac{6}{5} \\ -\frac{12}{5}x + \frac{12}{7}y = -\frac{17}{5} \end{cases}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{6}{5} & -\frac{4}{7} \\ -\frac{12}{5} & \frac{12}{7} \end{bmatrix}^{-1} \begin{bmatrix} \frac{6}{5} \\ -\frac{17}{5} \end{bmatrix} = \begin{bmatrix} \frac{5}{2} & \frac{5}{6} \\ \frac{7}{2} & \frac{7}{4} \end{bmatrix} \begin{bmatrix} \frac{6}{5} \\ -\frac{17}{5} \end{bmatrix} = \begin{bmatrix} \frac{1}{6} \\ -\frac{7}{4} \end{bmatrix}$$

Solution: $\left(\frac{1}{6}, -\frac{7}{4}\right)$

92. $\begin{cases} 5x + 10y = 7 \\ 2x + y = -98 \end{cases}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ 2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 7 \\ -98 \end{bmatrix} = \begin{bmatrix} -\frac{1}{15} & \frac{2}{3} \\ \frac{2}{15} & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} 7 \\ -98 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{329}{5} \\ \frac{168}{5} \end{bmatrix} = \begin{bmatrix} -65.8 \\ 33.6 \end{bmatrix}$$

Solution: $(-65.8, 33.6)$

93. $A = \begin{bmatrix} 2 & 5 \\ -4 & 3 \end{bmatrix}: \begin{vmatrix} 2 & 5 \\ -4 & 3 \end{vmatrix} = (2)(3) - (-4)(5) = 26$

94. $A = \begin{bmatrix} -3 & 1 \\ 5 & -2 \end{bmatrix}: \begin{vmatrix} -3 & 1 \\ 5 & -2 \end{vmatrix} = (-3)(-2) - (5)(1) = 1$

95. $A = \begin{bmatrix} 10 & -2 \\ 18 & 8 \end{bmatrix}: \begin{vmatrix} 10 & -2 \\ 18 & 8 \end{vmatrix} = (10)(8) - (18)(-2) = 116$

96. $A = \begin{bmatrix} -30 & 10 \\ 5 & 2 \end{bmatrix}: \begin{vmatrix} -30 & 10 \\ 5 & 2 \end{vmatrix} = (-30)(2) - (5)(10) = -110$

97. $\begin{bmatrix} 2 & -1 \\ 7 & 4 \end{bmatrix}$

(a) $M_{11} = 4$
$M_{12} = 7$
$M_{21} = -1$
$M_{22} = 2$

(b) $C_{11} = M_{11} = 4$
$C_{12} = -M_{12} = -7$
$C_{21} = -M_{21} = 1$
$C_{22} = M_{22} = 2$

98. $\begin{bmatrix} 3 & 6 \\ 5 & -4 \end{bmatrix}$

(a) $M_{11} = -4$
$M_{12} = 5$
$M_{21} = 6$
$M_{22} = 3$

(b) $C_{11} = M_{11} = -4$
$C_{12} = -M_{12} = -5$
$C_{21} = -M_{21} = -6$
$C_{22} = M_{22} = 3$

99. $\begin{bmatrix} 3 & 2 & -1 \\ -2 & 5 & 0 \\ 1 & 8 & 6 \end{bmatrix}$

(a) $M_{11} = \begin{vmatrix} 5 & 0 \\ 8 & 6 \end{vmatrix} = 30$

$M_{12} = \begin{vmatrix} -2 & 0 \\ 1 & 6 \end{vmatrix} = -12$

$M_{13} = \begin{vmatrix} -2 & 5 \\ 1 & 8 \end{vmatrix} = -21$

$M_{21} = \begin{vmatrix} 2 & -1 \\ 8 & 6 \end{vmatrix} = 20$

$M_{22} = \begin{vmatrix} 3 & -1 \\ 1 & 6 \end{vmatrix} = 19$

$M_{23} = \begin{vmatrix} 3 & 2 \\ 1 & 8 \end{vmatrix} = 22$

$M_{31} = \begin{vmatrix} 2 & -1 \\ 5 & 0 \end{vmatrix} = 5$

$M_{32} = \begin{vmatrix} 3 & -1 \\ -2 & 0 \end{vmatrix} = -2$

$M_{33} = \begin{vmatrix} 3 & 2 \\ -2 & 5 \end{vmatrix} = 19$

(b) $C_{11} = M_{11} = 30$

$C_{12} = -M_{12} = 12$

$C_{13} = M_{13} = -21$

$C_{21} = -M_{21} = -20$

$C_{22} = M_{22} = 19$

$C_{23} = -M_{23} = -22$

$C_{31} = M_{31} = 5$

$C_{32} = -M_{32} = 2$

$C_{33} = M_{33} = 19$

100. $\begin{bmatrix} 8 & 3 & 4 \\ 6 & 5 & -9 \\ -4 & 1 & 2 \end{bmatrix}$

(a) $M_{11} = \begin{vmatrix} 5 & -9 \\ 1 & 2 \end{vmatrix} = 19$

$M_{12} = \begin{vmatrix} 6 & -9 \\ -4 & 2 \end{vmatrix} = -24$

$M_{13} = \begin{vmatrix} 6 & 5 \\ -4 & 1 \end{vmatrix} = 26$

$M_{21} = \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix} = 2$

$M_{22} = \begin{vmatrix} 8 & 4 \\ -4 & 2 \end{vmatrix} = 32$

$M_{23} = \begin{vmatrix} 8 & 3 \\ -4 & 1 \end{vmatrix} = 20$

$M_{31} = \begin{vmatrix} 3 & 4 \\ 5 & -9 \end{vmatrix} = -47$

$M_{32} = \begin{vmatrix} 8 & 4 \\ 6 & -9 \end{vmatrix} = -96$

$M_{33} = \begin{vmatrix} 8 & 3 \\ 6 & 5 \end{vmatrix} = 22$

(b) $C_{11} = M_{11} = 19$

$C_{12} = -M_{12} = 24$

$C_{13} = M_{13} = 26$

$C_{21} = -M_{21} = -2$

$C_{22} = M_{22} = 32$

$C_{23} = -M_{23} = -20$

$C_{31} = M_{31} = -47$

$C_{32} = -M_{32} = 96$

$C_{33} = M_{33} = 22$

101. Expand using Row 1.

$$\begin{vmatrix} -2 & 0 & 0 \\ 2 & -1 & 0 \\ -1 & 1 & -3 \end{vmatrix} = -2\begin{vmatrix} -1 & 0 \\ 1 & -3 \end{vmatrix} - 0\begin{vmatrix} 2 & 0 \\ -1 & 3 \end{vmatrix} + 0\begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix}$$

$$= -2(3) - 0(6) + 0(1)$$

$$= -6$$

102. Expand using Column 1.

$$\begin{vmatrix} 0 & 1 & -2 \\ 0 & 1 & 2 \\ -1 & -1 & 3 \end{vmatrix} = 0\begin{vmatrix} 1 & 2 \\ -1 & 3 \end{vmatrix} - 0\begin{vmatrix} 1 & -2 \\ -1 & 3 \end{vmatrix} - 1\begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix}$$

$$= 0(5) - 0(1) - 1(4)$$

$$= -4$$

103. Expand using Row 3.

$$\begin{vmatrix} 4 & 1 & -1 \\ 2 & 3 & 2 \\ 1 & -1 & 0 \end{vmatrix} = 1\begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} + 1\begin{vmatrix} 4 & -1 \\ 2 & 2 \end{vmatrix} + 0\begin{vmatrix} 4 & 1 \\ 2 & 3 \end{vmatrix}$$

$$= 1(5) + 1(10) + 0(10)$$

$$= 15$$

104. Expand using Column 3.

$$\begin{vmatrix} -1 & -2 & 1 \\ 2 & 3 & 0 \\ -5 & -1 & 3 \end{vmatrix} = 1\begin{vmatrix} 2 & 3 \\ -5 & -1 \end{vmatrix} - 0\begin{vmatrix} -1 & -2 \\ -5 & -1 \end{vmatrix} + 3\begin{vmatrix} -1 & -2 \\ 2 & 3 \end{vmatrix}$$

$$= 1(13) - 0(11) + 3(1)$$

$$= 16$$

105. Expand using Column 2.

$$\begin{vmatrix} -2 & 4 & 1 \\ -6 & 0 & 2 \\ 5 & 3 & 4 \end{vmatrix} = -4\begin{vmatrix} -6 & 2 \\ 5 & 4 \end{vmatrix} - 3\begin{vmatrix} -2 & 1 \\ -6 & 2 \end{vmatrix}$$

$$= -4(-34) - 3(2) = 130$$

106. Expand using Row 3.

$$\begin{vmatrix} 1 & 1 & 4 \\ -4 & 1 & 2 \\ 0 & 1 & -1 \end{vmatrix} = 0\begin{vmatrix} 1 & 4 \\ 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & 4 \\ -4 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ -4 & 1 \end{vmatrix}$$

$$= 0(-2) - 1(18) - 1(5)$$

$$= -23$$

107. $\begin{cases} 5x - 2y = 6 \\ -11x + 3y = -23 \end{cases}$

$$x = \frac{\begin{vmatrix} 6 & -2 \\ -23 & 3 \end{vmatrix}}{\begin{vmatrix} 5 & -2 \\ -11 & 3 \end{vmatrix}} = \frac{-28}{-7} = 4, \qquad y = \frac{\begin{vmatrix} 5 & 6 \\ -11 & -23 \end{vmatrix}}{\begin{vmatrix} 5 & -2 \\ -11 & 3 \end{vmatrix}} = \frac{-49}{-7} = 7$$

Solution: $(4, 7)$

108. $\begin{cases} 3x + 8y = -7 \\ 9x - 5y = 37 \end{cases}$

$$x = \frac{\begin{vmatrix} -7 & 8 \\ 37 & -5 \end{vmatrix}}{\begin{vmatrix} 3 & 8 \\ 9 & -5 \end{vmatrix}} = \frac{-261}{-87} = 3, \qquad y = \frac{\begin{vmatrix} 3 & -7 \\ 9 & 37 \end{vmatrix}}{\begin{vmatrix} 3 & 8 \\ 9 & -5 \end{vmatrix}} = \frac{174}{-87} = -2$$

Solution: $(3, -2)$

109. $\begin{cases} -2x + 3y - 5z = -11 \\ 4x - y + z = -3 \\ -x - 4y + 6z = 15 \end{cases}$

$$D = \begin{vmatrix} -2 & 3 & -5 \\ 4 & -1 & 1 \\ -1 & -4 & 6 \end{vmatrix} = -2(-1)^2\begin{vmatrix} -1 & 1 \\ -4 & 6 \end{vmatrix} + 4(-1)^3\begin{vmatrix} 3 & -5 \\ -4 & 6 \end{vmatrix} - 1(-1)^4\begin{vmatrix} 3 & -5 \\ -1 & 1 \end{vmatrix} = -2(-2) - 4(-2) - (-2) = 14$$

$$x = \frac{\begin{vmatrix} -11 & 3 & -5 \\ -3 & -1 & 1 \\ 15 & -4 & 6 \end{vmatrix}}{14} = \frac{-11(-1)^2\begin{vmatrix} -1 & 1 \\ -4 & 6 \end{vmatrix} - 3(-1)^3\begin{vmatrix} 3 & -5 \\ -4 & 6 \end{vmatrix} + 15(-1)^4\begin{vmatrix} 3 & -5 \\ -1 & 1 \end{vmatrix}}{14} = \frac{-11(-2) + 3(-2) + 15(-2)}{14} = \frac{-14}{14} = -1$$

$$y = \frac{\begin{vmatrix} -2 & -11 & -5 \\ 4 & -3 & 1 \\ -1 & 15 & 6 \end{vmatrix}}{14} = \frac{-2(-1)^2\begin{vmatrix} -3 & 1 \\ 15 & 6 \end{vmatrix} + 4(-1)^3\begin{vmatrix} -11 & -5 \\ 15 & 6 \end{vmatrix} - 1(-1)^4\begin{vmatrix} -11 & -5 \\ -3 & 1 \end{vmatrix}}{14} = \frac{-2(-33) - 4(9) - 1(-26)}{14} = \frac{56}{14} = 4$$

$$z = \frac{\begin{vmatrix} -2 & 3 & -11 \\ 4 & -1 & -3 \\ -1 & -4 & 15 \end{vmatrix}}{14} = \frac{-2(-1)^2\begin{vmatrix} -1 & -3 \\ -4 & 15 \end{vmatrix} + 4(-1)^3\begin{vmatrix} 3 & -11 \\ -4 & 15 \end{vmatrix} - 1(-1)^4\begin{vmatrix} 3 & -11 \\ -1 & -3 \end{vmatrix}}{14} = \frac{-2(-27) - 4(1) - 1(-20)}{14} = \frac{70}{14} = 5$$

Solution: $(-1, 4, 5)$

110. $\begin{cases} 5x - 2y + z = 15 \\ 3x - 3y - z = -7, \\ 2x - y - 7z = -3 \end{cases}$ $\quad D = \begin{vmatrix} 5 & -2 & 1 \\ 3 & -3 & -1 \\ 2 & -1 & -7 \end{vmatrix} = 65$

$$x = \frac{\begin{vmatrix} 15 & -2 & 1 \\ -7 & -3 & -1 \\ -3 & -1 & -7 \end{vmatrix}}{65} = \frac{390}{65} = 6, \quad y = \frac{\begin{vmatrix} 5 & 15 & 1 \\ 3 & -7 & -1 \\ 2 & -3 & -7 \end{vmatrix}}{65} = \frac{520}{65} = 8, \quad z = \frac{\begin{vmatrix} 5 & -2 & 15 \\ 3 & -3 & -7 \\ 2 & -1 & -3 \end{vmatrix}}{65} = \frac{65}{65} = 1$$

Solution: $(6, 8, 1)$

111. $(1, 0), (5, 0), (5, 8)$

$$\text{Area} = \tfrac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 5 & 0 & 1 \\ 5 & 8 & 1 \end{vmatrix} = \tfrac{1}{2}\left(1\begin{vmatrix} 0 & 1 \\ 8 & 1 \end{vmatrix} + 1\begin{vmatrix} 5 & 0 \\ 5 & 8 \end{vmatrix}\right) = \tfrac{1}{2}(-8 + 40) = \tfrac{1}{2}(32) = 16 \text{ square units}$$

112. $(-4, 0), (4, 0), (0, 6)$

$$\text{Area} = \tfrac{1}{2}\begin{vmatrix} -4 & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 6 & 1 \end{vmatrix} = \tfrac{1}{2}(48) = 24 \text{ square units}$$

113. $(-1, 7), (3, -9), (-3, 15)$

$$\begin{vmatrix} -1 & 7 & 1 \\ 3 & -9 & 1 \\ -3 & 15 & 1 \end{vmatrix} = \begin{vmatrix} 3 & -9 \\ -3 & 15 \end{vmatrix} - \begin{vmatrix} -1 & 7 \\ -3 & 15 \end{vmatrix} + \begin{vmatrix} -1 & 7 \\ 3 & -9 \end{vmatrix}$$

$$= 18 - 6 - 12 = 0$$

The points are collinear.

114. $(0, -5), (-2, -6), (8, -1)$

$$\begin{vmatrix} 0 & -5 & 1 \\ -2 & -6 & 1 \\ 8 & -1 & 1 \end{vmatrix} = \begin{vmatrix} -2 & -6 \\ 8 & -1 \end{vmatrix} - \begin{vmatrix} 0 & -5 \\ 8 & -1 \end{vmatrix} + \begin{vmatrix} 0 & -5 \\ -2 & -6 \end{vmatrix}$$

$$= 50 - 40 - 10 = 0$$

The points are collinear.

115. $(-4, 0), (4, 4)$

$$\begin{vmatrix} x & y & 1 \\ -4 & 0 & 1 \\ 4 & 4 & 1 \end{vmatrix} = 0$$

$$1\begin{vmatrix} -4 & 0 \\ 4 & 4 \end{vmatrix} - 1\begin{vmatrix} x & y \\ 4 & 4 \end{vmatrix} + 1\begin{vmatrix} x & y \\ -4 & 0 \end{vmatrix} = 0$$

$$-16 - (4x - 4y) + 4y = 0$$

$$-4x + 8y - 16 = 0$$

$$x - 2y + 4 = 0$$

116. $(2, 5), (6, -1)$

$$\begin{vmatrix} x & y & 1 \\ 2 & 5 & 1 \\ 6 & -1 & 1 \end{vmatrix} = 0$$

$$x\begin{vmatrix} 5 & 1 \\ -1 & 1 \end{vmatrix} - y\begin{vmatrix} 2 & 1 \\ 6 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 5 \\ 6 & -1 \end{vmatrix} = 0$$

$$6x + 4y - 32 = 0$$

$$3x + 2y - 16 = 0$$

117. $\left(-\frac{5}{2}, 3\right), \left(\frac{7}{2}, 1\right)$

$$\begin{vmatrix} x & y & 1 \\ -\frac{5}{2} & 3 & 1 \\ \frac{7}{2} & 1 & 1 \end{vmatrix} = 0$$

$$1\begin{vmatrix} -\frac{5}{2} & 3 \\ \frac{7}{2} & 1 \end{vmatrix} - 1\begin{vmatrix} x & y \\ \frac{7}{2} & 1 \end{vmatrix} + 1\begin{vmatrix} x & y \\ -\frac{5}{2} & 3 \end{vmatrix} = 0$$

$$-13 - \left(x - \tfrac{7}{2}y\right) + \left(3x + \tfrac{5}{2}y\right) = 0$$

$$2x + 6y - 13 = 0$$

118. $(-0.8, 0.2), (0.7, 3.2)$

$$\begin{vmatrix} x & y & 1 \\ -0.8 & 0.2 & 1 \\ 0.7 & 3.2 & 1 \end{vmatrix} = 0$$

$$x\begin{vmatrix} 0.2 & 1 \\ 3.2 & 1 \end{vmatrix} - y\begin{vmatrix} -0.8 & 1 \\ 0.7 & 1 \end{vmatrix} + 1\begin{vmatrix} -0.8 & 0.2 \\ 0.7 & 3.2 \end{vmatrix} = 0$$

$$-3x + 1.5y - 2.7 = 0$$

Multiply both sides by $-\frac{10}{3}$.

$10x - 5y + 9 = 0$

119. The area of the parallelogram with vertices: $(0, 0), (2, 0), (1, 4)$ and $(3, 4) \Rightarrow a = 2, b = 0, c = 1$ and $d = 4$.

$$A = \left\| \begin{matrix} 2 & 0 \\ 1 & 4 \end{matrix} \right\| = |8 - 0| = 8 \text{ square units.}$$

120. The area of the parallelogram with vertices: $(0, 0), (-3, 0), (1, 3)$ and $(-2, 3) \Rightarrow a = -3, b = 0, c = 1$ and $d = 3$.

$$A = \left\| \begin{matrix} -3 & 0 \\ 1 & 3 \end{matrix} \right\| = |-9 - 0| = 9 \text{ square units.}$$

121. $A^{-1} = \begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix}$

$$\begin{bmatrix} -5 & 11 & -2 \end{bmatrix}\begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix} = \begin{bmatrix} 19 & 5 & 5 \end{bmatrix} \quad \text{S E E}$$

$$\begin{bmatrix} 370 & -265 & 225 \end{bmatrix}\begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 25 & 15 \end{bmatrix} \quad \text{\_ Y O}$$

$$\begin{bmatrix} -57 & 48 & -33 \end{bmatrix}\begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix} = \begin{bmatrix} 21 & 0 & 6 \end{bmatrix} \quad \text{U \_ F}$$

$$\begin{bmatrix} 32 & -15 & 20 \end{bmatrix}\begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix} = \begin{bmatrix} 18 & 9 & 4 \end{bmatrix} \quad \text{R I D}$$

$$\begin{bmatrix} 245 & -171 & 147 \end{bmatrix}\begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 25 & 0 \end{bmatrix} \quad \text{A Y \_}$$

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122. $A^{-1} = \begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix}$

$$\begin{bmatrix} 145 & -105 & 92 \\ 264 & -188 & 160 \\ 23 & -16 & 15 \\ 129 & -84 & 78 \\ -9 & 8 & -5 \\ 159 & -118 & 100 \\ 219 & -152 & 133 \\ 370 & -265 & 225 \\ -105 & 84 & -63 \end{bmatrix} \begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix} = \begin{bmatrix} 13 & 1 & 25 \\ 0 & 20 & 8 \\ 5 & 0 & 6 \\ 15 & 18 & 3 \\ 5 & 0 & 2 \\ 5 & 0 & 23 \\ 9 & 20 & 8 \\ 0 & 25 & 15 \\ 21 & 0 & 0 \end{bmatrix} \begin{matrix} \text{M} & \text{A} & \text{Y} \\ \_ & \text{T} & \text{H} \\ \text{E} & \_ & \text{F} \\ \text{O} & \text{R} & \text{C} \\ \text{E} & \_ & \text{B} \\ \text{E} & \_ & \text{W} \\ \text{I} & \text{T} & \text{H} \\ \_ & \text{Y} & \text{O} \\ \text{U} & \_ & \_ \end{matrix}$$

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123. False. The matrix must be square.

124. True. Expand along Row 3.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} + c_1 & a_{32} + c_2 & a_{33} + c_3 \end{vmatrix} = (a_{31} + c_1)\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} - (a_{32} + c_2)\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} + (a_{33} + c_3)\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$= a_{31}\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} - a_{32}\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} + a_{33}\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} + c_1\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} - c_2\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} + c_3\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$= \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Note: Expand each of these matrices along Row 3 to see the previous step.

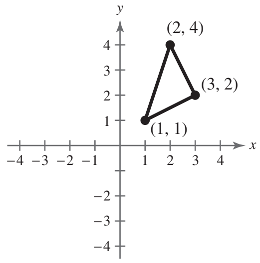
125. If A is a square matrix, the cofactor C_{ij} of the entry a_{ij} is $(-1)^{i+j}M_{ij}$, where M_{ij} is the determinant obtained by deleting the ith row and jth column of A. The determinant of A is the sum of the entries of any row or column of A multiplied by their respective cofactors.

126. The part of the matrix corresponding to the coefficients of the system reduces to a matrix in which the number of rows with nonzero entries is the same as the number of variables.

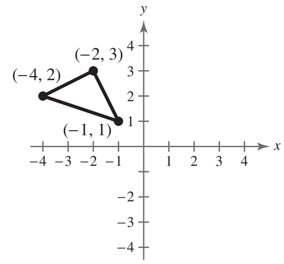
Problem Solving for Chapter 8

1. $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad T = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \end{bmatrix}$

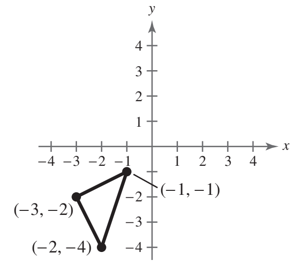
(a) $AT = \begin{bmatrix} -1 & -4 & -2 \\ 1 & 2 & 3 \end{bmatrix} \quad AAT = \begin{bmatrix} -1 & -2 & -3 \\ -1 & -4 & -2 \end{bmatrix}$



Original Triangle



AT Triangle



AAT Triangle

The transformation *A* interchanges the *x* and *y* coordinates and then takes the negative of the *x* coordinate. *A* represents a counterclockwise rotation by $90°$.

(b) *AAT* is rotated clockwise $90°$ to obtain *AT*. *AT* is then rotated clockwise $90°$ to obtain *T*.

2. (a) 2011

$$\left(\frac{1}{311{,}721{,}632}\right)\begin{bmatrix} 42{,}376{,}825 & 92{,}983{,}542 & 17{,}934{,}267 \\ 40{,}463{,}751 & 94{,}530{,}885 & 23{,}432{,}361 \end{bmatrix} \times 100\% = \begin{array}{c} \begin{matrix} \text{0-19} & \text{20-64} & \text{65 +} \end{matrix} \\ \begin{bmatrix} 13.59\% & 29.83\% & 5.75\% \\ 12.98\% & 30.33\% & 7.52\% \end{bmatrix} \end{array} \begin{matrix} \text{Male} \\ \text{Female} \end{matrix}$$

2014

$$\left(\frac{1}{318{,}857{,}919}\right)\begin{bmatrix} 41{,}969{,}399 & 94{,}615{,}796 & 20{,}351{,}292 \\ 40{,}166{,}203 & 95{,}862{,}447 & 25{,}891{,}919 \end{bmatrix} \times 100\% = \begin{array}{c} \begin{matrix} \text{0-19} & \text{20-64} & \text{65 +} \end{matrix} \\ \begin{bmatrix} 13.16\% & 29.67\% & 6.38\% \\ 12.60\% & 30.06\% & 8.12\% \end{bmatrix} \end{array} \begin{matrix} \text{Male} \\ \text{Female} \end{matrix}$$

(b) $\begin{bmatrix} 13.16 - 13.59 & 29.67 - 29.83 & 6.38 - 5.75 \\ 12.60 - 12.98 & 30.06 - 30.33 & 8.12 - 7.52 \end{bmatrix} = \begin{bmatrix} -0.43\% & -0.16\% & 0.63\% \\ -0.38\% & -0.27\% & 0.60\% \end{bmatrix} \begin{matrix} \text{Male} \\ \text{Female} \end{matrix}$

(c) Both male and female populations had percents that decreased for 0-19 and 20-64 age groups.

3. (a) $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = A$

A is idempotent.

(b) $A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \neq A$

A is *not* idempotent.

(c) $A^2 = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \neq A$

A is *not* idempotent.

(d) $A^2 = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} \neq A$

A is *not* idempotent.

(e) $A^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \neq A$

A is *not* idempotent.

(f) $A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \neq A$

A is *not* idempotent.

4. From Exercise 3, we have the singular matrix

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \text{ where } A^2 = A.$$

Also, $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ has this property.

5. $A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$

(a) $$A^2 - 2A + 5I = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} + 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} -3 & 4 \\ -4 & -3 \end{bmatrix} + \begin{bmatrix} -2 & -4 \\ 4 & -2 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

(b) $$A^{-1} = \frac{1}{(1) - (-4)}\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$$
$$\frac{1}{5}(2I - A) = \frac{1}{5}\left[\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\right] = \frac{1}{5}\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$$

So, $A^{-1} = \frac{1}{5}(2I - A)$.

(c) $$A^2 - 2A + 5I = 0$$
$$A^2 - 2A = -5I$$
$$(A - 2I)A = -5I$$
$$-\frac{1}{5}(A - 2I)A = I$$
$$\frac{1}{5}(2I - A)A = I$$

So, $A^{-1} = \frac{1}{5}(2I - A)$.

6. (a) $$\begin{bmatrix} 0.70 & 0.15 & 0.15 \\ 0.20 & 0.80 & 0.15 \\ 0.10 & 0.05 & 0.70 \end{bmatrix}\begin{bmatrix} 25{,}000 \\ 30{,}000 \\ 45{,}000 \end{bmatrix} = \begin{bmatrix} 28{,}750 \\ 35{,}750 \\ 35{,}500 \end{bmatrix}$$

Gold Satellite System: 28,750 subscribers
Galaxy Satellite Network: 35,750 subscribers
Nonsubscribers: 35,500
Answers will vary.

(b) $$\begin{bmatrix} 0.70 & 0.15 & 0.15 \\ 0.20 & 0.80 & 0.15 \\ 0.10 & 0.05 & 0.70 \end{bmatrix}\begin{bmatrix} 28{,}750 \\ 35{,}750 \\ 35{,}500 \end{bmatrix} \approx \begin{bmatrix} 30{,}813 \\ 39{,}675 \\ 29{,}513 \end{bmatrix}$$

Gold Satellite System: 30,813 subscribers
Galaxy Satellite Network: 39,675 subscribers
Nonsubscribers: 29,513
Answers will vary.

(c) $$\begin{bmatrix} 0.70 & 0.15 & 0.15 \\ 0.20 & 0.80 & 0.15 \\ 0.10 & 0.05 & 0.70 \end{bmatrix}\begin{bmatrix} 30{,}812.5 \\ 39{,}675 \\ 29{,}512.5 \end{bmatrix} \approx \begin{bmatrix} 31{,}947 \\ 42{,}329 \\ 25{,}724 \end{bmatrix}$$

Gold Satellite System: 31,947 subscribers
Galaxy Satellite Network: 42,329 subscribers
Nonsubscribers: 25,724
Answers will vary.

(d) Both satellite companies are increasing the number of subscribers, while the number of nonsubscribers is decreasing each year.

7. $A = \begin{bmatrix} -1 & 1 & -2 \\ 2 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 0 \\ 1 & 2 \\ 1 & -1 \end{bmatrix}$

$A^T = \begin{bmatrix} -1 & 2 \\ 1 & 0 \\ -2 & 1 \end{bmatrix}$, $B^T = \begin{bmatrix} -3 & 1 & 1 \\ 0 & 2 & -1 \end{bmatrix}$

$AB = \begin{bmatrix} 2 & 4 \\ -5 & -1 \end{bmatrix}$, $(AB)^T = \begin{bmatrix} 2 & -5 \\ 4 & -1 \end{bmatrix}$

$$B^T A^T = \begin{bmatrix} -3 & 1 & 1 \\ 0 & 2 & -1 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 1 & 0 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -5 \\ 4 & -1 \end{bmatrix}$$

So, $(AB)^T = B^T A^T$.

8. $A = \begin{bmatrix} 3 & x \\ -2 & -3 \end{bmatrix} \Rightarrow A^{-1} = \dfrac{1}{-9 + 2x}\begin{bmatrix} -3 & -x \\ 2 & 3 \end{bmatrix}$

If $A = A^{-1}$, then $\begin{bmatrix} \dfrac{-3}{-9 + 2x} & \dfrac{-x}{-9 + 2x} \\ \dfrac{2}{-9 + 2x} & \dfrac{3}{-9 + 2x} \end{bmatrix} = \begin{bmatrix} 3 & x \\ -2 & -3 \end{bmatrix}$.

Equating the first entry in Row 1 yields

$\dfrac{-3}{-9 + 2x} = 3 \Rightarrow -3 = -27 + 6x \Rightarrow x = 4.$

Now check $x = 4$ in the other entries:

$\dfrac{-4}{-9 + 2(4)} = 4$ ✓

$\dfrac{2}{-9 + 2(4)} = -2$ ✓

$\dfrac{3}{-9 + 2(4)} = -3$ ✓

So, $x = 4$.

9. If $A = \begin{bmatrix} 4 & x \\ -2 & -3 \end{bmatrix}$ is singular then

$ad - bc = -12 + 2x = 0.$

So, $x = 6$.

10. $(a - b)(b - c)(c - a) = -a^2b + a^2c + ab^2 - ac^2 - b^2c + bc^2$

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = \begin{vmatrix} b & c \\ b^2 & c^2 \end{vmatrix} - \begin{vmatrix} a & c \\ a^2 & c^2 \end{vmatrix} + \begin{vmatrix} a & b \\ a^2 & b^2 \end{vmatrix} = bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b$$

So, $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a - b)(b - c)(c - a).$

11. $(a - b)(b - c)(c - a)(a + b + c) = -a^3b + a^3c + ab^3 - ac^3 - b^3c + bc^3$

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = \begin{vmatrix} b & c \\ b^3 & c^3 \end{vmatrix} - \begin{vmatrix} a & c \\ a^3 & c^3 \end{vmatrix} + \begin{vmatrix} a & b \\ a^3 & b^3 \end{vmatrix} = bc^3 - b^3c - ac^3 + a^3c + ab^3 - a^3b$$

So, $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a - b)(b - c)(c - a)(a + b + c).$

12. $$\begin{vmatrix} x & 0 & c \\ -1 & x & b \\ 0 & -1 & a \end{vmatrix} = x\begin{vmatrix} x & b \\ -1 & a \end{vmatrix} + c\begin{vmatrix} -1 & x \\ 0 & -1 \end{vmatrix} = x(ax + b) + c(1 - 0) = ax^2 + bx + c$$

13. $$\begin{vmatrix} x & 0 & 0 & d \\ -1 & x & 0 & c \\ 0 & -1 & x & b \\ 0 & 0 & -1 & a \end{vmatrix} = x\begin{vmatrix} x & 0 & c \\ -1 & x & b \\ 0 & -1 & a \end{vmatrix} - d\begin{vmatrix} -1 & x & 0 \\ 0 & -1 & x \\ 0 & 0 & -1 \end{vmatrix} = x\underbrace{(ax^2 + bx + c)}_{\text{From Exercise 12}} - d\left(-\begin{vmatrix} -1 & x \\ 0 & -1 \end{vmatrix}\right) = ax^3 + bx^2 + cx + d$$

14. Let $A = \begin{bmatrix} 3 & -3 \\ 5 & -5 \end{bmatrix}$, then $|A| = 0$.

Let $A = \begin{bmatrix} 2 & 4 & -6 \\ -3 & 1 & 2 \\ 5 & -8 & 3 \end{bmatrix}$, then $|A| = 0$.

Let $A = \begin{bmatrix} 3 & -7 & 5 & -1 \\ -6 & 4 & 0 & 2 \\ 5 & 8 & -6 & -7 \\ 9 & 11 & -4 & -16 \end{bmatrix}$, then $|A| = 0$.

Conjecture: If A is an $n \times n$ matrix, each of whose rows add up to zero, then $|A| = 0$.

15.
$$\begin{aligned} 4S + 4N &= 184 \\ S + 6F &= 146 \\ 2N + 4F &= 104 \end{aligned}$$

$$D = \begin{vmatrix} 4 & 4 & 0 \\ 1 & 0 & 6 \\ 0 & 2 & 4 \end{vmatrix} = -64$$

$$S = \frac{\begin{vmatrix} 184 & 4 & 0 \\ 146 & 0 & 6 \\ 104 & 2 & 4 \end{vmatrix}}{-64} = \frac{-2048}{-64} = 32 \qquad N = \frac{\begin{vmatrix} 4 & 184 & 0 \\ 1 & 146 & 6 \\ 0 & 104 & 4 \end{vmatrix}}{-64} = \frac{-896}{-64} = 14 \qquad F = \frac{\begin{vmatrix} 4 & 4 & 184 \\ 1 & 0 & 146 \\ 0 & 2 & 104 \end{vmatrix}}{-64} = \frac{-1216}{-64} = 19$$

Element	*Atomic mass*
Sulfur	32
Nitrogen	14
Fluoride	19

16. Let x = cost of a transformer, y = cost per foot of wire, and z = cost of a light.

$$\begin{aligned} x + 25y + 5z &= 20 \\ x + 50y + 15z &= 35 \\ x + 100y + 20z &= 50 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 25 & 5 & 20 \\ 1 & 50 & 15 & 35 \\ 1 & 100 & 20 & 50 \end{array}\right] \underset{\rightarrow}{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 0.2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

By using the matrix capabilities of a graphing calculator to reduce the augmented matrix to reduced row-echelon form,
we have the following costs:

Transformer	\$10.00
Foot of wire	\$ 0.20
Light	\$ 1.00

17. $A = \begin{bmatrix} 1 & -2 & 2 \\ 1 & 1 & -3 \\ 1 & -1 & 4 \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} \frac{1}{11} & \frac{6}{11} & \frac{4}{11} \\ -\frac{7}{11} & \frac{2}{11} & \frac{5}{11} \\ -\frac{2}{11} & -\frac{1}{11} & \frac{3}{11} \end{bmatrix}$

$$\begin{bmatrix} 23 & 13 & -34 \\ 31 & -34 & 63 \\ 25 & -17 & 61 \\ 24 & 14 & -37 \\ 41 & -17 & -8 \\ 20 & -29 & 40 \\ 38 & -56 & 116 \\ 13 & -11 & 1 \\ 22 & -3 & -6 \\ 41 & -53 & 85 \\ 28 & -32 & 16 \end{bmatrix} \begin{bmatrix} \frac{1}{11} & \frac{6}{11} & \frac{4}{11} \\ -\frac{7}{11} & \frac{2}{11} & \frac{5}{11} \\ -\frac{2}{11} & -\frac{1}{11} & \frac{3}{11} \end{bmatrix} = \begin{bmatrix} 0 & 18 & 5 \\ 13 & 5 & 13 \\ 2 & 5 & 18 \\ 0 & 19 & 5 \\ 16 & 20 & 5 \\ 13 & 2 & 5 \\ 18 & 0 & 20 \\ 8 & 5 & 0 \\ 5 & 12 & 5 \\ 22 & 5 & 14 \\ 20 & 8 & 0 \end{bmatrix}$$

0 18 5 13 5 13 2 5 18 0
_ R E M E M B E R _

19 5 16 20 5 13 2 5 18 0
S E P T E M B E R _

20 8 5 0 5 12 5 22 5 14 20 8 0
T H E _ E L E V E N T H _

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18. (a) $\begin{bmatrix} 45 & -35 \end{bmatrix} \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} 10 & 15 \end{bmatrix}$

$\begin{bmatrix} 38 & -30 \end{bmatrix} \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} 8 & 14 \end{bmatrix}$

$45w - 35y = 10$

$45x - 35z = 15$

$38w - 30y = 8$

$38x - 30z = 14$

$\left. \begin{aligned} 45w - 35y &= 10 \\ 38w - 30y &= 8 \end{aligned} \right\} \Rightarrow w = 1, y = 1$

$\left. \begin{aligned} 45x - 35z &= 15 \\ 38x - 30z &= 14 \end{aligned} \right\} \Rightarrow x = -2, z = -3$

$A^{-1} = \begin{bmatrix} 1 & -2 \\ 1 & -3 \end{bmatrix}$

(b) $$\begin{bmatrix} 45 & -35 \\ 38 & -30 \\ 18 & -18 \\ 35 & -30 \\ 81 & -60 \\ 42 & -28 \\ 75 & -55 \\ 2 & -2 \\ 22 & -21 \\ 15 & -10 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 10 & 15 \\ 8 & 14 \\ 0 & 18 \\ 5 & 20 \\ 21 & 18 \\ 14 & 0 \\ 20 & 15 \\ 0 & 2 \\ 1 & 19 \\ 5 & 0 \end{bmatrix} \begin{matrix} J & O \\ H & N \\ \_ & R \\ E & T \\ U & R \\ N & \_ \\ T & O \\ \_ & B \\ A & S \\ E & \_ \end{matrix}$$

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19. $A = \begin{bmatrix} 6 & 4 & 1 \\ 0 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} \frac{1}{16} & -\frac{7}{16} & \frac{5}{8} \\ \frac{3}{16} & \frac{11}{16} & -\frac{9}{8} \\ -\frac{1}{8} & -\frac{1}{8} & \frac{3}{4} \end{bmatrix}$

$|A| = 16$ and $|A^{-1}| = \frac{1}{16}$

Conjecture: $|A^{-1}| = \frac{1}{|A|}$

20. (a) Answers will vary.

$A = \begin{bmatrix} 0 & 3 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 4 & -1 \\ 0 & 0 & 7 \\ 0 & 0 & 0 \end{bmatrix}$

(b) $A^2 = 0$, so $A^n = 0$ for an integer n where $n \geq 2$.

$B^2 = \begin{bmatrix} 0 & 0 & 28 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$B^3 = 0$, so $B^n = 0$ for an integer n where $n \geq 3$.

(c) $A^4 = 0$ if A is 4×4.

(d) Conjecture: If A is $n \times n$, then $A^n = 0$.

Practice Test for Chapter 8

1. Put the matrix in reduced row-echelon form.

$$\begin{bmatrix} 1 & -2 & 4 \\ 3 & -5 & 9 \end{bmatrix}$$

For Exercises 2–4, use matrices to solve the system of equations.

2. $\begin{cases} 3x + 5y = 3 \\ 2x - y = -11 \end{cases}$

3. $\begin{cases} 2x + 3y = -3 \\ 3x + 2y = 8 \\ x + y = 1 \end{cases}$

4. $\begin{cases} x + 3z = -5 \\ 2x + y = 0 \\ 3x + y - z = 3 \end{cases}$

5. Multiply $\begin{bmatrix} 1 & 4 & 5 \\ 2 & 0 & -3 \end{bmatrix}\begin{bmatrix} 1 & 6 \\ 0 & -7 \\ -1 & 2 \end{bmatrix}$.

6. Given $A = \begin{bmatrix} 9 & 1 \\ -4 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & -2 \\ 3 & 5 \end{bmatrix}$, find $3A - 5B$.

7. Find $f(A)$.

$f(x) = x^2 - 7x + 8, A = \begin{bmatrix} 3 & 0 \\ 7 & 1 \end{bmatrix}$

8. True or false:

$(A + B)(A + 3B) = A^2 + 4AB + 3B^2$ where A and B are matrices.

(Assume that A^2, AB, and B^2 exist.)

For Exercises 9–10, find the inverse of the matrix, if it exists.

9. $\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$

10. $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 6 & 5 \\ 6 & 10 & 8 \end{bmatrix}$

11. Use an inverse matrix to solve the systems.

(a) $\begin{cases} x + 2y = 4 \\ 3x + 5y = 1 \end{cases}$

(b) $\begin{cases} x + 2y = 3 \\ 3x + 5y = -2 \end{cases}$

For Exercises 12–14, find the determinant of the matrix.

12. $\begin{bmatrix} 6 & -1 \\ 3 & 4 \end{bmatrix}$

13. $\begin{bmatrix} 1 & 3 & -1 \\ 5 & 9 & 0 \\ 6 & 2 & -5 \end{bmatrix}$

14. $\begin{bmatrix} 1 & 4 & 2 & 3 \\ 0 & 1 & -2 & 0 \\ 3 & 5 & -1 & 1 \\ 2 & 0 & 6 & 1 \end{bmatrix}$

15. Evaluate $\begin{vmatrix} 6 & 4 & 3 & 0 & 6 \\ 0 & 5 & 1 & 4 & 8 \\ 0 & 0 & 2 & 7 & 3 \\ 0 & 0 & 0 & 9 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix}$.

16. Use a determinant to find the area of the triangle with vertices $(0, 7), (5, 0),$ and $(3, 9)$.

17. Use a determinant to find the equation of the line passing through $(2, 7)$ and $(-1, 4)$.

For Exercises 18–20, use Cramer's Rule to find the indicated value.

18. Find x.

$$\begin{cases} 6x - 7y = 4 \\ 2x + 5y = 11 \end{cases}$$

19. Find z.

$$\begin{cases} 3x + z = 1 \\ y + 4z = 3 \\ x - y = 2 \end{cases}$$

20. Find y.

$$\begin{cases} 721.4x - 29.1y = 33.77 \\ 45.9x + 105.6y = 19.85 \end{cases}$$