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Analytic Trigonometry

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CHAPTER 5
Analytic Trigonometry

Section 5.1 Using Fundamental Identities

1. $\tan u$

2. $\csc u$

3. $\cot u$

4. $\csc u$

5. 1

6. $-\sin u$

7. $\sec x = -\frac{5}{2}$, $\tan x < 0 \Rightarrow x$ is in Quadrant II.

$$\cos x = \frac{1}{\sec x} = \frac{1}{-\frac{5}{2}} = -\frac{2}{5}$$

$$\sin x = \sqrt{1 - \left(-\frac{2}{5}\right)^2} = \sqrt{1 - \frac{4}{25}} = \frac{\sqrt{21}}{5}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\frac{\sqrt{21}}{5}}{-\frac{2}{5}} = -\frac{\sqrt{21}}{2}$$

$$\csc x = \frac{1}{\sin x} = \frac{5}{\sqrt{21}} = \frac{5\sqrt{21}}{21}$$

$$\cot x = \frac{1}{\tan x} = -\frac{2}{\sqrt{21}} = -\frac{2\sqrt{21}}{21}$$

8. $\csc x = -\frac{7}{6}$, $\tan x > 0 \Rightarrow x$ is in Quadrant III.

$$\sin x = \frac{1}{\csc x} = \frac{1}{-\frac{7}{6}} = -\frac{6}{7}$$

$$\cos x = -\sqrt{1 - \left(-\frac{6}{7}\right)^2} = -\sqrt{1 - \frac{36}{49}} = -\frac{\sqrt{13}}{7}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{-\frac{6}{7}}{-\frac{\sqrt{13}}{7}} = \frac{6}{\sqrt{13}} = \frac{6\sqrt{13}}{13}$$

$$\sec x = \frac{1}{\cos x} = \frac{1}{-\frac{\sqrt{13}}{7}} = -\frac{7}{\sqrt{13}} = -\frac{7\sqrt{13}}{13}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\frac{6}{\sqrt{13}}} = \frac{\sqrt{13}}{6}$$

9. $\sin\theta = -\frac{3}{4}$, $\cos\theta > 0 \Rightarrow \theta$ is in Quadrant IV.

$$\cos\theta = \sqrt{1 - \left(-\frac{3}{4}\right)^2} = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\frac{3}{4}}{\frac{\sqrt{7}}{4}} = -\frac{3}{\sqrt{7}} = -\frac{3\sqrt{7}}{7}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{\sqrt{7}}{4}} = \frac{4}{\sqrt{7}} = \frac{4\sqrt{7}}{7}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-\frac{3}{\sqrt{7}}} = -\frac{\sqrt{7}}{3}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{-\frac{3}{4}} = -\frac{4}{3}$$

10. $\cos\theta = \frac{2}{3}$, $\sin\theta < 0 \Rightarrow \theta$ is in Quadrant IV.

$$\sin\theta = -\sqrt{1 - \left(\frac{2}{3}\right)^2} = -\sqrt{1 - \frac{4}{9}} = -\frac{\sqrt{5}}{3}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\frac{\sqrt{5}}{2}}{\frac{2}{3}} = -\frac{\sqrt{5}}{2}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-\frac{\sqrt{5}}{2}} = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{-\frac{\sqrt{5}}{3}} = -\frac{3}{\sqrt{5}} = -\frac{3\sqrt{5}}{5}$$

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11. $\tan x = \dfrac{2}{3}, \cos x > 0 \Rightarrow x$ is in Quadrant I.

$$\cot x = \frac{1}{\tan x} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$$

$$\sec x = \sqrt{1 + \left(\frac{2}{3}\right)^2} = \sqrt{1 + \frac{4}{9}} = \frac{\sqrt{13}}{3}$$

$$\csc x = \sqrt{1 + \left(\frac{3}{2}\right)^2} = \sqrt{1 + \frac{9}{4}} = \frac{\sqrt{13}}{2}$$

$$\sin x = \frac{1}{\csc x} = \frac{1}{\frac{\sqrt{13}}{2}} = \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\frac{\sqrt{13}}{3}} = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$$

12. $\cot x = \dfrac{7}{4}, \sin x < 0 \Rightarrow x$ is in Quadrant III.

$$\tan x = \frac{1}{\cot x} = \frac{1}{\frac{7}{4}} = \frac{4}{7}$$

$$\sec x = -\sqrt{1 + \left(\frac{4}{7}\right)^2} = -\sqrt{1 + \frac{16}{49}} = -\frac{\sqrt{65}}{7}$$

$$\csc x = -\sqrt{1 + \left(\frac{7}{4}\right)^2} = -\sqrt{1 + \frac{49}{16}} = -\frac{\sqrt{65}}{4}$$

$$\sin x = \frac{1}{\csc x} = \frac{1}{-\frac{\sqrt{65}}{4}} = -\frac{4}{\sqrt{65}} = -\frac{4\sqrt{65}}{65}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{-\frac{\sqrt{65}}{7}} = -\frac{7}{\sqrt{65}} = -\frac{7\sqrt{65}}{65}$$

13. $\sec x \cos x = \left(\dfrac{1}{\cancel{\cos x}}\right)\cancel{\cos x}$

$= 1$

Matches (c).

14. $\cot^2 x - \csc^2 x = (\csc^2 x - 1) - \csc^2 x$

$= -1$

Matches (b).

15. $\cos x(1 + \tan^2 x) = \cos x(\sec^2 x)$

$= \cos x\left(\dfrac{1}{\cos^2 x}\right)$

$= \dfrac{1}{\cos x}$

$= \sec x$

Matches (f).

16. $\cot x \sec x = \dfrac{\cos x}{\sin x} \cdot \dfrac{1}{\cos x} = \dfrac{1}{\sin x} = \csc x$

Matches (a).

17. $\dfrac{\sec^2 x - 1}{\sin^2 x} = \dfrac{\tan^2 x}{\sin^2 x} = \dfrac{\sin^2 x}{\cos^2 x} \cdot \dfrac{1}{\sin^2 x} = \sec^2 x$

Matches (e).

18. $\dfrac{\cos^2[(\pi/2) - x]}{\cos x} = \dfrac{\sin^2 x}{\cos x} = \dfrac{\sin x}{\cos x}\sin x = \tan x \sin x$

Matches (d).

19. $\dfrac{\tan\theta \cot\theta}{\sec\theta} = \dfrac{\tan\theta\left(\frac{1}{\tan\theta}\right)}{\frac{1}{\cos\theta}}$

$= \dfrac{1}{\frac{1}{\cos\theta}}$

$= \cos\theta$

20. $\cos\left(\dfrac{\pi}{2} - x\right)\sec x = \sin x \sec x$

$= \sin x\left(\dfrac{1}{\cos x}\right)$

$= \tan x$

21. $\tan^2 x - \tan^2 x \sin^2 x = \tan^2 x(1 - \sin^2 x)$

$= \tan^2 x \cos^2 x$

$= \dfrac{\sin^2 x}{\cos^2 x} \cdot \cos^2 x$

$= \sin^2 x$

22. $\sin^2 x \sec^2 x - \sin^2 x = \sin^2 x(\sec^2 x - 1)$

$= \sin^2 x \tan^2 x$

23. $\dfrac{\sec^2 x - 1}{\sec x - 1} = \dfrac{(\sec x + 1)(\sec x - 1)}{\sec x - 1}$

$= \sec x + 1$

24. $\dfrac{\cos x - 2}{\cos^2 x - 4} = \dfrac{\cos x - 2}{(\cos x + 2)(\cos x - 2)}$

$= \dfrac{1}{\cos x + 2}$

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25. $1 - 2\cos^2 x + \cos^4 x = (1 - \cos^2 x)^2$
$= (\sin^2 x)^2$
$= \sin^4 x$

26. $\sec^4 x - \tan^4 x = (\sec^2 x + \tan^2 x)(\sec^2 x - \tan^2 x)$
$= (\sec^2 x + \tan^2 x)(1)$
$= \sec^2 x + \tan^2 x$

27. $\cot^3 x + \cot^2 x + \cot x + 1 = \cot^2 x(\cot x + 1) + (\cot x + 1)$
$= (\cot x + 1)(\cot^2 x + 1)$
$= (\cot x + 1)\csc^2 x$

28. $\sec^3 x - \sec^2 x - \sec x + 1 = \sec^2 x(\sec x - 1) - (\sec x - 1)$
$= (\sec^2 x - 1)(\sec x - 1)$
$= \tan^2 x(\sec x - 1)$

29. $3\sin^2 x - 5\sin x - 2 = (3\sin x + 1)(\sin x - 2)$

30. $6\cos^2 x + 5\cos x - 6 = (3\cos x - 2)(2\cos x + 3)$

31. $\cot^2 x + \csc x - 1 = (\csc^2 x - 1) + \csc x - 1$
$= \csc^2 x + \csc x - 2$
$= (\csc x - 1)(\csc x + 2)$

32. $\sin^2 x + 3\cos x + 3 = (1 - \cos^2 x) + 3\cos x + 3$
$= -\cos^2 x + 3\cos x + 4$
$= -(\cos^2 x - 3\cos x - 4)$
$= -(\cos x + 1)(\cos x - 4)$

33. $\tan\theta\csc\theta = \dfrac{\sin\theta}{\cos\theta}\cdot\dfrac{1}{\sin\theta} = \dfrac{1}{\cos\theta} = \sec\theta$

34. $\tan(-x)\cos x = -\tan x\cos x$
$= -\dfrac{\sin x}{\cos x}\cdot\cos x$
$= -\sin x$

35. $\sin\phi(\csc\phi - \sin\phi) = (\sin\phi)\dfrac{1}{\sin\phi} - \sin^2\phi$
$= 1 - \sin^2\phi = \cos^2\phi$

36. $\cos x(\sec x - \cos x) = \cos x\left(\dfrac{1}{\cos x} - \cos x\right)$
$= 1 - \cos^2 x$
$= \sin^2 x$

37. $\sin\beta\tan\beta + \cos\beta = (\sin\beta)\dfrac{\sin\beta}{\cos\beta} + \cos\beta$
$= \dfrac{\sin^2\beta}{\cos\beta} + \dfrac{\cos^2\beta}{\cos\beta}$
$= \dfrac{\sin^2\beta + \cos^2\beta}{\cos\beta}$
$= \dfrac{1}{\cos\beta}$
$= \sec\beta$

38. $\cot u\sin u + \tan u\cos u = \dfrac{\cos u}{\sin u}(\sin u) + \dfrac{\sin u}{\cos u}(\cos u)$
$= \cos u + \sin u$

39. $\dfrac{1 - \sin^2 x}{\csc^2 x - 1} = \dfrac{\cos^2 x}{\cot^2 x} = \cos^2 x\tan^2 x = (\cos^2 x)\dfrac{\sin^2 x}{\cos^2 x}$
$= \sin^2 x$

40. $\dfrac{\cos^2 y}{1 - \sin y} = \dfrac{1 - \sin^2 y}{1 - \sin y}$
$= \dfrac{(1 + \sin y)(1 - \sin y)}{1 - \sin y} = 1 + \sin y$

41. $(\sin x + \cos x)^2 = \sin^2 x + 2\sin x\cos x + \cos^2 x$
$= (\sin^2 x + \cos^2 x) + 2\sin x\cos x$
$= 1 + 2\sin x\cos x$

42. $(2\csc x + 2)(2\csc x - 2) = 4\csc^2 x - 4$
$= 4(\csc^2 x - 1)$
$= 4\cot^2 x$

43. $\dfrac{1}{1 + \cos x} + \dfrac{1}{1 - \cos x} = \dfrac{1 - \cos x + 1 + \cos x}{(1 + \cos x)(1 - \cos x)}$
$= \dfrac{2}{1 - \cos^2 x}$
$= \dfrac{2}{\sin^2 x}$
$= 2\csc^2 x$

44. $\dfrac{1}{\sec x + 1} - \dfrac{1}{\sec x - 1} = \dfrac{\sec x - 1 - (\sec x + 1)}{(\sec x + 1)(\sec x - 1)}$
$= \dfrac{\sec x - 1 - \sec x - 1}{\sec^2 x - 1}$
$= \dfrac{-2}{\tan^2 x}$
$= -2\left(\dfrac{1}{\tan^2 x}\right)$
$= -2\cot^2 x$

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45. $\dfrac{\cos x}{1+\sin x} - \dfrac{\cos x}{1-\sin x} = \dfrac{\cos x(1-\sin x) - \cos x(1+\sin x)}{(1+\sin x)(1-\sin x)}$

$= \dfrac{\cos x - \sin x\cos x - \cos x - \sin x\cos x}{(1+\sin x)(1-\sin x)}$

$= \dfrac{-2\sin x\cos x}{1-\sin^2 x}$

$= \dfrac{-2\sin x\cos x}{\cos^2 x}$

$= \dfrac{-2\sin x}{\cos x}$

$= -2\tan x$

46. $\dfrac{\sin x}{1+\cos x} + \dfrac{\sin x}{1-\cos x} = \dfrac{\sin x(1-\cos x) + \sin x(1+\cos x)}{(1+\cos x)(1-\cos x)}$

$= \dfrac{\sin x - \sin x\cos x + \sin x + \sin x\cos x}{(1+\cos x)(1-\cos x)}$

$= \dfrac{2\sin x}{1-\cos^2 x}$

$= \dfrac{2\sin x}{\sin^2 x}$

$= \dfrac{2}{\sin x}$

$= 2\csc x$

47. $\tan x - \dfrac{\sec^2 x}{\tan x} = \dfrac{\tan^2 x - \sec^2 x}{\tan x}$

$= \dfrac{-1}{\tan x} = -\cot x$

48. $\dfrac{\cos x}{1+\sin x} + \dfrac{1+\sin x}{\cos x} = \dfrac{\cos^2 x + (1+\sin x)^2}{\cos x(1+\sin x)}$

$= \dfrac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{\cos x(1+\sin x)}$

$= \dfrac{2+2\sin x}{\cos x(1+\sin x)}$

$= \dfrac{2(1+\sin x)}{\cos x(1+\sin x)}$

$= \dfrac{2}{\cos x}$

$= 2\sec x$

49. $\dfrac{\sin^2 y}{1-\cos y} = \dfrac{1-\cos^2 y}{1-\cos y}$

$= \dfrac{(1+\cos y)(1-\cos y)}{1-\cos y} = 1+\cos y$

50. $\dfrac{5}{\tan x + \sec x} \cdot \dfrac{\tan x - \sec x}{\tan x - \sec x} = \dfrac{5(\tan x - \sec x)}{\tan^2 x - \sec^2 x}$

$= \dfrac{5(\tan x - \sec x)}{-1}$

$= 5(\sec x - \tan x)$

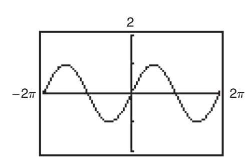
51. $y_1 = \dfrac{\tan x + 1}{\sec x + \csc x}$

$= \dfrac{\dfrac{\sin x}{\cos x} + 1}{\dfrac{1}{\cos x} + \dfrac{1}{\sin x}}$

$= \dfrac{\dfrac{\sin x + \cos x}{\cos x}}{\dfrac{\sin x + \cos x}{\sin x\cos x}}$

$= \left(\dfrac{\sin x + \cos x}{\cos x}\right)\left(\dfrac{\sin x\cos x}{\sin x + \cos x}\right)$

$= \sin x$



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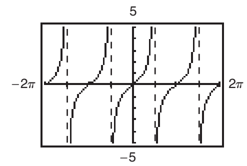
52. $y_1 = \dfrac{1}{\sin x}\left(\dfrac{1}{\cos x} - \cos x\right) = \tan x$



$$\frac{1}{\sin x}\left(\frac{1}{\cos x} - \cos x\right) = \frac{1}{\sin x \cos x} - \frac{\cos x}{\sin x}$$
$$= \frac{1 - \cos^2 x}{\sin x \cos x} = \frac{\sin^2 x}{\sin x \cos x} = \frac{\sin x}{\cos x} = \tan x$$

53. Let $x = 3\cos\theta$.

$$\begin{aligned}\sqrt{9 - x^2} &= \sqrt{9 - (3\cos\theta)^2}\\ &= \sqrt{9 - 9\cos^2\theta}\\ &= \sqrt{9(1 - \cos^2\theta)}\\ &= \sqrt{9\sin^2\theta} = 3\sin\theta\end{aligned}$$

54. Let $x = 7\sin\theta$.

$$\begin{aligned}\sqrt{49 - x^2} &= \sqrt{49 - (7\sin\theta)^2}\\ &= \sqrt{49 - 49\sin^2\theta}\\ &= \sqrt{49(1 - \sin^2\theta)}\\ &= \sqrt{49\cos^2\theta}\\ &= 7\cos\theta\end{aligned}$$

55. Let $x = 2\sec\theta$.

$$\begin{aligned}\sqrt{x^2 - 4} &= \sqrt{(2\sec\theta)^2 - 4}\\ &= \sqrt{4(\sec^2\theta - 1)}\\ &= \sqrt{4\tan^2\theta}\\ &= 2\tan\theta\end{aligned}$$

56. Let $3x = 5\tan\theta$.

$$\begin{aligned}\sqrt{9x^2 + 25} &= \sqrt{(3x)^2 + 25}\\ &= \sqrt{(5\tan\theta)^2 + 25}\\ &= \sqrt{25\tan^2\theta + 25}\\ &= \sqrt{25(\tan^2\theta + 1)}\\ &= \sqrt{25\sec^2\theta}\\ &= 5\sec\theta\end{aligned}$$

57. Let $x = 2\sin\theta$.

$$\begin{aligned}\sqrt{4 - x^2} &= \sqrt{2}\\ \sqrt{4 - (2\sin\theta)^2} &= \sqrt{2}\\ \sqrt{4 - 4\sin^2\theta} &= \sqrt{2}\\ \sqrt{4(1 - \sin^2\theta)} &= \sqrt{2}\\ \sqrt{4\cos^2\theta} &= \sqrt{2}\\ 2\cos\theta &= \sqrt{2}\\ \cos\theta &= \frac{\sqrt{2}}{2}\end{aligned}$$

$$\sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - \left(\frac{\sqrt{2}}{2}\right)^2} = \pm\frac{\sqrt{2}}{2}$$

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58. $x = 10\cos\theta$

$5\sqrt{3} = \sqrt{100 - x^2}$

$5\sqrt{3} = \sqrt{100 - (10\cos\theta)^2}$

$5\sqrt{3} = \sqrt{100(1 - \cos^2\theta)}$

$5\sqrt{3} = \sqrt{100\sin^2\theta}$

$5\sqrt{3} = 10\sin\theta$

$\sin\theta = \dfrac{5\sqrt{3}}{10} = \dfrac{\sqrt{3}}{2}$

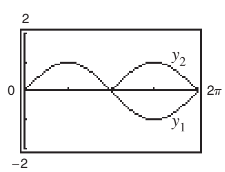
$\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \left(\dfrac{\sqrt{3}}{2}\right)^2} = \dfrac{1}{2}$

59. $\sin\theta = \sqrt{1 - \cos^2\theta}$

Let $y_1 = \sin x$ and $y_2 = \sqrt{1 - \cos^2 x}, 0 \le x \le 2\pi$.

$y_1 = y_2$ for $0 \le x \le \pi$.

So, $\sin\theta = \sqrt{1 - \cos^2\theta}$ for $0 \le \theta \le \pi$.

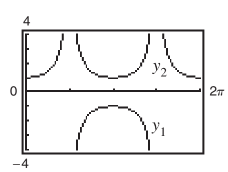


60. $\sec\theta = \sqrt{1 + \tan^2\theta}$

Let $y_1 = \dfrac{1}{\cos x}$ and $y_2 = \sqrt{1 + \tan^2 x}, 0 \le x \le 2\pi$.

$y_1 = y_2$ for $0 \le x < \dfrac{\pi}{2}$ and $\dfrac{3\pi}{2} < x \le 2\pi$.

So, $\sec\theta = \sqrt{1 + \tan^2\theta}$ for $0 \le \theta < \dfrac{\pi}{2}$ and $\dfrac{3\pi}{2} < \theta < 2\pi$.



61. $\ln|\sin x| + \ln|\cot x| = \ln|\sin x \cot x|$

$= \ln\left|\sin x \cdot \dfrac{\cos x}{\sin x}\right|$

$= \ln|\cos x|$

62. $\ln|\cos x| - \ln|\sin x| = \ln\dfrac{|\cos x|}{|\sin x|} = \ln|\cot x|$

63. $\ln|\tan t| - \ln(1 - \cos^2 t) = \ln\left[\dfrac{|\tan t|}{1 - \cos^2 t}\right]$

$= \ln\left|\dfrac{\tan t}{\sin^2 t}\right|$

$= \ln\left|\dfrac{\sin t}{\cos t} \cdot \dfrac{1}{\sin^2 t}\right|$

$= \ln\left|\dfrac{1}{\cos t \sin t}\right|$

$= \ln|\sec t \csc t|$

64. $\ln(\cos^2 t) + \ln(1 + \tan^2 t) = \ln[\cos^2 t(1 + \tan^2 t)]$

$= \ln[\cos^2 t \sec^2 t]$

$= \ln\left(\cos^2 t \cdot \dfrac{1}{\cos^2 t}\right)$

$= \ln(1) = 0$

65. $\mu W\cos\theta = W\sin\theta$

$\mu = \dfrac{W\sin\theta}{W\cos\theta} = \tan\theta$

66. $\sec x\tan x - \sin x = \dfrac{1}{\cos x} \cdot \dfrac{\sin x}{\cos x} - \sin x$

$= \dfrac{\sin x}{\cos^2 x} - \sin x$

$= \dfrac{\sin x - \sin x\cos^2 x}{\cos^2 x}$

$= \dfrac{\sin x(1 - \cos^2 x)}{\cos^2 x}$

$= \dfrac{\sin x\sin^2 x}{\cos^2 x}$

$= \sin x\tan^2 x$

67. True.

$\tan u = \dfrac{\sin u}{\cos u}$

$\cot u = \dfrac{\cos u}{\sin u}$

$\sec u = \dfrac{1}{\cos u}$

$\csc u = \dfrac{1}{\sin u}$

68. False. A cofunction identity can be used to transform a tangent function so that it can be represented by a cotangent function.

69. As $x \to \dfrac{\pi^-}{2}$, $\tan x \to \infty$ and $\cot x \to 0$.

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70. As $x \to \pi^+$, $\sin x \to 0$ and $\csc x = \dfrac{1}{\sin x} \to -\infty$.

71. $\cos(-\theta) \neq -\cos\theta$

$\cos(-\theta) = \cos\theta$

The correct identity is $\dfrac{\sin\theta}{\cos(-\theta)} = \dfrac{\sin\theta}{\cos\theta}$

$= \tan\theta$

72. Let $u = a\tan\theta$, then

$$\begin{aligned}\sqrt{a^2 + u^2} &= \sqrt{a^2 + (a\tan\theta)^2}\\ &= \sqrt{a^2 + a^2\tan^2\theta}\\ &= \sqrt{a^2(1 + \tan^2\theta)}\\ &= \sqrt{a^2\sec^2\theta}\\ &= a\sec\theta.\end{aligned}$$

73. Because $\sin^2\theta + \cos^2\theta = 1$, then $\cos^2\theta = 1 - \sin^2\theta$.

$\cos\theta = \pm\sqrt{1 - \sin\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\sin\theta}{\pm\sqrt{1 - \sin^2\theta}}$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{\pm\sqrt{1 - \sin^2\theta}}{\sin\theta}$

$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{\pm\sqrt{1 - \sin^2\theta}}$

$\csc\theta = \dfrac{1}{\sin\theta}$

74. To derive $\sin^2\theta + \cos^2\theta = 1$, let $\sin\theta = \dfrac{a}{\sqrt{a^2 + b^2}}$ and $\cos\theta = \dfrac{b}{\sqrt{a^2 + b^2}}$.

$$\begin{aligned}\text{So, } \sin^2\theta + \cos^2\theta &= \left(\frac{a}{\sqrt{a^2 + b^2}}\right)^2 + \left(\frac{b}{\sqrt{a^2 + b^2}}\right)^2 = \frac{a^2}{a^2 + b^2} + \frac{b^2}{a^2 + b^2}\\ &= \frac{a^2 + b^2}{a^2 + b^2}\\ &= 1.\end{aligned}$$

To derive $1 + \tan^2\theta = \sec^2\theta$, let $\tan\theta = \dfrac{a}{b}$ and $\sec\theta = \dfrac{\sqrt{a^2 + b^2}}{b}$.

$$\begin{aligned}\text{So, } 1 + \tan^2\theta &= 1 + \left(\frac{a}{b}\right)^2 = 1 + \frac{a^2}{b^2} = \frac{b^2 + a^2}{b^2}\\ &= \left(\sqrt{\frac{a^2 + b^2}{b^2}}\right)^2 = \left(\frac{\sqrt{a^2 + b^2}}{b}\right)^2\\ &= \sec^2\theta.\end{aligned}$$

To derive $1 + \cot^2\theta = \csc^2\theta$, let $\cot\theta = \dfrac{b}{a}$ and $\csc\theta = \dfrac{\sqrt{a^2 + b^2}}{a}$.

$$\begin{aligned}\text{So, } 1 + \cot^2\theta &= 1 + \left(\frac{b}{a}\right)^2 = 1 + \frac{b^2}{a^2}\\ &= \frac{a^2 + b^2}{a^2} = \left(\sqrt{\frac{a^2 + b^2}{a^2}}\right)^2\\ &= \left(\frac{\sqrt{a^2 + b^2}}{a}\right)^2 = \csc^2\theta.\end{aligned}$$

Answers will vary.

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75. $$\frac{\sec\theta(1+\tan\theta)}{\sec\theta+\csc\theta} = \frac{\left(\frac{1}{\cos\theta}\right)\left(1+\frac{\sin\theta}{\cos\theta}\right)}{\frac{1}{\cos\theta}+\frac{1}{\sin\theta}} = \frac{\frac{\cos\theta+\sin\theta}{\cos^2\theta}}{\frac{\sin\theta+\cos\theta}{\sin\theta\cos\theta}} = \left(\frac{\sin\theta+\cos\theta}{\cos^2\theta}\right)\left(\frac{\sin\theta\cos\theta}{\sin\theta+\cos\theta}\right) = \frac{\sin\theta}{\cos\theta}$$

Section 5.2 Verifying Trigonometric Identities

1. identity

2. conditional equation

3. $\tan u$

4. $\cot u$

5. $\sin u$

6. $\cot^2 u$

7. $-\csc u$

8. $\sec u$

9. $\tan t\cot t = \frac{\sin t}{\cos t}\cdot\frac{\cos t}{\sin t} = 1$

10. $\frac{\tan x\cot x}{\cos x} = \frac{1}{\cos x} = \sec x$

11. $(1+\sin\alpha)(1-\sin\alpha) = 1-\sin^2\alpha = \cos^2\alpha$

12. $$\cos^2\beta-\sin^2\beta = \cos^2\beta-(1-\cos^2\beta) = 2\cos^2\beta-1$$

13. $$\cos^2\beta-\sin^2\beta = (1-\sin^2\beta)-\sin^2\beta = 1-2\sin^2\beta$$

14. $$\sin^2\alpha-\sin^4\alpha = \sin^2\alpha(1-\sin^2\alpha) = (1-\cos^2\alpha)(\cos^2\alpha) = \cos^2\alpha-\cos^4\alpha$$

15. $$\tan\left(\frac{\pi}{2}-\theta\right)\tan\theta = \cot\theta\tan\theta = \left(\frac{1}{\tan\theta}\right)\tan\theta = 1$$

16. $$\frac{\cos[(\pi/2)-x]}{\sin[(\pi/2)-x]} = \frac{\sin x}{\cos x} = \tan x$$

17. $$\sin t\csc\left(\frac{\pi}{2}-t\right) = \sin t\sec t = \sin t\left(\frac{1}{\cos t}\right) = \frac{\sin t}{\cos t} = \tan t$$

18. $$\sec^2 y-\cot^2\left(\frac{\pi}{2}-y\right) = \sec^2 y-\tan^2 y = 1$$

19. $$\frac{1}{\tan x}+\frac{1}{\cot x} = \frac{\cot x+\tan x}{\tan x\cot x} = \frac{\cot x+\tan x}{1} = \tan x+\cot x$$

20. $$\frac{1}{\sin x}-\frac{1}{\csc x} = \frac{\csc x-\sin x}{\sin x\csc x} = \frac{\csc x-\sin x}{1} = \csc x-\sin x$$

21. $$\frac{1+\sin\theta}{\cos\theta}+\frac{\cos\theta}{1+\sin\theta} = \frac{(1+\sin\theta)^2+\cos^2\theta}{\cos\theta(1+\sin\theta)} = \frac{1+2\sin\theta+\sin^2\theta+\cos^2\theta}{\cos\theta(1+\sin\theta)} = \frac{2+2\sin\theta}{\cos\theta(1+\sin\theta)} = \frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)} = \frac{2}{\cos\theta} = 2\sec\theta$$

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22. $$\frac{\cos\theta\cot\theta}{1-\sin\theta}-1=\frac{\cos\theta\cot\theta-(1-\sin\theta)}{1-\sin\theta}$$
$$=\frac{\cos\theta\left(\dfrac{\cos\theta}{\sin\theta}\right)-1+\sin\theta}{1-\sin\theta}\cdot\frac{\sin\theta}{\sin\theta}$$
$$=\frac{\cos^2\theta-\sin\theta+\sin^2\theta}{\sin\theta(1-\sin\theta)}$$
$$=\frac{1-\sin\theta}{\sin\theta(1-\sin\theta)}$$
$$=\frac{1}{\sin\theta}$$
$$=\csc\theta$$

23. $$\frac{1}{\cos x+1}+\frac{1}{\cos x-1}=\frac{\cos x-1+\cos x+1}{(\cos x+1)(\cos x-1)}$$
$$=\frac{2\cos x}{\cos^2 x-1}$$
$$=\frac{2\cos x}{-\sin^2 x}$$
$$=-2\cdot\frac{1}{\sin x}\cdot\frac{\cos x}{\sin x}$$
$$=-2\csc x\cot x$$

24. $$\cos x-\frac{\cos x}{1-\tan x}=\frac{\cos x(1-\tan x)-\cos x}{1-\tan x}$$
$$=\frac{-\cos x\tan x}{1-\tan x}$$
$$=\frac{-\cos x(\sin x/\cos x)}{1-(\sin x/\cos x)}\cdot\frac{\cos x}{\cos x}$$
$$=\frac{-\sin x\cos x}{\cos x-\sin x}$$
$$=\frac{\sin x\cos x}{\sin x-\cos x}$$

25. $$\sec y\cos y=\left(\frac{1}{\cos y}\right)\cos y=1$$

26. $$\cot^2 y(\sec^2 y-1)=\cot^2 y\tan^2 y=1$$

27. $$\frac{\tan^2\theta}{\sec\theta}=\frac{(\sin\theta/\cos\theta)\tan\theta}{1/\cos\theta}=\sin\theta\tan\theta$$

28. $$\frac{\cot^3 t}{\csc t}=\frac{\cot t\cot^2 t}{\csc t}$$
$$=\frac{\cot t(\csc^2 t-1)}{\csc t}$$
$$=\frac{\dfrac{\cos t}{\sin t}(\csc^2 t-1)}{\dfrac{1}{\sin t}}$$
$$=\frac{\cos t\sin t}{\sin t}(\csc^2 t-1)$$
$$=\cos t(\csc^2 t-1)$$

29. $$\frac{1}{\tan\beta}+\tan\beta=\frac{1+\tan^2\beta}{\tan\beta}$$
$$=\frac{\sec^2\beta}{\tan\beta}$$

30. $$\frac{\sec\theta-1}{1-\cos\theta}=\frac{\sec\theta-1}{1-(1/\sec\theta)}\cdot\frac{\sec\theta}{\sec\theta}=\frac{\sec\theta(\sec\theta-1)}{\sec\theta-1}=\sec\theta$$

31. $$\frac{\cot^2 t}{\csc t}=\frac{\cos^2 t/\sin^2 t}{1/\sin t}=\frac{\cos^2 t}{\sin t}=\frac{1-\sin^2 t}{\sin t}$$

32. $$\cos x+\sin x\tan x=\cos x+\sin x\left(\frac{\sin x}{\cos x}\right)$$
$$=\frac{\cos^2 x+\sin^2 x}{\cos x}$$
$$=\frac{1}{\cos x}$$
$$=\sec x$$

33. $$\sec x-\cos x=\frac{1}{\cos x}-\cos x$$
$$=\frac{1-\cos^2 x}{\cos x}$$
$$=\frac{\sin^2 x}{\cos x}$$
$$=\sin x\cdot\frac{\sin x}{\cos x}$$
$$=\sin x\tan x$$

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34. $$\begin{aligned}\cot x - \tan x &= \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}\\ &= \frac{\cos^2 x - \sin^2 x}{\sin x \cos x}\\ &= \frac{1 - \sin^2 x - \sin^2 x}{\sin x \cos x}\\ &= \frac{1 - 2\sin^2 x}{\sin x \cos x}\\ &= \frac{1}{\cos x}\left(\frac{1 - 2\sin^2 x}{\sin x}\right)\\ &= \frac{1}{\cos x}\left(\frac{1}{\sin x} - \frac{2\sin^2 x}{\sin x}\right)\\ &= \sec x(\csc x - 2\sin x)\end{aligned}$$

35. $$\frac{\cot x}{\sec x} = \frac{\cos x/\sin x}{1/\cos x} = \frac{\cos^2 x}{\sin x} = \frac{1 - \sin^2 x}{\sin x} = \frac{1}{\sin x} - \frac{\sin^2 x}{\sin x} = \csc x - \sin x$$

36. $$\begin{aligned}\frac{\csc(-x)}{\sec(-x)} &= \frac{1/\sin(-x)}{1/\cos(-x)}\\ &= \frac{\cos(-x)}{\sin(-x)}\\ &= \frac{\cos x}{-\sin x}\\ &= -\cot x\end{aligned}$$

37. $$\sin^{1/2} x \cos x - \sin^{5/2} x \cos x = \sin^{1/2} x \cos x(1 - \sin^2 x) = \sin^{1/2} x \cos x \cdot \cos^2 x = \cos^3 x\sqrt{\sin x}$$

38. $$\sec^6 x(\sec x \tan x) - \sec^4 x(\sec x \tan x) = \sec^4 x(\sec x \tan x)(\sec^2 x - 1) = \sec^4 x(\sec x \tan x)\tan^2 x = \sec^5 x \tan^3 x$$

39. $$\begin{aligned}(1 + \sin y)[1 + \sin(-y)] &= (1 + \sin y)(1 - \sin y)\\ &= 1 - \sin^2 y\\ &= \cos^2 y\end{aligned}$$

40. $$\begin{aligned}\frac{\tan x + \tan y}{1 - \tan x \tan y} &= \frac{\dfrac{1}{\cot x} + \dfrac{1}{\cot y}}{1 - \dfrac{1}{\cot x}\cdot\dfrac{1}{\cot y}} \cdot \frac{\cot x \cot y}{\cot x \cot y}\\ &= \frac{\cot y + \cot x}{\cot x \cot y - 1}\end{aligned}$$

41. $$\begin{aligned}\sqrt{\frac{1 + \sin\theta}{1 - \sin\theta}} &= \sqrt{\frac{1 + \sin\theta}{1 - \sin\theta}\cdot\frac{1 + \sin\theta}{1 + \sin\theta}}\\ &= \sqrt{\frac{(1 + \sin\theta)^2}{1 - \sin^2\theta}}\\ &= \sqrt{\frac{(1 + \sin\theta)^2}{\cos^2\theta}}\\ &= \frac{1 + \sin\theta}{|\cos\theta|}\end{aligned}$$

42. $$\begin{aligned}\frac{\cos x - \cos y}{\sin x + \sin y} + \frac{\sin x - \sin y}{\cos x + \cos y} &= \frac{(\cos x - \cos y)(\cos x + \cos y) + (\sin x - \sin y)(\sin x + \sin y)}{(\sin x + \sin y)(\cos x + \cos y)}\\ &= \frac{\cos^2 x - \cos^2 y + \sin^2 x - \sin^2 y}{(\sin x + \sin y)(\cos x + \cos y)}\\ &= \frac{(\cos^2 x + \sin^2 x) - (\cos^2 y + \sin^2 y)}{(\sin x + \sin y)(\cos x + \cos y)}\\ &= 0\end{aligned}$$

43. $\cot(-x) \neq \cot x$

The correct substitution is $\cot(-x) = -\cot x$.

$$\frac{1}{\tan x} + \cot(-x) = \cot x - \cot x = 0$$

44. The first line claims that $\sec(-\theta) = -\sec\theta$ and $\sin(-\theta) = \sin\theta$. The correct substitutions are $\sec(-\theta) = \sec\theta$ and $\sin(-\theta) = -\sin\theta$.

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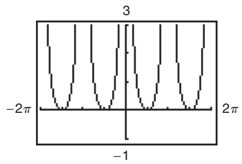
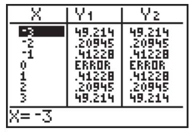
45. (a)



Identity

(b)



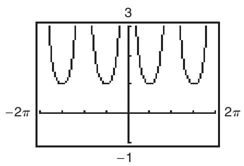
X	Y1	Y2
-3	49.214	49.214
-2	.20945	.20945
-1	.41228	.41228
0	ERROR	ERROR
1	.41228	.41228
2	.20945	.20945
3	49.214	49.214

X=-3

Identity

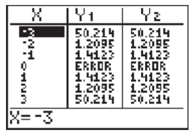
(c) $\left(1 + \cot^2 x\right)\left(\cos^2 x\right) = \csc^2 x \cos^2 x = \dfrac{1}{\sin^2 x} \cdot \cos^2 x = \cot^2 x$

46. (a)



Identity

(b)



X	Y1	Y2
-3	50.214	50.214
-2	1.2095	1.2095
-1	1.4123	1.4123
0	ERROR	ERROR
1	1.4123	1.4123
2	1.2095	1.2095
3	50.214	50.214

X=-3

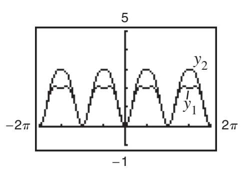
Identity

(c) $\csc x\left(\csc x - \sin x\right) + \dfrac{\sin x - \cos x}{\sin x} + \cot x = \csc^2 x - \csc x \sin x + 1 - \dfrac{\cos x}{\sin x} + \cot x$

$= \csc^2 x - 1 + 1 - \cot x + \cot x$

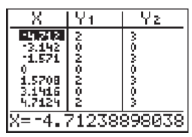
$= \csc^2 x$

47. (a)



Not an identity

(b)



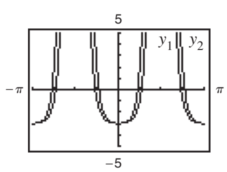
X	Y1	Y2
[illegible]	2	3
-3.142	0	0
-1.571	2	3
0	0	0
1.5708	2	3
3.1416	0	0
4.7124	2	3

X=-4.71238898038

Not an identity

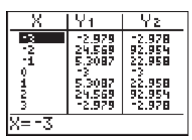
(c) $2 + \cos^2 x - 3\cos^4 x = \left(1 - \cos^2 x\right)\left(2 + 3\cos^2 x\right) = \sin^2 x\left(2 + 3\cos^2 x\right) \neq \sin^2 x\left(3 + 2\cos^2 x\right)$

48. (a)



Not an identity

(b)



X	Y1	Y2
-3	-2.979	-2.978
-2	24.569	92.954
-1	5.3087	22.958
0	-3	-3
1	5.3087	22.958
2	24.569	92.954
3	-2.979	-2.978

X=-3

Not an identity

(c) $\tan^4 x + \tan^2 x - 3 = \dfrac{\sin^4 x}{\cos^4 x} + \dfrac{\sin^2 x}{\cos^2 x} - 3$

$= \dfrac{1}{\cos^2 x}\left(\dfrac{\sin^4 x}{\cos^2 x} + \sin^2 x\right) - 3$

$= \dfrac{1}{\cos^2 x}\left(\dfrac{\sin^4 x + \sin^2 x \cos^2 x}{\cos^2 x}\right) - 3$

$= \dfrac{1}{\cos^2 x}\left(\dfrac{\sin^2 x}{\cos^2 x}\right)\left(\sin^2 x + \cos^2 x\right) - 3$

$= \dfrac{1}{\cos^2 x}\left(\dfrac{\sin^2 x}{\cos^2 x} \cdot 1\right) - 3$

$= \sec^2 x \tan^2 x - 3$

$\neq \sec^2 x\left(4\tan^2 x - 3\right)$

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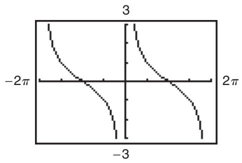
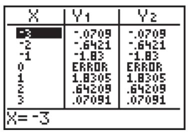
49. (a)



Identity

(b)



X	Y1	Y2
-3	-.0709	-.0709
-2	-.6421	-.6421
-1	-1.83	-1.83
0	ERROR	ERROR
1	1.8305	1.8305
2	.64209	.64209
3	.07091	.07091

X=-3

Identity

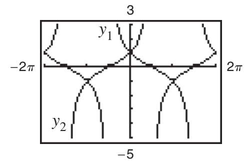
(c) $\dfrac{1 + \cos x}{\sin x} = \dfrac{(1 + \cos x)(1 - \cos x)}{\sin x(1 - \cos x)}$

$$= \frac{1 - \cos^2 x}{\sin x(1 - \cos x)}$$

$$= \frac{\sin^2 x}{\sin x(1 - \cos x)}$$

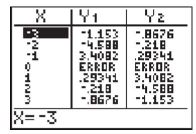
$$= \frac{\sin x}{1 - \cos x}$$

50. (a)



Not an identity

(b)



X	Y1	Y2
-3	-1.153	-.8676
-2	-4.588	-.218
-1	3.4082	.29341
0	ERROR	ERROR
1	.29341	3.4082
2	-.218	-4.588
3	-.8676	-1.153

X=-3

Not an identity

(c) $\dfrac{\cot \alpha}{\csc \alpha + 1}$ is the reciprocal of $\dfrac{\csc \alpha + 1}{\cot \alpha}$.

They will only be equivalent at isolated points in their respective domains. So, not an identity.

51. $\tan^3 x \sec^2 x - \tan^3 x = \tan^3 x(\sec^2 x - 1)$

$$= \tan^3 x \tan^2 x$$

$$= \tan^5 x$$

52. $(\tan^2 x + \tan^4 x)\sec^2 x = \left(\dfrac{\sin^2 x}{\cos^2 x} + \dfrac{\sin^4 x}{\cos^4 x}\right)\dfrac{1}{\cos^2 x}$

$$= \frac{1}{\cos^4 x}\left(\sin^2 x + \frac{\sin^4 x}{\cos^2 x}\right)$$

$$= \frac{1}{\cos^4 x}\left(\frac{\sin^2 x \cos^2 x + \sin^4 x}{\cos^2 x}\right)$$

$$= \frac{1}{\cos^4 x}\left(\frac{\sin^2 x(\cos^2 x + \sin^2 x)}{\cos^2 x}\right)$$

$$= \frac{1}{\cos^4 x}\left(\frac{\sin^2 x}{\cos^2 x} \cdot 1\right) = \sec^4 x \cdot \tan^2 x$$

53. $(\sin^2 x - \sin^4 x)\cos x = \sin^2 x(1 - \sin^2 x)\cos x$

$$= \sin^2 x \cos^2 x \cos x$$

$$= \sin^2 x \cos^3 x$$

54. $\sin^4 x + \cos^4 x = \sin^2 x \sin^2 x + \cos^4 x$

$$= (1 - \cos^2 x)(1 - \cos^2 x) + \cos^4 x$$

$$= 1 - 2\cos^2 x + \cos^4 x + \cos^4 x$$

$$= 1 - 2\cos^2 x + 2\cos^4 x$$

55. $\sin^2 25° + \sin^2 65° = \sin^2 25° + \cos^2(90° - 65°)$

$$= \sin^2 25° + \cos^2 25°$$

$$= 1$$

56. $\tan^2 63° + \cot^2 16° - \sec^2 74° - \csc^2 27° = \tan^2 63° + \cot^2 16° - \csc^2(90° - 74°) - \sec^2(90° - 27°)$

$$= \tan^2 63° + \cot^2 16° - \csc^2 16° - \sec^2 63°$$

$$= (\tan^2 63° - \sec^2 63°) + (\cot^2 16° - \csc^2 16°)$$

$$= -1 + (-1)$$

$$= -2$$

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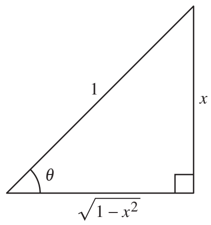
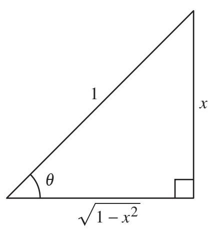
57. Let $\theta = \sin^{-1} x \Rightarrow \sin\theta = x = \dfrac{x}{1}$.



From the diagram,

$$\tan\left(\sin^{-1} x\right) = \tan\theta = \frac{x}{\sqrt{1 - x^2}}.$$

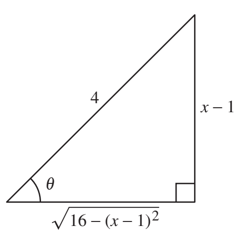
58. Let $\theta = \sin^{-1} x \Rightarrow \sin\theta = x = \dfrac{x}{1}$.



From the diagram,

$$\cos\left(\sin^{-1} x\right) = \cos\theta = \frac{\sqrt{1 - x^2}}{1} = \sqrt{1 - x^2}.$$

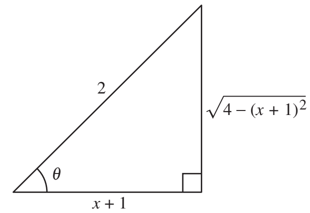
59. Let $\theta = \sin^{-1} \dfrac{x - 1}{4} \Rightarrow \sin\theta = \dfrac{x - 1}{4}$.



From the diagram,

$$\tan\left(\sin^{-1}\frac{x - 1}{4}\right) = \tan\theta = \frac{x - 1}{\sqrt{16 - (x - 1)^2}}.$$

60. Let $\theta = \cos^{-1} \dfrac{x + 1}{2} \Rightarrow \cos\theta = \dfrac{x + 1}{2}$.



From the diagram,

$$\tan\left(\cos^{-1}\frac{x + 1}{2}\right) = \tan\theta = \frac{\sqrt{4 - (x + 1)^2}}{x + 1}.$$

61.
$$\begin{aligned} \cos x - \csc x \cot x &= \cos x - \frac{1}{\sin x}\frac{\cos x}{\sin x} \\ &= \cos x\left(1 - \frac{1}{\sin^2 x}\right) \\ &= \cos x\left(1 - \csc^2 x\right) \\ &= -\cos x\left(\csc^2 x - 1\right) \\ &= -\cos x \cot^2 x \end{aligned}$$

62. (a) $\dfrac{h\sin(90^\circ - \theta)}{\sin\theta} = \dfrac{h\cos\theta}{\sin\theta} = h\cot\theta$

(b)

θ	15°	30°	45°	60°	75°	90°
s	18.66	8.66	5	2.89	1.34	0

(c) Maximum: 15°

Minimum: 90°

(d) Noon

63. False. $\tan x^2 = \tan(x \cdot x)$ and $\tan^2 x = (\tan x)(\tan x)$, $\tan x^2 \neq \tan^2 x$.

64. True. Cosine is an even function,

$$\begin{aligned} \cos\left(\theta - \frac{\pi}{2}\right) &= \cos\left[-\left(\frac{\pi}{2} - \theta\right)\right] \\ &= \cos\left(\frac{\pi}{2} - \theta\right) \\ &= \sin\theta. \end{aligned}$$

65. False. For the equation to be an identity, it must be true for all values of θ in the domain.

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66. If $\sin\theta = \frac{a}{c}$, $\sec\theta = \frac{c}{b}$, and

$a^2 + b^2 = c^2 \Rightarrow a^2 = c^2 - b^2$, then

$$\frac{\sec^2\theta - 1}{\sec^2\theta} = \frac{\left(\frac{c}{b}\right)^2 - 1}{\left(\frac{c}{b}\right)^2} = \frac{\frac{c^2}{b^2} - 1}{\frac{c^2}{b^2}} = \frac{\frac{c^2 - b^2}{b^2}}{\frac{c^2}{b^2}} = \frac{c^2 - b^2}{b^2}\cdot\frac{b^2}{c^2} = \frac{c^2 - b^2}{c^2} = \frac{a^2}{c^2} = \left(\frac{a}{c}\right)^2 = \sin^2\theta.$$

67. Because $\sin^2\theta = 1 - \cos^2\theta$, then

$\sin\theta = \pm\sqrt{1 - \cos^2\theta}$; $\sin\theta \neq \sqrt{1 - \cos^2\theta}$ if θ lies in Quadrant III or IV.

One such angle is $\theta = \frac{7\pi}{4}$.

68. $\tan\theta = \sqrt{\sec^2\theta - 1}$

True identity: $\tan\theta = \pm\sqrt{\sec^2\theta - 1}$

$\tan\theta = \sqrt{\sec^2\theta - 1}$ is not true for $\pi/2 < \theta < \pi$ or $3\pi/2 < \theta < 2\pi$. So, the equation is not true for $\theta = 3\pi/4$.

69.

$$1 - \cos\theta = \sin\theta$$
$$(1 - \cos\theta)^2 = (\sin\theta)^2$$
$$1 - 2\cos\theta + \cos^2\theta = \sin^2\theta$$
$$1 - 2\cos\theta + \cos^2\theta = 1 - \cos^2\theta$$
$$2\cos^2\theta - 2\cos\theta = 0$$
$$2\cos\theta(\cos\theta - 1) = 0$$

The equation is not an identity because it is only true when $\cos\theta = 0$ or $\cos\theta = 1$. So, one angle for which the equation is not true is $-\frac{\pi}{2}$.

70.

$$1 + \tan\theta = \sec\theta$$
$$(1 + \tan\theta)^2 = (\sec\theta)^2$$
$$1 + 2\tan\theta + \tan^2\theta = \sec^2\theta$$
$$1 + 2\tan\theta + \tan^2\theta = 1 + \tan^2\theta$$
$$2\tan\theta = 0$$
$$\tan\theta = 0$$

This equation is not an identity because it is only true when $\tan\theta = 0$. So, one angle for which the equation is not true is $\frac{\pi}{6}$.

Section 5.3 Solving Trigonometric Equations

1. isolate

2. general

3. quadratic

4. extraneous

5. $\tan x - \sqrt{3} = 0$

(a) $x = \frac{\pi}{3}$

$$\tan\frac{\pi}{3} - \sqrt{3} = \sqrt{3} - \sqrt{3} = 0$$

(b) $x = \frac{4\pi}{3}$

$$\tan\frac{4\pi}{3} - \sqrt{3} = \sqrt{3} - \sqrt{3} = 0$$

6. $\sec x - 2 = 0$

(a) $x = \frac{\pi}{3}$

$$\sec\frac{\pi}{3} - 2 = \frac{1}{\cos(\pi/3)} - 2 = \frac{1}{1/2} - 2 = 2 - 2 = 0$$

(b) $x = \frac{5\pi}{3}$

$$\sec\frac{5\pi}{3} - 2 = \frac{1}{\cos(5\pi/3)} - 2 = \frac{1}{1/2} - 2 = 2 - 2 = 0$$

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7. $3\tan^2 2x - 1 = 0$

(a) $x = \dfrac{\pi}{12}$

$$3\left[\tan 2\left(\frac{\pi}{12}\right)\right]^2 - 1 = 3\tan^2\frac{\pi}{6} - 1 = 3\left(\frac{1}{\sqrt{3}}\right)^2 - 1 = 0$$

(b) $x = \dfrac{5\pi}{12}$

$$3\left[\tan 2\left(\frac{5\pi}{12}\right)\right]^2 - 1 = 3\tan^2\frac{5\pi}{6} - 1 = 3\left(-\frac{1}{\sqrt{3}}\right)^2 - 1 = 0$$

8. $2\cos^2 4x - 1 = 0$

(a) $x = \dfrac{\pi}{16}$

$$2\cos^2\left[4\left(\frac{\pi}{16}\right)\right] - 1 = 2\cos^2\frac{\pi}{4} - 1 = 2\left(\frac{\sqrt{2}}{2}\right)^2 - 1 = 2\left(\frac{1}{2}\right) - 1 = 1 - 1 = 0$$

(b) $x = \dfrac{3\pi}{16}$

$$2\cos^2\left[4\left(\frac{3\pi}{16}\right)\right] - 1 = 2\cos^2\frac{3\pi}{4} - 1 = 2\left(-\frac{\sqrt{2}}{2}\right)^2 - 1 = 2\left(\frac{1}{2}\right) - 1 = 0$$

9. $2\sin^2 x - \sin x - 1 = 0$

(a) $x = \dfrac{\pi}{2}$

$$2\sin^2\frac{\pi}{2} - \sin\frac{\pi}{2} - 1 = 2(1)^2 - 1 - 1 = 0$$

(b) $x = \dfrac{7\pi}{6}$

$$2\sin^2\frac{7\pi}{6} - \sin\frac{7\pi}{6} - 1 = 2\left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right) - 1 = \frac{1}{2} + \frac{1}{2} - 1 = 0$$

10. $\csc^4 x - 4\csc^2 x = 0$

(a) $x = \dfrac{\pi}{6}$

$$\csc^4\frac{\pi}{6} - 4\csc^2\frac{\pi}{6} = \frac{1}{\sin^4(\pi/6)} - \frac{4}{\sin^2(\pi/6)} = \frac{1}{(1/2)^4} - \frac{4}{(1/2)^2} = 16 - 16 = 0$$

(b) $x = \dfrac{5\pi}{6}$

$$\csc^4\frac{5\pi}{6} - 4\csc\frac{5\pi}{6} = \frac{1}{\sin^4(5\pi/6)} - \frac{4}{\sin^2(5\pi/6)} = \frac{1}{(1/2)^4} - \frac{4}{(1/2)^2} = 16 - 16 = 0$$

11. $\sqrt{3}\csc x - 2 = 0$

$$\sqrt{3}\csc x = 2$$
$$\csc x = \frac{2}{\sqrt{3}}$$
$$x = \frac{\pi}{3} + 2n\pi$$
$$\text{or } x = \frac{2\pi}{3} + 2n\pi$$

12. $\tan x + \sqrt{3} = 0$

$$\tan x = -\sqrt{3}$$
$$x = \frac{2\pi}{3} + n\pi$$

13. $\cos x + 1 = -\cos x$

$$2\cos x + 1 = 0$$
$$\cos x = -\frac{1}{2}$$
$$x = \frac{2\pi}{3} + 2n\pi \text{ or } x = \frac{4\pi}{3} + 2n\pi$$

14. $3\sin x + 1 = \sin x$

$$2\sin x + 1 = 0$$
$$\sin x = -\frac{1}{2}$$
$$x = \frac{7\pi}{6} + 2n\pi \text{ or}$$
$$x = \frac{11\pi}{6} + 2n\pi$$

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15. $3\sec^2 x - 4 = 0$

$$\sec^2 x = \frac{4}{3}$$
$$\sec x = \pm\frac{2}{\sqrt{3}}$$
$$x = \frac{\pi}{6} + n\pi$$
$$\text{or } x = \frac{5\pi}{6} + n\pi$$

16. $3\cot^2 x - 1 = 0$

$$\cot^2 x = \frac{1}{3}$$
$$\cot x = \pm\frac{1}{\sqrt{3}}$$
$$x = \frac{\pi}{3} + n\pi$$
$$\text{or } x = \frac{2\pi}{3} + n\pi$$

17. $4\cos^2 x - 1 = 0$

$$\cos^2 x = \frac{1}{4}$$
$$\cos x = \pm\frac{1}{2}$$
$$x = \frac{\pi}{3} + n\pi \quad \text{or} \quad x = \frac{2\pi}{3} + n\pi$$

18. $2 - 4\sin^2 x = 0$

$$\sin^2 x = \frac{1}{2}$$
$$\sin x = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$$
$$x = \frac{\pi}{4} + 2n\pi$$
$$x = \frac{3\pi}{4} + 2n\pi$$
$$x = \frac{5\pi}{4} + 2n\pi$$
$$x = \frac{7\pi}{4} + 2n\pi$$

These answers can be represented as $x = \dfrac{\pi}{4} + \dfrac{n\pi}{2}$.

19. $\sin x(\sin x + 1) = 0$

$\sin x = 0$ or $\sin x = -1$

$x = n\pi \qquad x = \dfrac{3\pi}{2} + 2n\pi$

20. $(2\sin^2 x - 1)(\tan^2 x - 3) = 0$

$2\sin^2 x - 1 = 0$ or $\tan^2 x = 3$

$\sin^2 x = \dfrac{1}{2} \qquad \tan x = \pm\sqrt{3}$

$\sin x = \pm\dfrac{1}{\sqrt{2}} \qquad x = \dfrac{\pi}{3} + n\pi$

$\sin x = \pm\dfrac{\sqrt{2}}{2} \qquad x = \dfrac{2\pi}{3} + n\pi$

$$x = \frac{\pi}{4} + 2n\pi$$
$$x = \frac{3\pi}{4} + 2n\pi$$
$$x = \frac{5\pi}{4} + 2n\pi$$
$$x = \frac{7\pi}{4} + 2n\pi$$

21. $\cos^3 x - \cos x = 0$

$\cos x(\cos^2 x - 1) = 0$

$\cos x = 0$ or $\cos^2 x - 1 = 0$

$x = \dfrac{\pi}{2} + n\pi \qquad \cos x = \pm 1$

$x = n\pi$

Both of these answers can be represented as $x = \dfrac{n\pi}{2}$.

22. $\sec^2 x - 1 = 0$

$$\sec^2 x = 1$$
$$\sec x = \pm 1$$
$$x = n\pi$$

23. $3\tan^3 x = \tan x$

$3\tan^3 x - \tan x = 0$

$\tan x(3\tan^2 x - 1) = 0$

$\tan x = 0$ or $3\tan^2 x - 1 = 0$

$x = n\pi \qquad \tan x = \pm\dfrac{\sqrt{3}}{3}$

$x = \dfrac{\pi}{6} + n\pi, \dfrac{5\pi}{6} + n\pi$

24. $\sec x \csc x = 2\csc x$

$\sec x \csc x - 2\csc x = 0$

$\csc x(\sec x - 2) = 0$

$\csc x = 0$ or $\sec x - 2 = 0$

No solution $\qquad \sec x = 2$

$x = \dfrac{\pi}{3} + 2n\pi, \dfrac{5\pi}{3} + 2n\pi$

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25. $2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$2\cos x - 1 = 0$ or $\cos x + 1 = 0$

$\cos x = \frac{1}{2}$ $\qquad$ $\cos x = -1$

$x = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi$ $\qquad$ $x = \pi + 2n\pi$

26. $2\sin^2 x + 3\sin x + 1 = 0$

$(2\sin x + 1)(\sin x + 1) = 0$

$2\sin x + 1 = 0$ or $\sin x + 1 = 0$

$\sin x = -\frac{1}{2}$ $\qquad$ $\sin x = -1$

$x = \frac{7\pi}{6} + 2n\pi, \frac{11\pi}{6} + 2n\pi$ $\qquad$ $x = \frac{3\pi}{2} + 2n\pi$

27. $\sec^2 x - \sec x = 2$

$\sec^2 x - \sec x - 2 = 0$

$(\sec x - 2)(\sec x + 1) = 0$

$\sec x - 2 = 0$ or $\sec x + 1 = 0$

$\sec x = 2$ $\qquad$ $\sec x = -1$

$x = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi$ $\qquad$ $x = \pi + 2n\pi$

28. $\csc^2 x + \csc x = 2$

$\csc^2 x + \csc x - 2 = 0$

$(\csc x + 2)(\csc x - 1) = 0$

$\csc x + 2 = 0$ or $\csc x - 1 = 0$

$\csc x = -2$ $\qquad$ $\csc x = 1$

$x = \frac{7\pi}{6} + 2n\pi, \frac{11\pi}{6} + 2n\pi$ $\qquad$ $x = \frac{\pi}{2} + 2n\pi$

29. $\sin x - 2 = \cos x - 2$

$\sin x = \cos x$

$\frac{\sin x}{\cos x} = 1$

$\tan x = 1$

$x = \tan^{-1} 1$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

30. $\cos x + \sin x \tan x = 2$

$\cos x + \sin x\left(\frac{\sin x}{\cos x}\right) = 2$

$\frac{\cos^2 x + \sin^2 x}{\cos x} = 2$

$\frac{1}{\cos x} = 2$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

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31.
$$2\sin^2 x = 2 + \cos x$$
$$2 - 2\cos^2 x = 2 + \cos x$$
$$2\cos^2 x + \cos x = 0$$
$$\cos x(2\cos x + 1) = 0$$
$$\cos x = 0 \quad \text{or} \quad 2\cos x + 1 = 0$$
$$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad 2\cos x = -1$$
$$\cos x = -\frac{1}{2}$$
$$x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

32.
$$\tan^2 x = \sec x - 1$$
$$\sec^2 x - 1 = \sec x - 1$$
$$\sec^2 x - \sec x = 0$$
$$\sec x(\sec x - 1) = 0$$
$$\sec x = 0 \quad \text{or} \quad \sec x - 1 = 0$$
No Solutions
$$\sec x = 1$$
$$x = 0$$

33.
$$\sin^2 x = 3\cos^2 x$$
$$\sin^2 x - 3\cos^2 x = 0$$
$$\sin^2 x - 3(1 - \sin^2 x) = 0$$
$$4\sin^2 x = 3$$
$$\sin x = \pm\frac{\sqrt{3}}{2}$$
$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

34.
$$2\sec^2 x + \tan^2 x - 3 = 0$$
$$2(\tan^2 x + 1) + \tan^2 x - 3 = 0$$
$$3\tan^2 x - 1 = 0$$
$$\tan x = \pm\frac{\sqrt{3}}{3}$$
$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

35.
$$2\sin x + \csc x = 0$$
$$2\sin x + \frac{1}{\sin x} = 0$$
$$2\sin^2 x + 1 = 0$$
$$\sin^2 x = -\frac{1}{2} \Rightarrow \text{No solution}$$

36.
$$3\sec x - 4\cos x = 0$$
$$\frac{3}{\cos x} - 4\cos x = 0$$
$$\frac{3 - 4\cos^2 x}{\cos x} = 0$$
$$3 - 4\cos^2 x = 0$$
$$\cos^2 x = \frac{3}{4}$$
$$\cos x = \pm\frac{\sqrt{3}}{2}$$
$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

37.
$$\csc x + \cot x = 1$$
$$(\csc x + \cot x)^2 = 1^2$$
$$\csc^2 x + 2\csc x\cot x + \cot^2 x = 1$$
$$\cot^2 x + 1 + 2\csc x\cot x + \cot^2 x = 1$$
$$2\cot^2 x + 2\csc x\cot x = 0$$
$$2\cot x(\cot x + \csc x) = 0$$
$$2\cot x = 0 \quad \text{or} \quad \cot x + \csc x = 0$$
$$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad \frac{\cos x}{\sin x} = -\frac{1}{\sin x}$$
$\left(\frac{3\pi}{2} \text{ is extraneous.}\right)$
$$\cos x = -1$$
$$x = \pi$$
(π is extraneous.)

$x = \pi/2$ is the only solution.

38.
$$\sec x + \tan x = 1$$
$$\frac{1}{\cos x} + \frac{\sin x}{\cos x} = 1$$
$$1 + \sin x = \cos x$$
$$(1 + \sin x)^2 = \cos^2 x$$
$$1 + 2\sin x + \sin^2 x = \cos^2 x$$
$$1 + 2\sin x + \sin^2 x = 1 - \sin^2 x$$
$$2\sin^2 x + 2\sin x = 0$$
$$2\sin x(\sin x + 1) = 0$$
$$\sin x = 0 \quad \text{or} \quad \sin x + 1 = 0$$
$$x = 0, \pi \qquad \sin x = -1$$
(π is extraneous.)
$$x = \frac{3\pi}{2}$$
$\left(\frac{3\pi}{2} \text{ is extraneous.}\right)$

$x = 0$ is the only solution.

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39. $2\cos 2x - 1 = 0$

$$\cos 2x = \frac{1}{2}$$

$$2x = \frac{\pi}{3} + 2n\pi \quad \text{or} \quad 2x = \frac{5\pi}{3} + 2n\pi$$

$$x = \frac{\pi}{6} + n\pi \qquad x = \frac{5\pi}{6} + n\pi$$

40. $2\sin 2x + \sqrt{3} = 0$

$$\sin 2x = -\frac{\sqrt{3}}{2}$$

$$2x = \frac{4\pi}{3} + 2n\pi \quad \text{or} \quad 2x = \frac{5\pi}{3} + 2n\pi$$

$$x = \frac{2\pi}{3} + n\pi \qquad x = \frac{5\pi}{6} + n\pi$$

41. $\tan 3x - 1 = 0$

$$\tan 3x = 1$$

$$3x = \frac{\pi}{4} + n\pi$$

$$x = \frac{\pi}{12} + \frac{n\pi}{3}$$

42. $\sec 4x - 2 = 0$

$$\sec 4x = 2$$

$$\cos 4x = \frac{1}{2}$$

$$4x = \frac{\pi}{3} + 2n\pi \quad \text{or} \quad 4x = \frac{5\pi}{3} + 2n\pi$$

$$x = \frac{\pi}{12} + \frac{n\pi}{2} \qquad x = \frac{5\pi}{12} + \frac{n\pi}{2}$$

43. $2\cos\frac{x}{2} = \sqrt{2} = 0$

$$\cos\frac{x}{2} = \frac{\sqrt{2}}{2}$$

$$\frac{x}{2} = \frac{\pi}{4} + 2n\pi \quad \text{or} \quad \frac{x}{2} = \frac{7\pi}{4} + 2n\pi$$

$$x = \frac{\pi}{2} + 4n\pi \qquad x = \frac{7\pi}{2} + 4n\pi$$

44. $2\sin\frac{x}{2} = \sqrt{3} = 0$

$$\sin\frac{x}{2} = -\frac{\sqrt{3}}{2}$$

$$\frac{x}{2} = \frac{4\pi}{3} + 2n\pi \quad \text{or} \quad \frac{x}{2} = \frac{5\pi}{3} + 2n\pi$$

$$x = \frac{8\pi}{3} + 4n\pi \qquad x = \frac{10\pi}{3} + 4n\pi$$

45. $3\tan\frac{x}{2} - \sqrt{3} = 0$

$$\tan\frac{x}{2} = \frac{\sqrt{3}}{3}$$

$$\frac{x}{2} = \frac{\pi}{6} + n\pi \Rightarrow x = \frac{\pi}{3} + 2n\pi$$

46. $\tan\frac{x}{2} + \sqrt{3} = 0$

$$\tan\frac{x}{2} = -\sqrt{3}$$

$$\frac{x}{2} = \frac{2\pi}{3} + n\pi \Rightarrow x = \frac{4\pi}{3} + 2n\pi$$

47. $y = \sin\frac{\pi x}{2} + 1$

$$\sin\left(\frac{\pi x}{2}\right) + 1 = 0$$

$$\sin\left(\frac{\pi x}{2}\right) = -1$$

$$\frac{\pi x}{2} = \frac{3\pi}{2} + 2n\pi$$

$$x = 3 + 4n$$

For $-2 < x < 4$, the intercepts are -1 and 3.

48. $y = \sin \pi x + \cos \pi x$

$$\sin \pi x + \cos \pi x = 0$$

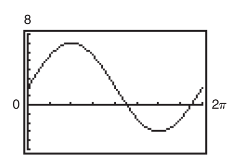
$$\sin \pi x = -\cos \pi x$$

$$\pi x = -\frac{\pi}{4} + n\pi$$

$$x = -\frac{1}{4} + n$$

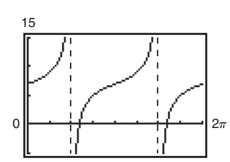
For $-1 < x < 3$, the intercepts are $-\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{11}{4}$.

49. $5\sin x + 2 = 0$



$x \approx 3.553$ and $x \approx 5.872$

50. $2\tan x + 7 = 0$



$x \approx 1.849$ and $x \approx 4.991$

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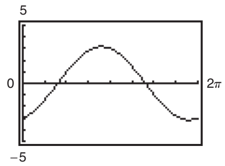
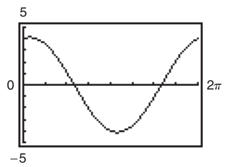
51. $\sin x - 3\cos x = 0$



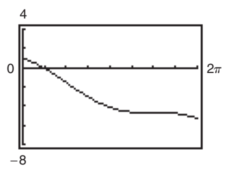
$x \approx 1.249$ and $x \approx 4.391$

52. $\sin x + 4\cos x = 0$



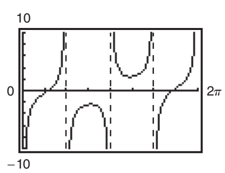
$x \approx 1.816$ and $x \approx 4.957$

53. $\cos x = x$



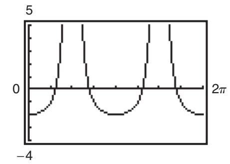
$x \approx 0.739$

54. $\tan x = \csc x$



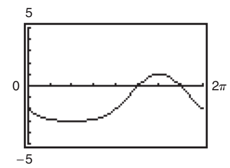
$x \approx 0.905$ and $x \approx 5.379$

55. $\sec^2 x - 3 = 0$



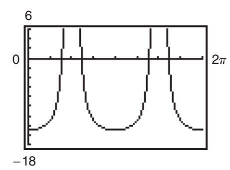
$x \approx 0.955, x \approx 2.186, x \approx 4.097$ and $x \approx 5.328$

56. $\csc^2 x - 5 = 0$



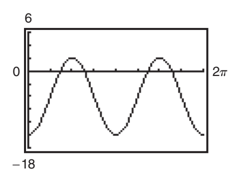
$x \approx 0.464, x \approx 2.678, x = 3.605$ and $x \approx 5.820$

57. $2\tan^2 x = 15$



$x \approx 1.221, x \approx 1.921, x \approx 4.362$ and $x \approx 5.062$

58. $6\sin^2 x = 5$



$x \approx 1.150, x \approx 1.991, x \approx 4.292$ and $x \approx 5.133$

59.
$$\tan^2 x + \tan x - 12 = 0$$
$$(\tan x + 4)(\tan x - 3) = 0$$

$\tan x + 4 = 0$ or $\tan x - 3 = 0$

$\tan x = -4$ $\qquad$ $\tan x = 3$

$x = \arctan(-4) + n\pi$ $\qquad$ $x = \arctan 3 + n\pi$

60.
$$\tan^2 x - \tan x - 2 = 0$$
$$(\tan x + 1)(\tan x - 2) = 0$$

$\tan x + 1 = 0$ or $\tan x - 2 = 0$

$\tan x = -1$ $\qquad$ $\tan x = 2$

$x = \dfrac{3\pi}{4} + n\pi$ $\qquad$ $x = \arctan 2 + n\pi$

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61.
$$\sec^2 x - 6\tan x = -4$$
$$1 + \tan^2 x - 6\tan x + 4 = 0$$
$$\tan^2 x - 6\tan x + 5 = 0$$
$$(\tan x - 1)(\tan x - 5) = 0$$
$$\tan x - 1 = 0 \qquad \tan x - 5 = 0$$
$$\tan x = 1 \qquad \tan x = 5$$
$$x = \frac{\pi}{4} + n\pi \qquad x = \arctan 5 + n\pi$$

62.
$$\sec^2 x + \tan x - 3 = 0$$
$$1 + \tan^2 x + \tan x - 3 = 0$$
$$\tan^2 x + \tan x - 2 = 0$$
$$(\tan x + 2)(\tan x - 1) = 0$$
$$\tan x + 2 = 0 \qquad \tan x - 1 = 0$$
$$\tan x = -2 \qquad \tan x = 1$$
$$x = \arctan(-2) + n\pi \qquad x = \arctan(1) + n\pi$$
$$\approx -1.1071 + n\pi \qquad = \frac{\pi}{4} + n\pi$$

63.
$$2\sin^2 x + 5\cos x = 4$$
$$2(1 - \cos^2 x) + 5\cos x - 4 = 0$$
$$-2\cos^2 x + 5\cos x - 2 = 0$$
$$-(2\cos x - 1)(\cos x - 2) = 0$$
$$2\cos x - 1 = 0 \quad \text{or} \quad \cos x - 2 = 0$$
$$\cos x = \frac{1}{2} \qquad \cos x = 2$$
$$x = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi \qquad \text{No solution}$$

64.
$$2\cos^2 x + 7\sin x = 5$$
$$2(1 - \sin^2 x) + 7\sin x - 5 = 0$$
$$-2\sin^2 x + 7\sin x - 3 = 0$$
$$-(2\sin x - 1)(\sin x - 3) = 0$$
$$2\sin x - 1 = 0 \quad \text{or} \quad \sin x - 3 = 0$$
$$\sin x = \frac{1}{2} \qquad \sin x = 3$$
$$x = \frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi \qquad \text{No solution}$$

65.
$$\cot^2 x - 9 = 0$$
$$\cot^2 x = 9$$
$$\frac{1}{9} = \tan^2 x$$
$$\pm\frac{1}{3} = \tan x$$
$$x = \arctan\tfrac{1}{3} + n\pi, \arctan\left(-\tfrac{1}{3}\right) + n\pi$$

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66. $\cot^2 x - 6\cot x + 5 = 0$

$(\cot x - 5)(\cot x - 1) = 0$

$\cot x - 5 = 0$ or $\cot x - 1 = 0$

$\cot x = 5$ $\cot x = 1$

$\frac{1}{5} = \tan x$ $1 = \tan x$

$x = \arctan\frac{1}{5} + n\pi$ $x = \frac{\pi}{4} + n\pi$

67. $\sec^2 x - 4\sec x = 0$

$\sec x(\sec x - 4) = 0$

$\sec x = 0$ $\sec x - 4 = 0$

No solution $\sec x = 4$

$\frac{1}{4} = \cos x$

$x = \arccos\frac{1}{4} + 2n\pi, -\arccos\frac{1}{4} + 2n\pi$

68. $\sec^2 x + 2\sec x - 8 = 0$

$(\sec x + 4)(\sec x - 2) = 0$

$\sec x + 4 = 0$ or $\sec x - 2 = 0$

$\sec x = -4$ $\sec x = 2$

$-\frac{1}{4} = \cos x$ $\frac{1}{2} = \cos x$

$x = \arccos\left(-\frac{1}{4}\right) + 2n\pi, -\arccos\left(-\frac{1}{4}\right) + 2n\pi$ $x = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi$

69. $\csc^2 x + 3\csc x - 4 = 0$

$(\csc x + 4)(\csc x - 1) = 0$

$\csc x + 4 = 0$ or $\csc x - 1 = 0$

$\csc x = -4$ $\csc x = 1$

$-\frac{1}{4} = \sin x$ $1 = \sin x$

$x = \arcsin\left(\frac{1}{4}\right) + 2n\pi, \arcsin\left(-\frac{1}{4}\right) + 2n\pi$ $x = \frac{\pi}{2} + 2n\pi$

70. $\csc^2 x - 5\csc x = 0$

$\csc x(\csc x - 5) = 0$

$\csc x = 0$ or $\csc x - 5 = 0$

No solution $\csc x = 5$

$\frac{1}{5} = \sin x$

$x = \arcsin\left(\frac{1}{5}\right) + 2n\pi, \arcsin\left(-\frac{1}{5}\right) + 2n\pi$

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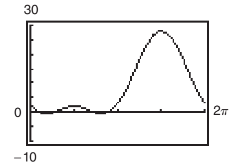
71. $12\sin^2 x - 13\sin x + 3 = 0$

$$\sin x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(12)(3)}}{2(12)} = \frac{13 \pm 5}{24}$$

$\sin x = \frac{1}{3}$ or $\sin x = \frac{3}{4}$

$x \approx 0.3398, 2.8018$ $\quad x \approx 0.8481, 2.2935$



The x-intercepts occur at $x \approx 0.3398$, $x \approx 0.8481$, $x \approx 2.2935$, and $x \approx 2.8018$.

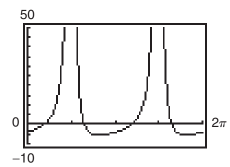
72. $3\tan^2 x + 4\tan x - 4 = 0$

$$\tan x = \frac{-4 \pm \sqrt{4^2 - 4(3)(-4)}}{2(3)} = \frac{-4 \pm \sqrt{64}}{6} = -2, \frac{2}{3}$$

$\tan x = -2$ $\qquad \tan x = \frac{2}{3}$

$x = \arctan(-2) + n\pi$ $\qquad x = \arctan\left(\frac{2}{3}\right) + n\pi$

$\approx -1.1071 + n\pi$ $\qquad \approx 0.5880 + n\pi$



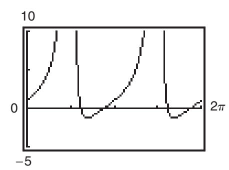
The values of x in $[0, 2\pi)$ are 0.5880, 3.7296, 2.0344, 5.1760.

73. $\tan^2 x + 3\tan x + 1 = 0$

$$\tan x = \frac{-3 \pm \sqrt{3^2 - 4(1)(1)}}{2(1)} = \frac{-3 \pm \sqrt{5}}{2}$$

$\tan x = \frac{-3 - \sqrt{5}}{2}$ or $\tan x = \frac{-3 + \sqrt{5}}{2}$

$x \approx 1.9357, 5.0773$ $\qquad x \approx 2.7767, 5.9183$



The x-intercepts occur at $x \approx 1.9357$, $x \approx 2.7767$, $x \approx 5.0773$, and $x \approx 5.9183$.

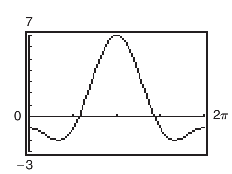
74. $4\cos^2 x - 4\cos x - 1 = 0$

$$\cos x = \frac{4 \pm \sqrt{(-4)^2 - 4(4)(-1)}}{2(4)} = \frac{4 \pm \sqrt{32}}{8} = \frac{1 \pm \sqrt{2}}{2}$$

$\cos x = \frac{1 - \sqrt{2}}{2}$ $\qquad \cos x = \frac{1 + \sqrt{2}}{2}$

$x = \arccos\left(\frac{1 - \sqrt{2}}{2}\right)$ $\qquad$ No solution

≈ 1.7794 $\qquad \left(\frac{1 + \sqrt{2}}{2} > 1\right)$



Solutions in $[0, 2\pi)$ are $\arccos\left(\frac{1 - \sqrt{2}}{2}\right)$ and $2\pi - \arccos\left(\frac{1 - \sqrt{2}}{2}\right)$: 1.7794, 4.5038.

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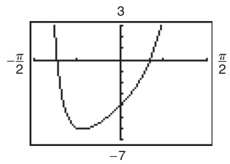
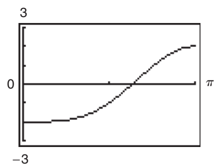
75. $3\tan^2 x + 5\tan x - 4 = 0, \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



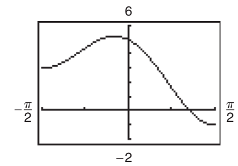
$x \approx -1.154, 0.534$

76. $\cos^2 x - 2\cos x - 1 = 0, [0, \pi]$



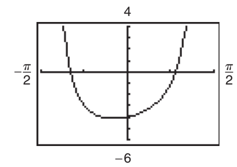
$x \approx 1.998$

77. $4\cos^2 x - 2\sin x + 1 = 0, \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



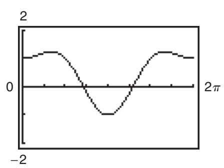
$x \approx 1.110$

78. $2\sec^2 x + \tan x - 6 = 0, \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



$x \approx -1.035, 0.870$

79. (a) $f(x) = \sin^2 x + \cos x$



Maximum: $(1.0472, 1.25)$

Maximum: $(5.2360, 1.25)$

Minimum: $(0, 1)$

Minimum: $(3.1416, -1)$

(b) $2\sin x\cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

$\sin x = 0$ or $2\cos x - 1 = 0$

$x = 0, \pi$

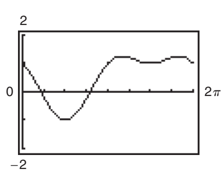
$\approx 0, 3.1416$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

$\approx 1.0472, 5.2360$

80. (a) $f(x) = \cos^2 x - \sin x$



Maximum: $(3.6652, 1.25)$

Maximum: $(5.7596, 1.25)$

Minimum: $(1.5708, -1)$

Minimum: $(4.7124, 1)$

(b) $-2\sin x\cos x - \cos x = 0$

$-\cos x(2\sin x + 1) = 0$

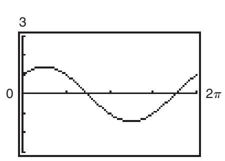
$-\cos x = 0$ $\quad$ $2\sin x + 1 = 0$

$\cos x = 0$ $\quad$ $\sin x = -\frac{1}{2}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad$ $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\approx 1.5708, 4.7124$ $\quad$ $\approx 3.6652, 5.7596$

81. (a) $f(x) = \sin x + \cos x$



Maximum: $(0.7854, 1.4142)$

Minimum: $(3.9270, -1.4142)$

(b) $\cos x - \sin x = 0$

$\cos x = \sin x$

$1 = \frac{\sin x}{\cos x}$

$\tan x = 1$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

$\approx 0.7854, 3.9270$

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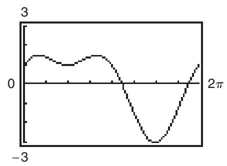
82. (a) $f(x) = 2\sin x + \cos 2x$



Maximum: $(0.5236, 1.5)$

Maximum: $(2.6180, 1.5)$

Minimum: $(1.5708, 1.0)$

Minimum: $(4.7124, -3.0)$

(b) $2\cos x - 4\sin x\cos x = 0$

$2\cos x(1 - 2\sin x) = 0$

$2\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\approx 1.5708, 4.7124$

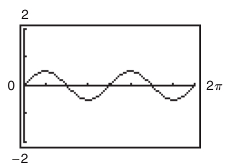
$1 - 2\sin x = 0$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\approx 0.5236, 2.6180$

83. (a) $f(x) = \sin x\cos x$



Maximum: $(0.7854, 0.5)$

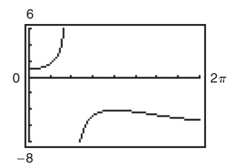
Maximum: $(3.9270, 0.5)$

Minimum: $(2.3562, -0.5)$

Minimum: $(5.4978, -0.5)$

(b)
$$-\sin^2 x + \cos^2 x = 0$$
$$-\sin^2 x + 1 - \sin^2 x = 0$$
$$-2\sin^2 x + 1 = 0$$
$$\sin^2 x = \frac{1}{2}$$
$$\sin x = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$$
$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$
$$\approx 0.7854, 2.3562, 3.9270, 5.4978$$

84. (a) $f(x) = \sec x + \tan x - x$



Maximum: $(3.1416, -4.1416)$

Minimum: $(0, 1)$

(b)
$$\sec x\tan x + \sec^2 x - 1 = 0$$
$$\frac{1}{\cos x}\cdot\frac{\sin x}{\cos x} + \frac{1}{\cos^2 x} - 1 = 0$$
$$\frac{\sin x + 1}{\cos^2 x} - 1 = 0$$
$$\frac{\sin x + 1}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x} = 0$$
$$\frac{\sin x + 1 - \cos^2 x}{\cos^2 x} = 0$$
$$\frac{\sin x + \sin^2 x}{\cos^2 x} = 0$$
$$\sin x + \sin^2 x = 0$$
$$\sin x(1 + \sin x) = 0$$

$\sin x = 0$ or $1 + \sin x = 0$

$x = 0, \pi$ $\quad$ $\sin x = -1$

$\approx 0, 3.1416$ $\quad$ $x = \frac{3\pi}{2}$

$\frac{3\pi}{2}$ is undefined in original function. So, it is not a solution.

85. The graphs of $y_1 = 2\sin x$ and $y_2 = 3x + 1$ appear to have one point of intersection. This implies there is one solution to the equation $2\sin x = 3x + 1$.

86. The graphs of $y_1 = 2\sin x$ and $y_2 = \frac{1}{2}x + 1$ appear to have three points of intersection. This implies there are three solutions to the equation $2\sin x = \frac{1}{2}x + 1$.

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87. $f(x) = \frac{\sin x}{x}$

(a) Domain: all real numbers except $x = 0$.

(b) The graph has y-axis symmetry and a horizontal asymptote at $y = 0$.

(c) As $x \to 0, f(x) \to 1$.

(d) $\frac{\sin x}{x} = 0$ has four solutions in the interval $[-8, 8]$.

$$\sin x\left(\frac{1}{x}\right) = 0$$
$$\sin x = 0$$
$$x = -2\pi, -\pi, \pi, 2\pi$$

88. $f(x) = \cos \frac{1}{x}$

(a) Domain: all real numbers x except $x = 0$.

(b) The graph has y-axis symmetry and a horizontal asymptote at $y = 1$.

(c) As $x \to 0, f(x)$ oscillates between -1 and 1.

(d) There are infinitely many solutions in the interval $[-1, 1]$. They occur at $x = \frac{2}{(2n + 1)\pi}$ where n is any integer.

(e) The greatest solution appears to occur at $x \approx 0.6366$.

89. $$y = \frac{1}{12}(\cos 8t - 3 \sin 8t)$$

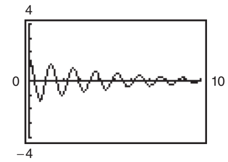
$$\frac{1}{12}(\cos 8t - 3 \sin 8t) = 0$$
$$\cos 8t = 3 \sin 8t$$
$$\frac{1}{3} = \tan 8t$$
$$8t \approx 0.32175 + n\pi$$
$$t \approx 0.04 + \frac{n\pi}{8}$$

In the interval $0 \le t \le 1, t \approx 0.04, 0.43,$ and 0.83.

90. $y_1 = 1.56e^{-0.22t} \cos 4.9t$

Right-most point of intersection: $(1.96, -1)$

The displacement does not exceed one foot from equilibrium after $t = 1.96$ seconds.



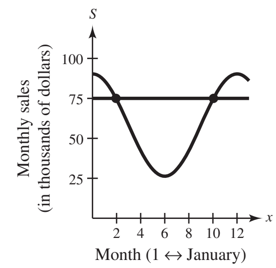
91. Graph $y_1 = 58.3 + 32 \cos\left(\frac{\pi t}{6}\right)$

$y_2 = 75$.

Left point of intersection: $(1.95, 75)$

Right point of intersection: $(10.05, 75)$

So, sales exceed 7500 in January, November, and December.



92. $$\text{Range} = 300 \text{ feet}$$
$$v_0 = 100 \text{ feet per second}$$
$$r = \tfrac{1}{32}v_0^2 \sin 2\theta$$

$$\tfrac{1}{32}(100)^2 \sin 2\theta = 300$$
$$\sin 2\theta = 0.96$$
$$2\theta = \arcsin(0.96) \approx 73.74°$$
$$\theta \approx 36.9°$$

or

$$2\theta = 180° - \arcsin(0.96) \approx 106.26°$$
$$\theta \approx 53.1°$$

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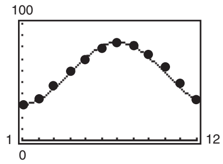
93. (a) and (c)



The model fits the data well.

(b) $C = a\cos(bt - c) + d$

$a = \frac{1}{2}[\text{high} - \text{low}] = \frac{1}{2}[84.1 - 31.0] = 26.55$

$p = 2[\text{high time} - \text{low time}] = 2[7 - 1] = 12$

$b = \frac{2\pi}{p} = \frac{2\pi}{12} = \frac{\pi}{6}$

The maximum occurs at 7, so the left end point is

$\frac{c}{b} = 7 \Rightarrow c = 7\left(\frac{\pi}{6}\right) = \frac{7\pi}{6}$

$d = \frac{1}{2}[\text{high} + \text{low}] = \frac{1}{2}[93.6 + 62.3] = 57.55$

$C = 26.55\cos\left(\frac{\pi}{6}t - \frac{7\pi}{6}\right) + 57.55$

(d) The constant term, *d*, gives the average maximum temperature.

The average maximum temperature in Chicago is 57.55°F.

(e) The average maximum temperature is above 72°F from June through September. The average maximum temperature is below 70°F from October through May.

94. $h(t) = 53 + 50\sin\left(\frac{\pi}{16}t - \frac{\pi}{2}\right)$

(a) $h(t) = 53$ when $50\sin\left(\frac{\pi}{16}t - \frac{\pi}{2}\right) = 0.$

$\frac{\pi}{16}t - \frac{\pi}{2} = 0$ or $\frac{\pi}{16}t - \frac{\pi}{2} = \pi$

$\frac{\pi}{16}t = \frac{\pi}{2}$ $\qquad \frac{\pi}{16}t = \frac{3\pi}{2}$

$t = 8$ $\qquad t = 24$

A person on the Ferris wheel will be 53 feet above ground at 8 seconds and at 24 seconds

(b) The person will be at the top of the Ferris wheel when

$\sin\left(\frac{\pi}{16}t - \frac{\pi}{2}\right) = 1$

$\frac{\pi}{16}t - \frac{\pi}{2} = \frac{\pi}{2}$

$\frac{\pi}{16}t = \pi$

$t = 16.$

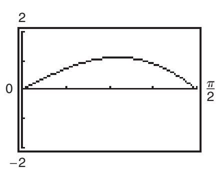
The first time this occurs is after 16 seconds.

The period of this function is $\frac{2\pi}{\pi/16} = 32.$

During 160 seconds, 5 cycles will take place and the person will be at the top of the ride 5 times, spaced 32 seconds apart. The times are: 16 seconds, 48 seconds, 80 seconds, 112 seconds, and 144 seconds.

95. $A = 2x\cos x, 0 < x < \frac{\pi}{2}$

(a)



The maximum area of $A \approx 1.12$ occurs when $x \approx 0.86.$

(b) $A \geq 1$ for $0.6 < x < 1.1$

96. $f(x) = 3\sin(0.6x - 2)$

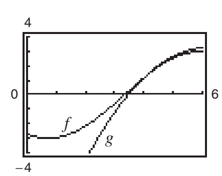
(a) Zero: $\sin(0.6x - 2) = 0$

$0.6x - 2 = 0$

$0.6x = 2$

$x = \frac{2}{0.6} = \frac{10}{3}$

(b) $g(x) = -0.45x^2 + 5.52x - 13.70$



For $3.5 \leq x \leq 6$ the approximation appears to be good.

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(c) $-0.45x^2 + 5.52x - 13.70 = 0$

$$x = \frac{-5.52 \pm \sqrt{(5.52)^2 - 4(-0.45)(-13.70)}}{2(-0.45)}$$

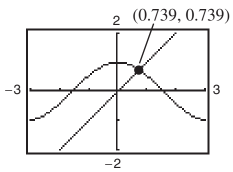
$$x \approx 3.46, 8.81$$

The zero of g on $[0, 6]$ is 3.46. The zero is close to the zero $\frac{10}{3} \approx 3.33$ of f.

97. $f(x) = \tan \frac{\pi x}{4}$

Because $\tan \pi/4 = 1$, $x = 1$ is the smallest nonnegative fixed point.

98. Graph $y = \cos x$ and $y = x$ on the same set of axes. Their point of intersection gives the value of c such that $f(c) = c \Rightarrow \cos c = c$.



$c \approx 0.739$

99. True. The period of $2 \sin 4t - 1$ is $\frac{\pi}{2}$ and the period of $2 \sin t - 1$ is 2π.

In the interval $[0, 2\pi)$ the first equation has four cycles whereas the second equation has only one cycle, so the first equation has four times the x-intercepts (solutions) as the second equation.

100. False.

$\sin x = 3.4$ has no solution because 3.4 is outside the range of sine.

101. $\cot x \cos^2 x = 2 \cot x$

$\cos^2 x = 2$

$\cos x = \pm\sqrt{2}$

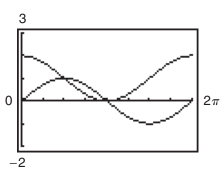
No solution

Because you solved this problem by first dividing by $\cot x$, you do not get the same solution as Example 3.

When solving equations, you do not want to divide each side by a variable expression that will cancel out because you may accidentally remove one of the solutions.

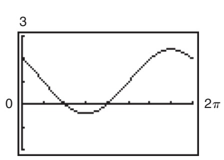
102. The equation $2 \cos x - 1 = 0$ is equivalent to $\cos x = \frac{1}{2}$. So, the points of intersection of $y = \cos x$ and $y = \frac{1}{2}$ represent the solutions of the equation $2 \cos x - 1 = 0$. In the interval $(-2\pi, 2\pi)$ the solutions of the equation are $x = -\frac{5\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}$, and $\frac{5\pi}{3}$.

103. (a)



The graphs intersect when $x = \frac{\pi}{2}$ and $x = \pi$.

(b)



The x-intercepts are $\left(\frac{\pi}{2}, 0\right)$ and $(\pi, 0)$.

(c) Both methods produce the same x-values. Answers will vary on which method is preferred.

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Section 5.4 Sum and Difference Formulas

1. $\sin u \cos v - \cos u \sin v$

2. $\cos u \cos v - \sin u \sin v$

3. $\dfrac{\tan u + \tan v}{1 - \tan u \tan v}$

4. $\sin u \cos v + \cos u \sin v$

5. $\cos u \cos v + \sin u \sin v$

6. $\dfrac{\tan u - \tan v}{1 + \tan u \tan v}$

7. (a) $\cos\left(\dfrac{\pi}{4} + \dfrac{\pi}{3}\right) = \cos\dfrac{\pi}{4}\cos\dfrac{\pi}{3} - \sin\dfrac{\pi}{4}\sin\dfrac{\pi}{3}$

$= \dfrac{\sqrt{2}}{2}\cdot\dfrac{1}{2} - \dfrac{\sqrt{2}}{2}\cdot\dfrac{\sqrt{3}}{2}$

$= \dfrac{\sqrt{2} - \sqrt{6}}{4}$

(b) $\cos\dfrac{\pi}{4} + \cos\dfrac{\pi}{3} = \dfrac{\sqrt{2}}{2} + \dfrac{1}{2} = \dfrac{\sqrt{2} + 1}{2}$

8. (a) $\sin\left(\dfrac{7\pi}{6} - \dfrac{\pi}{3}\right) = \sin\dfrac{5\pi}{6} = \sin\dfrac{\pi}{6} = \dfrac{1}{2}$

(b) $\sin\dfrac{7\pi}{6} - \sin\dfrac{\pi}{3} = -\dfrac{1}{2} - \dfrac{\sqrt{3}}{2} = \dfrac{-1 - \sqrt{3}}{2}$

9. (a) $\sin(135° - 30°) = \sin 135° \cos 30° - \cos 135° \sin 30°$

$= \left(\dfrac{\sqrt{2}}{2}\right)\left(\dfrac{\sqrt{3}}{2}\right) - \left(-\dfrac{\sqrt{2}}{2}\right)\left(\dfrac{1}{2}\right) = \dfrac{\sqrt{6} + \sqrt{2}}{4}$

(b) $\sin 135° - \cos 30° = \dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{3}}{2} = \dfrac{\sqrt{2} - \sqrt{3}}{2}$

10. (a) $\cos(120° + 45°) = \cos 120° \cos 45° - \sin 120° \sin 45°$

$= \left(-\dfrac{1}{2}\right)\left(\dfrac{\sqrt{2}}{2}\right) - \left(\dfrac{\sqrt{3}}{2}\right)\left(\dfrac{\sqrt{2}}{2}\right) = \dfrac{-\sqrt{2} - \sqrt{6}}{4}$

(b) $\cos 120° + \cos 45° = -\dfrac{1}{2} + \dfrac{\sqrt{2}}{2} = \dfrac{-1 + \sqrt{2}}{2}$

11. $\sin\dfrac{11\pi}{12} = \sin\left(\dfrac{3\pi}{4} + \dfrac{\pi}{6}\right)$

$= \sin\dfrac{3\pi}{4}\cos\dfrac{\pi}{6} + \cos\dfrac{3\pi}{4}\sin\dfrac{\pi}{6}$

$= \dfrac{\sqrt{2}}{2}\cdot\dfrac{\sqrt{3}}{2} + \left(-\dfrac{\sqrt{2}}{2}\right)\dfrac{1}{2}$

$= \dfrac{\sqrt{2}}{4}\left(\sqrt{3} - 1\right)$

$\cos\dfrac{11\pi}{12} = \cos\left(\dfrac{3\pi}{4} + \dfrac{\pi}{6}\right)$

$= \cos\dfrac{3\pi}{4}\cos\dfrac{\pi}{6} - \sin\dfrac{3\pi}{4}\sin\dfrac{\pi}{6}$

$= -\dfrac{\sqrt{2}}{2}\cdot\dfrac{\sqrt{3}}{2} - \dfrac{\sqrt{2}}{2}\cdot\dfrac{1}{2} = -\dfrac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$

$\tan\dfrac{11\pi}{4} = \tan\left(\dfrac{3\pi}{4} + \dfrac{\pi}{6}\right)$

$= \dfrac{\tan\dfrac{3\pi}{4} + \tan\dfrac{\pi}{6}}{1 - \tan\dfrac{3\pi}{4}\tan\dfrac{\pi}{6}}$

$= \dfrac{-1 + \dfrac{\sqrt{3}}{3}}{1 - (-1)\dfrac{\sqrt{3}}{3}}$

$= \dfrac{-3 + \sqrt{3}}{3 + \sqrt{3}}\cdot\dfrac{3 - \sqrt{3}}{3 - \sqrt{3}}$

$= \dfrac{-12 + 6\sqrt{3}}{6} = -2 + \sqrt{3}$

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12. $\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$

$$\sin\frac{7\pi}{12} = \sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \sin\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\frac{\pi}{3} = \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$$

$$\cos\frac{7\pi}{12} = \cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \cos\frac{\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{3}\sin\frac{\pi}{4} = \frac{1}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4}\left(1 - \sqrt{3}\right)$$

$$\tan\frac{7\pi}{12} = \tan\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \frac{\tan\frac{\pi}{3} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{3}\tan\frac{\pi}{4}} = \frac{\sqrt{3} + 1}{1 - \sqrt{3}} = -2 - \sqrt{3}$$

13. $\sin\frac{17\pi}{12} = \sin\left(\frac{9\pi}{4} - \frac{5\pi}{6}\right)$

$$= \sin\frac{9\pi}{4}\cos\frac{5\pi}{6} - \cos\frac{9\pi}{4}\sin\frac{5\pi}{6} = \frac{\sqrt{2}}{2}\left(-\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = -\frac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$$

$$\cos\frac{17\pi}{12} = \cos\left(\frac{9\pi}{4} - \frac{5\pi}{6}\right) = \cos\frac{9\pi}{4}\cos\frac{5\pi}{6} + \sin\frac{9\pi}{4}\sin\frac{5\pi}{6} = \frac{\sqrt{2}}{2}\left(-\frac{\sqrt{3}}{2}\right) + \frac{\sqrt{2}}{2}\left(\frac{1}{2}\right) = \frac{\sqrt{2}}{4}\left(1 - \sqrt{3}\right)$$

$$\tan\frac{17\pi}{12} = \tan\left(\frac{9\pi}{4} - \frac{5\pi}{6}\right) = \frac{\tan(9\pi/4) - \tan(5\pi/6)}{1 + \tan(9\pi/4)\tan(5\pi/6)} = \frac{1 - \left(-\sqrt{3}/3\right)}{1 + \left(-\sqrt{3}/3\right)} = \frac{3 + \sqrt{3}}{3 - \sqrt{3}}\cdot\frac{3 + \sqrt{3}}{3 + \sqrt{3}} = \frac{12 + 6\sqrt{3}}{6} = 2 + \sqrt{3}$$

14. $-\frac{\pi}{12} = \frac{\pi}{6} - \frac{\pi}{4}$

$$\sin\left(-\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{6} - \frac{\pi}{4}\right) = \sin\frac{\pi}{6}\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\cos\frac{\pi}{6} = \frac{1}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{2}}{4}\left(1 - \sqrt{3}\right)$$

$$\cos\left(-\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{6} - \frac{\pi}{4}\right) = \cos\frac{\pi}{6}\cos\frac{\pi}{4} + \sin\frac{\pi}{6}\sin\frac{\pi}{4} = \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$$

$$\tan\left(-\frac{\pi}{12}\right) = \tan\left(\frac{\pi}{6} - \frac{\pi}{4}\right) = \frac{\tan\frac{\pi}{6} - \tan\frac{\pi}{4}}{1 + \tan\frac{\pi}{6}\tan\frac{\pi}{4}} = \frac{\frac{\sqrt{3}}{3} - 1}{1 + \frac{\sqrt{3}}{3}} = -2 + \sqrt{3}$$

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15. $\sin 105° = \sin(60° + 45°)$

$= \sin 60° \cos 45° + \cos 60° \sin 45°$

$= \dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{2}}{2} + \dfrac{1}{2} \cdot \dfrac{\sqrt{2}}{2}$

$= \dfrac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$

$\cos 105° = \cos(60° + 45°)$

$= \cos 60° \cos 45° - \sin 60° \sin 45°$

$= \dfrac{1}{2} \cdot \dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{2}}{2}$

$= \dfrac{\sqrt{2}}{4}\left(1 - \sqrt{3}\right)$

$\tan 105° = \tan(60° + 45°)$

$= \dfrac{\tan 60° + \tan 45°}{1 - \tan 60° \tan 45°}$

$= \dfrac{\sqrt{3} + 1}{1 - \sqrt{3}} = \dfrac{\sqrt{3} + 1}{1 - \sqrt{3}} \cdot \dfrac{1 + \sqrt{3}}{1 + \sqrt{3}}$

$= \dfrac{4 + 2\sqrt{3}}{-2} = -2 - \sqrt{3}$

16. $165° = 135° + 30°$

$\sin 165° = \sin(135° + 30°)$

$= \sin 135° \cos 30° + \sin 30° \cos 135°$

$= \sin 45° \cos 30° - \sin 30° \cos 45°$

$= \dfrac{\sqrt{2}}{2} \cdot \dfrac{\sqrt{3}}{2} - \dfrac{1}{2} \cdot \dfrac{\sqrt{2}}{2}$

$= \dfrac{\sqrt{2}}{4}\left(\sqrt{3} - 1\right)$

$\cos 165° = \cos(135° + 30°)$

$= \cos 135° \cos 30° - \sin 135° \sin 30°$

$= -\cos 45° \cos 30° - \sin 45° \cos 30°$

$= -\dfrac{\sqrt{2}}{2} \cdot \dfrac{\sqrt{3}}{2} - \dfrac{\sqrt{2}}{2} \cdot \dfrac{1}{2}$

$= -\dfrac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$

$\tan 165° = \tan(135° + 30°)$

$= \dfrac{\tan 135° + \tan 30°}{1 - \tan 135° \tan 30°}$

$= \dfrac{-\tan 45° + \tan 30°}{1 + \tan 45° \tan 30°}$

$= \dfrac{-1 + \dfrac{\sqrt{3}}{3}}{1 + \dfrac{\sqrt{3}}{3}}$

$= -2 + \sqrt{3}$

17. $\sin(-195°) = \sin(30° - 225°)$

$= \sin 30° \cos 225° - \cos 30° \sin 225°$

$= \sin 30°(-\cos 45°) - \cos 30°(-\sin 45°)$

$= \dfrac{1}{2}\left(-\dfrac{\sqrt{2}}{2}\right) - \dfrac{\sqrt{3}}{2}\left(-\dfrac{\sqrt{2}}{2}\right)$

$= -\dfrac{\sqrt{2}}{4}\left(1 - \sqrt{3}\right)$

$= \dfrac{\sqrt{2}}{4}\left(\sqrt{3} - 1\right)$

$\cos(-195°) = \cos(30° - 225°)$

$= \cos 30° \cos 225° + \sin 30° \sin 225°$

$= \cos 30°(-\cos 45°) + \sin 30°(-45°)$

$= \dfrac{\sqrt{3}}{2}\left(-\dfrac{\sqrt{2}}{2}\right) + \dfrac{1}{2}\left(-\dfrac{\sqrt{2}}{2}\right)$

$= -\dfrac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$

$\tan(-195°) = \tan(30° - 225°)$

$= \dfrac{\tan 30° - \tan 225°}{1 + \tan 30° \tan 225°}$

$= \dfrac{\tan 30° - \tan 45°}{1 + \tan 30° \tan 45°}$

$= \dfrac{\left(\dfrac{\sqrt{3}}{3}\right) - 1}{1 + \left(\dfrac{\sqrt{3}}{3}\right)} = \dfrac{\sqrt{3} - 3}{3 + \sqrt{3}} \cdot \dfrac{3 - \sqrt{3}}{3 - \sqrt{3}}$

$= \dfrac{-12 + 6\sqrt{3}}{6} = -2 + \sqrt{3}$

18. $225° = 300° - 45°$

$\sin 255° = \sin(300° - 45°)$

$= \sin 300° \cos 45° - \sin 45° \cos 300°$

$= -\sin 60° \cos 45° - \sin 45° \cos 60°$

$= -\dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{2}}{2} \cdot \dfrac{1}{2} = -\dfrac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$

$\cos 255° = \cos(300° - 45°)$

$= \cos 300° \cos 45° + \sin 300° \sin 45°$

$= \cos 60° \cos 45° - \sin 60° \sin 45°$

$= \dfrac{1}{2} \cdot \dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{2}}{2} = \dfrac{\sqrt{2}}{4}\left(1 - \sqrt{3}\right)$

$\tan 255° = \tan(300° - 45°)$

$= \dfrac{\tan 300° - \tan 45°}{1 + \tan 300° \tan 45°}$

$= \dfrac{-\tan 60° - \tan 45°}{1 - \tan 60° \tan 45°}$

$= \dfrac{-\sqrt{3} - 1}{1 - \sqrt{3}} = 2 + \sqrt{3}$

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19. $\dfrac{13\pi}{12} = \dfrac{3\pi}{4} + \dfrac{\pi}{3}$

$$\sin\frac{13\pi}{12} = \sin\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)$$
$$= \sin\frac{3\pi}{4}\cos\frac{\pi}{3} + \cos\frac{3\pi}{4}\sin\frac{\pi}{3}$$
$$= \frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$
$$= \frac{\sqrt{2}}{4}\left(1 - \sqrt{3}\right)$$

$$\cos\frac{13\pi}{12} = \cos\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)$$
$$= \cos\frac{3\pi}{4}\cos\frac{\pi}{3} - \sin\frac{3\pi}{4}\sin\frac{\pi}{3}$$
$$= -\frac{\sqrt{2}}{2}\cdot\frac{1}{2} - \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} = -\frac{\sqrt{2}}{4}\left(1 + \sqrt{3}\right)$$

$$\tan\frac{13\pi}{12} = \tan\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)$$
$$= \frac{\tan\left(\frac{3\pi}{4}\right) + \tan\left(\frac{\pi}{3}\right)}{1 - \tan\left(\frac{3\pi}{4}\right)\tan\left(\frac{\pi}{3}\right)}$$
$$= \frac{-1 + \sqrt{3}}{1 - (-1)\left(\sqrt{3}\right)}$$
$$= -\frac{1 - \sqrt{3}}{1 + \sqrt{3}}\cdot\frac{1 - \sqrt{3}}{1 - \sqrt{3}}$$
$$= -\frac{4 - 2\sqrt{3}}{-2}$$
$$= 2 - \sqrt{3}$$

20. $\dfrac{19\pi}{12} = \dfrac{\pi}{3} + \dfrac{5\pi}{4}$

$$\sin\frac{19\pi}{12} = \sin\left(\frac{\pi}{3} + \frac{5\pi}{4}\right)$$
$$= \sin\frac{\pi}{3}\cos\frac{5\pi}{4} + \sin\frac{5\pi}{4}\cos\frac{\pi}{3}$$
$$= \frac{\sqrt{3}}{2}\left(-\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\cdot\frac{1}{2}$$
$$= -\frac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$$

$$\cos\frac{19\pi}{12} = \cos\left(\frac{\pi}{3} + \frac{5\pi}{4}\right)$$
$$= \cos\frac{\pi}{3}\cos\frac{5\pi}{4} - \sin\frac{\pi}{3}\sin\frac{5\pi}{4}$$
$$= \frac{1}{2}\left(-\frac{\sqrt{2}}{2}\right) - \frac{\sqrt{3}}{2}\left(-\frac{\sqrt{2}}{2}\right)$$
$$= \frac{\sqrt{2}}{4}\left(-1 + \sqrt{3}\right)$$

$$\tan\frac{19\pi}{12} = \tan\left(\frac{\pi}{3} + \frac{5\pi}{4}\right)$$
$$= \frac{\tan\frac{\pi}{3} + \tan\frac{5\pi}{4}}{1 - \tan\frac{\pi}{3}\tan\frac{5\pi}{4}}$$
$$= \frac{\tan\frac{\pi}{3} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{3}\tan\frac{\pi}{4}}$$
$$= \frac{\sqrt{3} + 1}{1 - \sqrt{3}}\cdot\frac{1 + \sqrt{3}}{1 + \sqrt{3}}$$
$$= \frac{4 + 2\sqrt{3}}{-2} = -2 - \sqrt{3}$$

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21. $-\dfrac{5\pi}{12} = -\dfrac{\pi}{4} - \dfrac{\pi}{6}$

$$\sin\left(-\frac{\pi}{4} - \frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{4}\right)\cos\frac{\pi}{6} - \cos\left(-\frac{\pi}{4}\right)\sin\frac{\pi}{6}$$

$$= \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = -\frac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$$

$$\cos\left(-\frac{\pi}{4} - \frac{\pi}{6}\right) = \cos\left(-\frac{\pi}{4}\right)\cos\frac{\pi}{6} + \sin\left(-\frac{\pi}{4}\right)\sin\frac{\pi}{6}$$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{2}}{4}\left(\sqrt{3} - 1\right)$$

$$\tan\left(-\frac{\pi}{4} - \frac{\pi}{6}\right) = \frac{\tan\left(-\frac{\pi}{4}\right) - \tan\frac{\pi}{6}}{1 + \tan\left(-\frac{\pi}{4}\right)\tan\frac{\pi}{6}}$$

$$= \frac{-1 - \frac{\sqrt{3}}{3}}{1 + (-1)\left(\frac{\sqrt{3}}{3}\right)} = \frac{-3 - \sqrt{3}}{3 - \sqrt{3}}$$

$$= \frac{-3 - \sqrt{3}}{3 - \sqrt{3}} \cdot \frac{3 + \sqrt{3}}{3 + \sqrt{3}}$$

$$= \frac{-12 - 6\sqrt{3}}{6} = -2 - \sqrt{3}$$

22. $-\dfrac{7\pi}{12} = -\dfrac{\pi}{3} - \dfrac{\pi}{4}$

$$\sin\left(-\frac{7\pi}{12}\right) = \sin\left(-\frac{\pi}{3} - \frac{\pi}{4}\right) = \sin\left(-\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \cos\left(-\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$$

$$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = -\frac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$$

$$\cos\left(-\frac{7\pi}{12}\right) = \cos\left(-\frac{\pi}{3} - \frac{\pi}{4}\right) = \cos\left(-\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \sin\left(-\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$$

$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{4}\left(1 - \sqrt{3}\right)$$

$$\tan\left(-\frac{7\pi}{12}\right) = \tan\left(-\frac{\pi}{3} - \frac{\pi}{4}\right) = \frac{\tan\left(-\frac{\pi}{3}\right) - \tan\left(\frac{\pi}{4}\right)}{1 + \tan\left(-\frac{\pi}{3}\right)\tan\left(\frac{\pi}{4}\right)} = \frac{-\sqrt{3} - 1}{1 + \left(-\sqrt{3}\right)(1)} = 2 + \sqrt{3}$$

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23. $285° = 225° + 60°$

$\sin 285° = \sin(225° + 60°) = \sin 225° \cos 60° + \cos 225° \sin 60°$

$$= -\frac{\sqrt{2}}{2}\left(\frac{1}{2}\right) - \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$$

$\cos 285° = \cos(225° + 60°) = \cos 225° \cos 60° - \sin 225° \sin 60°$

$$= -\frac{\sqrt{2}}{2}\left(\frac{1}{2}\right) - \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{2}}{4}\left(\sqrt{3} - 1\right)$$

$\tan 285° = \tan(225° + 60°) = \dfrac{\tan 225° + \tan 60°}{1 - \tan 225° \tan 60°}$

$$= \frac{1 + \sqrt{3}}{1 - \sqrt{3}} \cdot \frac{1 + \sqrt{3}}{1 + \sqrt{3}} = \frac{4 + 2\sqrt{3}}{-2} = -2 - \sqrt{3} = -\left(2 + \sqrt{3}\right)$$

24. $15° = 45° - 30°$

$\sin 15° = \sin(45° - 30°) = \sin 45° \cos 30° - \cos 45° \sin 30°$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{2}\left(\sqrt{3} - 1\right)}{4} = \frac{\sqrt{2}}{4}\left(\sqrt{3} - 1\right)$$

$\cos 15° = \cos(45° - 30°) = \cos 45° \cos 30° + \sin 45° \sin 30°$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{2}\left(\sqrt{3} + 1\right)}{4} = \frac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$$

$\tan 15° = \tan(45° - 30°) = \dfrac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$

$$= \frac{1 - \frac{\sqrt{3}}{3}}{1 + (1)\left(\frac{\sqrt{3}}{3}\right)} = \frac{\frac{3 - \sqrt{3}}{3}}{\frac{3 + \sqrt{3}}{3}} = \frac{3 - \sqrt{3}}{3 + \sqrt{3}} \cdot \frac{3 - \sqrt{3}}{3 - \sqrt{3}} = \frac{12 - 6\sqrt{3}}{6} = 2 - \sqrt{3}$$

25. $-165° = -(120° + 45°)$

$\sin(-165°) = \sin\left[-(120° + 45°)\right] = -\sin(120° + 45°) = -\left[\sin 120° \cos 45° + \cos 120° \sin 45°\right]$

$$= -\left[\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}\right] = -\frac{\sqrt{2}}{4}\left(\sqrt{3} - 1\right)$$

$\cos(-165°) = \cos\left[-(120° + 45°)\right] = \cos(120° + 45°) = \cos 120° \cos 45° - \sin 120° \sin 45°$

$$= -\frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{4}\left(1 + \sqrt{3}\right)$$

$\tan(-165°) = \tan\left[-(120° + 45°)\right] = -\tan(120° + \tan 45°) = -\dfrac{\tan 120° + \tan 45°}{1 - \tan 120° \tan 45°}$

$$= -\frac{-\sqrt{3} + 1}{1 - \left(-\sqrt{3}\right)(1)} = -\frac{1 - \sqrt{3}}{1 + \sqrt{3}} \cdot \frac{1 - \sqrt{3}}{1 - \sqrt{3}} = -\frac{4 - 2\sqrt{3}}{-2} = 2 - \sqrt{3}$$

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26. $-105 = 30° - 135°$

$$\sin(30° - 135°) = \sin 30° \cos 135° - \cos 30° \sin 135° = \sin 30°(-\cos 45°) - \cos 30° \sin 45°$$
$$= \left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = -\frac{\sqrt{2}}{4}\left(1 + \sqrt{3}\right)$$

$$\cos(30° - 135°) = \cos 30° \cos 135° + \sin 30° \sin 135° = \cos 30°(-\cos 45°) + \sin 30° \sin 45°$$
$$= \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{4}\left(1 - \sqrt{3}\right)$$

$$\tan(30° - 135°) = \frac{\tan 30° - \tan 135°}{1 + \tan 30° \tan 135°} = \frac{\tan 30° - (-\tan 45°)}{1 + \tan 30°(-\tan 45°)}$$
$$= \frac{\frac{\sqrt{3}}{3} - (-1)}{1 + \left(\frac{\sqrt{3}}{3}\right)(-1)} = 2 + \sqrt{3}$$

27. $\sin 3 \cos 1.2 - \cos 3 \sin 1.2 = \sin(3 - 1.2) = \sin 1.8$

28. $\cos\frac{\pi}{7}\cos\frac{\pi}{5} - \sin\frac{\pi}{7}\sin\frac{\pi}{5} = \cos\left(\frac{\pi}{7} + \frac{\pi}{5}\right)$

$= \cos\frac{12\pi}{35}$

29. $\sin 60° \cos 15° + \cos 60° \sin 15° = \sin(60° + 15°)$

$= \sin 75°$

30. $\cos 130° \cos 40° - \sin 130° \sin 40° = \cos(130° + 40°)$

$= \cos 170°$

31. $\dfrac{\tan(\pi/15) + \tan(2\pi/5)}{1 - \tan(\pi/15)\tan(2\pi/5)} = \tan(\pi/15 + 2\pi/5)$

$= \tan(7\pi/15)$

32. $\dfrac{\tan 1.1 - \tan 4.6}{1 + \tan 1.1 \tan 4.6} = \tan(1.1 - 4.6) = \tan(-3.5)$

33. $\cos 3x \cos 2y + \sin 3x \sin 2y = \cos(3x - 2y)$

34. $\sin x \cos 2x + \cos x \sin 2x = \sin(x + 2x) = \sin(3x)$

35. $\sin\frac{\pi}{12}\cos\frac{\pi}{4} + \cos\frac{\pi}{12}\sin\frac{\pi}{4} = \sin\left(\frac{\pi}{12} + \frac{\pi}{4}\right)$

$= \sin\frac{\pi}{3}$

$= \frac{\sqrt{3}}{2}$

36. $\cos\frac{\pi}{16}\cos\frac{3\pi}{16} - \sin\frac{\pi}{16}\sin\frac{3\pi}{16} = \cos\left(\frac{\pi}{16} + \frac{3\pi}{16}\right)$

$= \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

37. $\cos 130° \cos 10° + \sin 130° \sin 10° = \cos(130° - 10°)$

$= \cos 120°$

$= -\frac{1}{2}$

38. $\sin 100° \cos 40° - \cos 100° \sin 40° = \sin(100° - 40°)$

$= \sin 60°$

$= \frac{\sqrt{3}}{2}$

39. $\dfrac{\tan(9\pi/8) - \tan(\pi/8)}{1 + \tan(9\pi/8)\tan(\pi/8)} = \tan\left(\frac{9\pi}{8} - \frac{\pi}{8}\right)$

$= \tan\pi$

$= 0$

40. $\dfrac{\tan 25° + \tan 110°}{1 - \tan 25° \tan 110°} = \tan(25° + 110°)$

$= \tan 135°$

$= -1$

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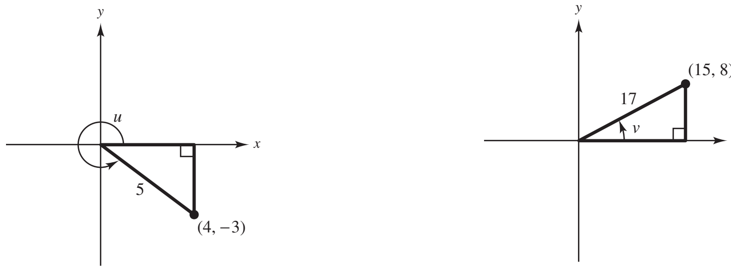
For Exercises 41–46, you have:

$\sin u = -\frac{3}{5}$, u in Quadrant IV $\Rightarrow \cos u = \frac{4}{5}$, $\tan u = -\frac{4}{3}$

$\cos v = \frac{15}{17}$, v in Quadrant I $\Rightarrow \sin v = \frac{8}{17}$, $\tan v = \frac{8}{15}$



Figures for Exercises 41–46

41. $$\sin(u+v) = \sin u \cos v + \cos u \sin v = \left(-\frac{3}{5}\right)\left(\frac{15}{17}\right) + \left(\frac{4}{5}\right)\left(\frac{8}{17}\right) = -\frac{13}{85}$$

42. $$\cos(u-v) = \cos u \cos v + \sin u \sin v = \left(\frac{4}{5}\right)\left(\frac{15}{17}\right) + \left(-\frac{3}{5}\right)\left(\frac{8}{17}\right) = \frac{60}{85} + \frac{-24}{85} = \frac{36}{85}$$

43. $$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} = \frac{-\frac{3}{4} + \left(\frac{8}{15}\right)}{1 - \left(-\frac{3}{4}\right)\left(\frac{8}{15}\right)} = \frac{-\frac{13}{60}}{1 + \frac{32}{60}} = \left(-\frac{13}{60}\right)\left(\frac{5}{7}\right) = -\frac{13}{84}$$

44. $$\csc(u-v) = \frac{1}{\sin(u-v)} = \frac{1}{\sin u \cos v - \cos u \sin v} = \frac{1}{\left(-\frac{3}{5}\right)\left(\frac{15}{17}\right) - \left(\frac{4}{5}\right)\left(\frac{8}{17}\right)} = \frac{1}{-\frac{77}{85}} = -\frac{85}{77}$$

45. $$\sec(v-u) = \frac{1}{\cos(v-u)} = \frac{1}{\cos v \cos u + \sin v \sin u} = \frac{1}{\left(\frac{15}{17}\right)\left(\frac{4}{5}\right) + \left(\frac{8}{17}\right)\left(-\frac{3}{5}\right)} = \frac{1}{\left(\frac{60}{85}\right) + \left(-\frac{24}{85}\right)} = \frac{1}{\frac{36}{85}} = \frac{85}{36}$$

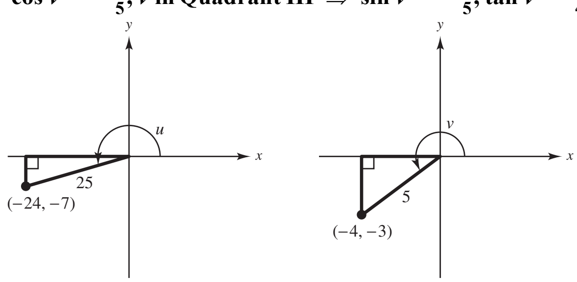
46. $$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} = \frac{\left(-\frac{3}{4}\right) + \left(\frac{8}{15}\right)}{1 - \left(-\frac{3}{4}\right)\left(\frac{8}{15}\right)} = \frac{-\frac{13}{60}}{\frac{7}{5}} = -\frac{13}{84}$$

$$\cot(u+v) = \frac{1}{\tan(u+v)} = \frac{1}{-\frac{13}{84}} = -\frac{84}{13}$$

For Exercises 47–52, you have:

$\sin u = -\frac{7}{25}$, u in Quadrant III $\Rightarrow \cos u = -\frac{24}{25}$, $\tan u = \frac{7}{24}$

$\cos v = -\frac{4}{5}$, v in Quadrant III $\Rightarrow \sin v = -\frac{3}{5}$, $\tan v = \frac{3}{4}$



Figures for Exercises 47–52

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47. $\cos(u + v) = \cos u \cos v - \sin u \sin v$

$= \left(-\frac{24}{25}\right)\left(-\frac{4}{5}\right) - \left(-\frac{7}{25}\right)\left(-\frac{3}{5}\right)$

$= \frac{3}{5}$

48. $\sin(u + v) = \sin u \cos v + \cos u \sin v$

$= \left(-\frac{7}{25}\right)\left(-\frac{4}{5}\right) + \left(-\frac{24}{25}\right)\left(-\frac{3}{5}\right)$

$= \frac{28}{125} + \frac{72}{125} = \frac{100}{125} = \frac{4}{5}$

49. $\tan(u - v) = \dfrac{\tan u - \tan v}{1 + \tan u \tan v}$

$= \dfrac{\frac{7}{24} - \frac{3}{4}}{1 + \left(\frac{7}{24}\right)\left(\frac{3}{4}\right)} = \dfrac{-\frac{11}{24}}{\frac{39}{32}} = -\dfrac{44}{117}$

50. $\tan(v - u) = \dfrac{\tan v - \tan u}{1 + \tan v \tan u} = \dfrac{\left(\frac{3}{4}\right) - \left(\frac{7}{24}\right)}{1 + \left(\frac{3}{4}\right)\left(\frac{7}{24}\right)}$

$= \dfrac{\frac{11}{24}}{\frac{39}{32}} = \dfrac{44}{117}$

$\cot(v - u) = \dfrac{1}{\tan(v - u)} = \dfrac{1}{\frac{44}{117}} = \dfrac{117}{44}$

51. $\csc(u - v) = \dfrac{1}{\sin(u - v)} = \dfrac{1}{\sin u \cos v - \cos u \sin v}$

$= \dfrac{1}{\left(-\frac{7}{25}\right)\left(-\frac{4}{5}\right) - \left(-\frac{24}{25}\right)\left(-\frac{3}{5}\right)}$

$= \dfrac{1}{-\frac{44}{125}}$

$= -\dfrac{125}{44}$

52. $\sec(v - u) = \dfrac{1}{\cos(v - u)}$

$= \dfrac{1}{\cos v \cos u + \sin v \sin u}$

$= \dfrac{1}{\left(-\frac{4}{5}\right)\left(-\frac{24}{25}\right) + \left(-\frac{3}{5}\right)\left(-\frac{7}{25}\right)}$

$= \dfrac{1}{\frac{117}{125}}$

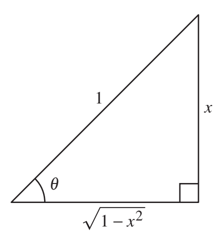
$= \dfrac{125}{117}$

53. $\sin(\arcsin x + \arccos x) = \sin(\arcsin x)\cos(\arccos x) + \sin(\arccos x)\cos(\arcsin x)$

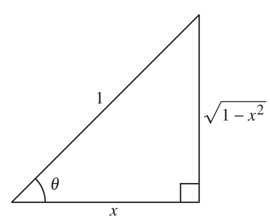
$= x \cdot x + \sqrt{1 - x^2} \cdot \sqrt{1 - x^2}$

$= x^2 + 1 - x^2$

$= 1$



$\theta = \arcsin x$



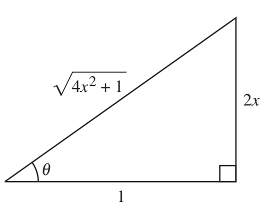
$\theta = \arccos x$

54. $\sin(\arctan 2x - \arccos x) = \sin(\arctan 2x - \arccos x)$

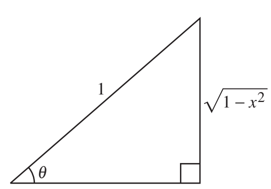
$= \sin(\arctan 2x)\cos(\arccos x) - \cos(\arctan 2x)\sin(\arccos x)$

$= \dfrac{2x}{\sqrt{4x^2 + 1}}(x) - \dfrac{1}{\sqrt{4x^2 + 1}}\left(\sqrt{1 - x^2}\right)$

$= \dfrac{2x^2 - \sqrt{1 - x^2}}{\sqrt{4x^2 + 1}}$



$\theta = \arctan 2x$



$\theta = \arccos x$

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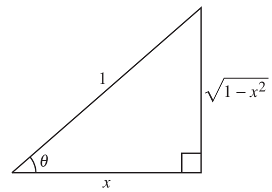
55. $\cos(\arccos x + \arcsin x) = \cos(\arccos x)\cos(\arcsin x) - \sin(\arccos x)\sin(\arcsin x)$

$$= x \cdot \sqrt{1 - x^2} - \sqrt{1 - x^2} \cdot x$$
$$= 0$$

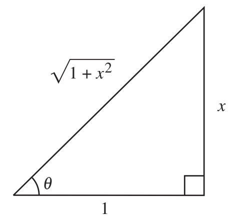
(Use the triangles in Exercise 53.)

56. $\cos(\arccos x - \arctan x) = \cos(\arccos x - \arctan x)$

$$= \cos(\arccos x)\cos(\arctan x) + \sin(\arccos x)\sin(\arctan x)$$
$$= (x)\left(\frac{1}{\sqrt{1 + x^2}}\right) + \left(\sqrt{1 - x^2}\right)\left(\frac{x}{\sqrt{1 + x^2}}\right)$$
$$= \frac{x + x\sqrt{1 - x^2}}{\sqrt{1 + x^2}}$$



$\theta = \arccos x$



$\theta = \arctan x$

57. $\sin\left(\frac{\pi}{2} - x\right) = \sin\frac{\pi}{2}\cos x - \cos\frac{\pi}{2}\sin x$

$$= (1)(\cos x) - (0)(\sin x)$$
$$= \cos x$$

58. $\sin\left(\frac{\pi}{2} + x\right) = \sin\frac{\pi}{2}\cos x + \sin x\cos\frac{\pi}{2}$

$$= (1)(\cos x) + (\sin x)(0)$$
$$= \cos x$$

59. $\sin\left(\frac{\pi}{6} + x\right) = \sin\frac{\pi}{6}\cos x + \cos\frac{\pi}{6}\sin x$

$$= \frac{1}{2}\left(\cos x + \sqrt{3}\sin x\right)$$

60. $\cos\left(\frac{5\pi}{4} - x\right) = \cos\frac{5\pi}{4}\cos x + \sin\frac{5\pi}{4}\sin x$

$$= -\frac{\sqrt{2}}{2}(\cos x + \sin x)$$

61. $\tan(\theta + \pi) = \dfrac{\tan\theta + \tan\pi}{1 - \tan\theta\tan\pi} = \dfrac{\tan\theta + 0}{1 - (\tan\theta)(0)} = \dfrac{\tan\theta}{1} = \tan\theta$

62. $\tan\left(\frac{\pi}{4} - \theta\right) = \dfrac{\tan\frac{\pi}{4} - \tan\theta}{1 + \tan\frac{\pi}{4}\tan\theta} = \dfrac{1 - \tan\theta}{1 + \tan\theta}$

63. $\cos(\pi - \theta) + \sin\left(\frac{\pi}{2} + \theta\right) = \cos\pi\cos\theta + \sin\pi\sin\theta + \sin\frac{\pi}{2}\cos\theta + \cos\frac{\pi}{2}\sin\theta$

$$= (-1)(\cos\theta) + (0)(\sin\theta) + (1)(\cos\theta) + (\sin\theta)(0)$$
$$= -\cos\theta + \cos\theta$$
$$= 0$$

64. $\cos(x + y)\cos(x - y) = (\cos x\cos y - \sin x\sin y)(\cos x\cos y + \sin x\sin y)$

$$= \cos^2 x\cos^2 y - \sin^2 x\sin^2 y$$
$$= \cos^2 x\left(1 - \sin^2 y\right) - \sin^2 x\sin^2 y$$
$$= \cos^2 x - \cos^2 x\sin^2 y - \sin^2 x\sin^2 y$$
$$= \cos^2 x - \sin^2 y\left(\cos^2 x + \sin^2 x\right)$$
$$= \cos^2 x - \sin^2 y$$

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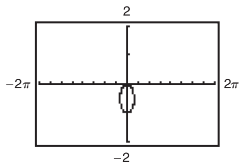
65. $\cos\left(\dfrac{3\pi}{2} - \theta\right) = \cos\dfrac{3\pi}{2}\cos\theta + \sin\dfrac{3\pi}{2}\sin\theta$

$= (0)(\cos\theta) + (-1)(\sin\theta)$

$= -\sin\theta$

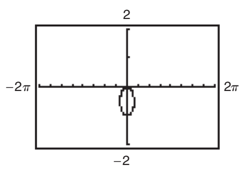


The graphs appear to coincide, so $\cos\left(\dfrac{3\pi}{2} - \theta\right) = -\sin\theta.$

66. $\sin(\pi + \theta) = \sin\pi\cos\theta + \cos\pi\sin\theta$

$= (0)\cos\theta + (-1)\sin\theta$

$= -\sin\theta$



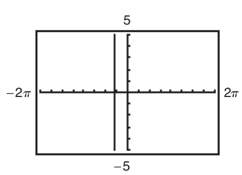
The graphs appear to coincide, so $\sin(\pi + \theta) = -\sin(\theta).$

67. $\sin\left(\dfrac{3\pi}{2} + \theta\right) = \sin\dfrac{3\pi}{2}\cos\theta + \cos\dfrac{3\pi}{2}\sin\theta$

$= (-1)(\cos\theta) + (0)(\sin\theta)$

$= -\cos\theta$

$\csc\left(\dfrac{3\pi}{2} + \theta\right) = \dfrac{1}{\sin\left(\dfrac{3\pi}{2} + \theta\right)} = \dfrac{1}{-\cos\theta} = -\sec\theta$



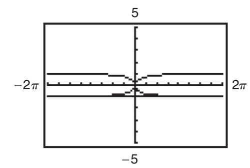
The graphs appear to coincide, so $\csc\left(\dfrac{3\pi}{2} + \theta\right) = -\sec\theta.$

68. $\tan(\pi + \theta) = \dfrac{\tan\pi + \tan\theta}{1 - \tan\pi\tan\theta}$

$= \dfrac{0 + \tan\theta}{1 - (0)\tan\theta}$

$= \tan\theta$

$\cot(\pi + \theta) = \dfrac{1}{\tan(\pi + \theta)} = \dfrac{1}{\tan\theta} = \cot\theta$



The graphs appear to coincide, so $\cot(\pi + \theta) = \cot\theta$

69.

$$\begin{aligned} \sin(x + \pi) - \sin x + 1 &= 0 \\ \sin x\cos\pi + \cos x\sin\pi - \sin x + 1 &= 0 \\ (\sin x)(-1) + (\cos x)(0) - \sin x + 1 &= 0 \\ -2\sin x + 1 &= 0 \\ \sin x &= \frac{1}{2} \\ x &= \frac{\pi}{6}, \frac{5\pi}{6} \end{aligned}$$

70.

$$\begin{aligned} \cos(x + \pi) - \cos x - 1 &= 0 \\ \cos x\cos\pi - \sin x\sin\pi - \cos x - 1 &= 0 \\ (\cos x)(-1) - (\sin x)(0) - \cos x - 1 &= 0 \\ -2\cos x - 1 &= 0 \\ \cos x &= -\frac{1}{2} \\ x &= \frac{2\pi}{3}, \frac{4\pi}{3} \end{aligned}$$

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71.

$$\cos\left(x + \frac{\pi}{4}\right) - \cos\left(x - \frac{\pi}{4}\right) = 1$$

$$\cos x \cos\frac{\pi}{4} - \sin x \sin\frac{\pi}{4} - \left(\cos x \cos\frac{\pi}{4} + \sin x \sin\frac{\pi}{4}\right) = 1$$

$$-2 \sin x\left(\frac{\sqrt{2}}{2}\right) = 1$$

$$-\sqrt{2}\sin x = 1$$

$$\sin x = -\frac{1}{\sqrt{2}}$$

$$\sin x = -\frac{\sqrt{2}}{2}$$

$$x = \frac{5\pi}{4}, \frac{7\pi}{4}$$

72.

$$\sin\left(x + \frac{\pi}{6}\right) - \sin\left(x - \frac{7\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$\sin x \cos\frac{\pi}{6} + \cos x \sin\frac{\pi}{6} - \left(\sin x \cos\frac{7\pi}{6} - \cos x \sin\frac{7\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$(\sin x)\left(\frac{\sqrt{3}}{2}\right) + (\cos x)\left(\frac{1}{2}\right) - (\sin x)\left(-\frac{\sqrt{3}}{2}\right) + (\cos x)\left(-\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$$

$$\sqrt{3}\sin x = \frac{\sqrt{3}}{2}$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

73.

$$\tan(x + \pi) + 2\sin(x + \pi) = 0$$

$$\frac{\tan x + \tan\pi}{1 - \tan x \tan\pi} + 2(\sin x \cos\pi + \cos x \sin\pi) = 0$$

$$\frac{\tan x + 0}{1 - \tan x(0)} + 2\left[\sin x(-1) + \cos x(0)\right] = 0$$

$$\frac{\tan x}{1} - 2\sin x = 0$$

$$\frac{\sin x}{\cos x} = 2\sin x$$

$$\sin x = 2\sin x\cos x$$

$$\sin x(1 - 2\cos x) = 0$$

$$\sin x = 0 \quad \text{or} \quad \cos x = \frac{1}{2}$$

$$x = 0, \pi \qquad x = \frac{\pi}{3}, \frac{5\pi}{3}$$

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74. $$\sin\left(x + \frac{\pi}{2}\right) - \cos^2 x = 0$$

$$\sin x \cos \frac{\pi}{2} + \cos x \sin \frac{\pi}{2} - \cos^2 x = 0$$

$$(\sin x)(0) + (\cos x)(1) - \cos^2 x = 0$$

$$\cos x - \cos^2 x = 0$$

$$\cos x(1 - \cos x) = 0$$

$\cos x = 0$ or $1 - \cos x = 0$

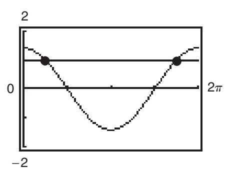
$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\qquad \cos x = 1$

$x = 0$

75. $\cos\left(x + \frac{\pi}{4}\right) + \cos\left(x - \frac{\pi}{4}\right) = 1$

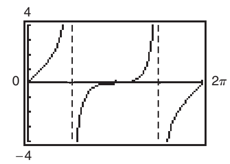
Graph $y_1 = \cos\left(x + \frac{\pi}{4}\right) + \cos\left(x - \frac{\pi}{4}\right)$ and $y_2 = 1$.

$x = \frac{\pi}{4}, \frac{7\pi}{4}$

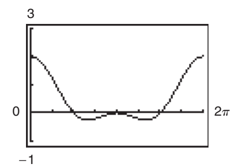


76. $\tan(x + \pi) - \cos\left(x + \frac{\pi}{2}\right) = 0$

$x = 0, \pi$

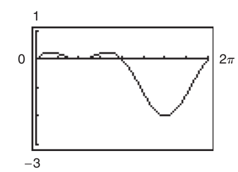


77. $\sin\left(x + \frac{\pi}{2}\right) + \cos^2 x = 0$



$x = \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

78. $\cos\left(x - \frac{\pi}{2}\right) - \sin^2 x = 0$



$x = 0, \frac{\pi}{2}, \pi$

79. $y = \frac{1}{3}\sin 2t + \frac{1}{4}\cos 2t$

(a) $a = \frac{1}{3}, b = \frac{1}{4}, B = 2$

$$C = \arctan \frac{b}{a} = \arctan \frac{3}{4} \approx 0.6435$$

$$y \approx \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{4}\right)^2}\sin(2t + 0.6435) = \frac{5}{12}\sin(2t + 0.6435)$$

(b) Amplitude: $\frac{5}{12}$ feet

(c) Frequency: $\frac{1}{\text{period}} = \frac{B}{2\pi} = \frac{2}{2\pi} = \frac{1}{\pi}$ cycle per second

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80.
$$y_1 = A\cos 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$
$$y_2 = A\cos 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)$$
$$y_1 + y_2 = A\cos 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + A\cos 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)$$
$$y_1 + y_2 = A\left[\cos 2\pi\frac{t}{T}\cos 2\pi\frac{x}{\lambda} + \sin 2\pi\frac{t}{T}\sin 2\pi\frac{x}{\lambda}\right] + A\left[\cos 2\pi\frac{t}{T}\cos 2\pi\frac{x}{\lambda} - \sin 2\pi\frac{t}{T}\sin 2\pi\frac{x}{\lambda}\right] = 2A\cos 2\pi\frac{t}{T}\cos 2\pi\frac{x}{\lambda}$$

81. True.

$\sin(u + v) = \sin u\cos v + \cos u\sin v$

$\sin(u - v) = \sin u\cos v - \cos u\sin v$

So, $\sin(u \pm v) = \sin u\cos v \pm \cos u\sin v$.

82. False.

$\cos(u + v) = \cos u\cos v - \sin u\sin v$

$\cos(u - v) = \cos u\cos v + \sin u\sin v$

So, $\cos(u \pm v) = \cos u\cos v \mp \sin u\sin v$.

83.
$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha = 0$$
$$\sin\alpha\cos\beta + \sin\beta\cos\alpha = 0$$
$$\sin\alpha\cos\beta = -\sin\beta\cos\alpha$$

False. When α and β are supplementary, $\sin\alpha\cos\beta = -\cos\alpha\sin\beta$.

84.
$$\begin{aligned}\cos(A + B) &= \cos(180° - C)\\ &= \cos(180°)\cos(C) + \sin(180°)\sin(C)\\ &= (-1)\cos(C) + (0)\sin(C)\\ &= -\cos(C)\end{aligned}$$

True. $\cos(A + B) = -\cos C$. When A, B and C form ΔABC, $A + B + C = 180°$, so $A + B = 180° - C$.

85. The denominator should be $1 + \tan x\tan(\pi/4)$.
$$\tan\left(x - \frac{\pi}{4}\right) = \frac{\tan x - \tan(\pi/4)}{1 + \tan x\tan(\pi/4)} = \frac{\tan x - 1}{1 + \tan x}$$

86. (a) Using the graph, $\sin(u + v) \approx 0$ and $\sin u + \sin v \approx 0.7 + 0.7 = 1.4$. Because $0 \neq 1.4$, $\sin(u + v) \neq \sin u + \sin v$.

(b) Using the graph, $\sin(u - v) \approx -1$ and $\sin u - \sin v \approx 0.7 - 0.7 = 0$. Because $-1 \neq 0$, $\sin(u - v) \neq \sin u - \sin v$.

87.
$$\begin{aligned}\cos(n\pi + \theta) &= \cos n\pi\cos\theta - \sin n\pi\sin\theta\\ &= (-1)^n(\cos\theta) - (0)(\sin\theta)\\ &= (-1)^n(\cos\theta),\text{ where } n \text{ is an integer.}\end{aligned}$$

88.
$$\begin{aligned}\sin(n\pi + \theta) &= \sin n\pi\cos\theta + \sin\theta\cos n\pi\\ &= (0)(\cos\theta) + (\sin\theta)(-1)^n\\ &= (-1)^n(\sin\theta),\text{ where } n \text{ is an integer.}\end{aligned}$$

89. $C = \arctan\dfrac{b}{a} \Rightarrow \sin C = \dfrac{b}{\sqrt{a^2 + b^2}}, \cos C = \dfrac{a}{\sqrt{a^2 + b^2}}$
$$\sqrt{a^2 + b^2}\sin(B\theta + C) = \sqrt{a^2 + b^2}\left(\sin B\theta\cdot\frac{a}{\sqrt{a^2 + b^2}} + \frac{b}{\sqrt{a^2 + b^2}}\cdot\cos B\theta\right) = a\sin B\theta + b\cos B\theta$$

90. $C = \arctan\dfrac{a}{b} \Rightarrow \sin C = \dfrac{a}{\sqrt{a^2 + b^2}}, \cos C = \dfrac{b}{\sqrt{a^2 + b^2}}$
$$\begin{aligned}\sqrt{a^2 + b^2}\cos(B\theta - C) &= \sqrt{a^2 + b^2}\left(\cos B\theta\cdot\frac{b}{\sqrt{a^2 + b^2}} + \sin B\theta\cdot\frac{a}{\sqrt{a^2 + b^2}}\right)\\ &= b\cos B\theta + a\sin B\theta = a\sin B\theta + b\cos B\theta\end{aligned}$$

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91. $\sin\theta + \cos\theta$

$a = 1, b = 1, B = 1$

(a) $C = \arctan\dfrac{b}{a} = \arctan 1 = \dfrac{\pi}{4}$

$$\sin\theta + \cos\theta = \sqrt{a^2 + b^2}\sin(B\theta + C) = \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right)$$

(b) $C = \arctan\dfrac{a}{b} = \arctan 1 = \dfrac{\pi}{4}$

$$\sin\theta + \cos\theta = \sqrt{a^2 + b^2}\cos(B\theta - C) = \sqrt{2}\cos\left(\theta - \frac{\pi}{4}\right)$$

92. $3\sin 2\theta + 4\cos 2\theta$

$a = 3, b = 4, B = 2$

(a) $C = \arctan\dfrac{b}{a} = \arctan\dfrac{4}{3} \approx 0.9273$

$$3\sin 2\theta + 4\cos 2\theta = \sqrt{a^2 + b^2}\sin(B\theta + C) \approx 5\sin(2\theta + 0.9273)$$

(b) $C = \arctan\dfrac{a}{b} = \arctan\dfrac{3}{4} \approx 0.6435$

$$3\sin 2\theta + 4\cos 2\theta = \sqrt{a^2 + b^2}\cos(B\theta - C) \approx 5\cos(2\theta - 0.6435)$$

93. $12\sin 3\theta + 5\cos 3\theta$

$a = 12, b = 5, B = 3$

(a) $C = \arctan\dfrac{b}{a} = \arctan\dfrac{5}{12} \approx 0.3948$

$$12\sin 3\theta + 5\cos 3\theta = \sqrt{a^2 + b^2}\sin(B\theta + C) \approx 13\sin(3\theta + 0.3948)$$

(b) $C = \arctan\dfrac{a}{b} = \arctan\dfrac{12}{5} \approx 1.1760$

$$12\sin 3\theta + 5\cos 3\theta = \sqrt{a^2 + b^2}\cos(B\theta - C) \approx 13\cos(3\theta - 1.1760)$$

94. $\sin 2\theta + \cos 2\theta$

$a = 1, b = 1, B = 2$

(a) $C = \arctan\dfrac{b}{a} = \arctan(1) = \dfrac{\pi}{4}$

$$\sin 2\theta + \cos 2\theta = \sqrt{a^2 + b^2}\sin(B\theta + C) = \sqrt{2}\sin\left(2\theta + \frac{\pi}{4}\right)$$

(b) $C = \arctan\dfrac{a}{b} = \arctan(1) = \dfrac{\pi}{4}$

$$\sin 2\theta + \cos 2\theta = \sqrt{a^2 + b^2}\cos(B\theta - C) = \sqrt{2}\cos\left(2\theta - \frac{\pi}{4}\right)$$

95. $C = \arctan\dfrac{b}{a} = \dfrac{\pi}{4} \Rightarrow a = b, a > 0, b > 0$

$\sqrt{a^2 + b^2} = 2 \Rightarrow a = b = \sqrt{2}$

$B = 1$

$2\sin\left(\theta + \dfrac{\pi}{4}\right) = \sqrt{2}\sin\theta + \sqrt{2}\cos\theta$

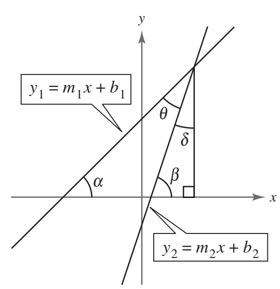
96. $C = \arctan\dfrac{b}{a} = \dfrac{\pi}{4} \Rightarrow a = b, a > 0, b > 0$

$\sqrt{a^2 + b^2} = 5 \Rightarrow a = b = \dfrac{5\sqrt{2}}{2}$

$B = 1$

$5\cos\left(\theta - \dfrac{\pi}{4}\right) = \dfrac{5\sqrt{2}}{2}\sin\theta + \dfrac{5\sqrt{2}}{2}\cos\theta$

97.



$m_1 = \tan\alpha$ and $m_2 = \tan\beta$

$\beta + \delta = 90° \Rightarrow \delta = 90° - \beta$

$$\alpha + \theta + \delta = 90° \Rightarrow \alpha + \theta + (90° - \beta) = 90° \Rightarrow \theta = \beta - \alpha$$

So, $\theta = \arctan m_2 - \arctan m_1$. For $y = x$ and $y = \sqrt{3}x$ you have $m_1 = 1$ and $m_2 = \sqrt{3}$.

$\theta = \arctan\sqrt{3} - \arctan 1 = 60° - 45° = 15°$

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98. For $m_2 > m_1 > 0$, the angle θ between the lines is:

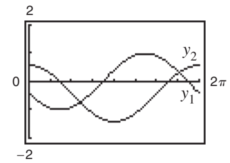
$$\theta = \arctan\left(\frac{m_2 - m_1}{1 + m_1 m_2}\right)$$

$$m_2 = 1$$

$$m_1 = \frac{1}{\sqrt{3}}$$

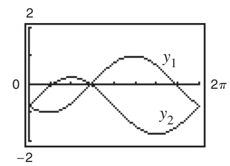
$$\theta = \arctan\left(\frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}\right) = \arctan\left(2 - \sqrt{3}\right) = 15°$$

99. $y_1 = \cos(x + 2),\ y_2 = \cos x + \cos 2$



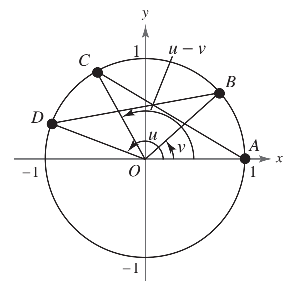
No, $y_1 \neq y_2$ because their graphs are different.

100. $y_1 = \sin(x + 4),\ y_2 = \sin x + \sin 4$



No, $y_1 \neq y_2$ because their graphs are different.

101. (a) To prove the identity for $\sin(u + v)$ you first need to prove the identity for $\cos(u - v)$.
Assume $0 < v < u < 2\pi$ and locate u, v, and $u - v$ on the unit circle.
The coordinates of the points on the circle are:
$A = (1, 0),\ B = (\cos v, \sin v),\ C = \left(\cos(u - v), \sin(u - v)\right)$, and $D = (\cos u, \sin u)$.
Because $\angle DOB = \angle COA$, chords AC and BD are equal. By the Distance Formula:



$$\sqrt{\left[\cos(u - v) - 1\right]^2 + \left[\sin(u - v) - 0\right]^2} = \sqrt{(\cos u - \cos v)^2 + (\sin u - \sin v)^2}$$

$$\cos^2(u - v) - 2\cos(u - v) + 1 + \sin^2(u - v) = \cos^2 u - 2\cos u\cos v + \cos^2 v + \sin^2 u - 2\sin u\sin v + \sin^2 v$$

$$\left[\cos^2(u - v) + \sin^2(u - v)\right] + 1 - 2\cos(u - v) = \left(\cos^2 u + \sin^2 u\right) + \left(\cos^2 v + \sin^2 v\right) - 2\cos u\cos v - 2\sin u\sin v$$

$$2 - 2\cos(u - v) = 2 - 2\cos u\cos v - 2\sin u\sin v$$

$$-2\cos(u - v) = -2(\cos u\cos v + \sin u\sin v)$$

$$\cos(u - v) = \cos u\cos v + \sin u\sin v$$

Now, to prove the identity for $\sin(u + v)$, use cofunction identities.

$$\sin(u + v) = \cos\left[\frac{\pi}{2} - (u + v)\right] = \cos\left[\left(\frac{\pi}{2} - u\right) - v\right]$$

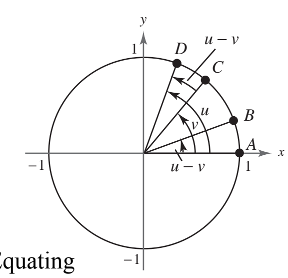
$$= \cos\left(\frac{\pi}{2} - u\right)\cos v + \sin\left(\frac{\pi}{2} - u\right)\sin v$$

$$= \sin u\cos v + \cos u\sin v$$

(b) First, prove $\cos(u - v) = \cos u\cos v + \sin u\sin v$ using the figure containing points

$A(1, 0)$
$B\left(\cos(u - v), \sin(u - v)\right)$
$C(\cos v, \sin v)$
$D(\cos u, \sin u)$

on the unit circle.



Because chords AB and CD are each subtended by angle $u - v$, their lengths are equal. Equating $\left[d(A, B)\right]^2 = \left[d(C, D)\right]^2$ you have $\left(\cos(u - v) - 1\right)^2 + \sin^2(u - v) = (\cos u - \cos v)^2 + (\sin u - \sin v)^2$.
Simplifying and solving for $\cos(u - v)$, you have $\cos(u - v) = \cos u\cos v + \sin u\sin v$.

Using $\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$,

$$\sin(u - v) = \cos\left[\frac{\pi}{2} - (u - v)\right] = \cos\left[\left(\frac{\pi}{2} - u\right) - (-v)\right] = \cos\left(\frac{\pi}{2} - u\right)\cos(-v) + \sin\left(\frac{\pi}{2} - u\right)\sin(-v)$$

$$= \sin u\cos v - \cos u\sin v$$

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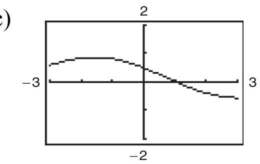
102. (a) The domains of f and g are the same, all real numbers h, except $h = 0$.

(b)

h	0.5	0.2	0.1	0.05	0.02	0.01
$f(h)$	0.267	0.410	0.456	0.478	0.491	0.496
$g(h)$	0.267	0.410	0.456	0.478	0.491	0.496

(c)



(d) As $h \to 0^*$, $f \to 0.5$ and $g \to 0.5$.

Section 5.5 Multiple-Angle and Product-to-Sum Formulas

1. $2 \sin u \cos u$

2. $\cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$

3. $\frac{1}{2}\left[\sin(u + v) + \sin(u - v)\right]$

4. $\tan^2 u$

5. $\pm\sqrt{\frac{1 - \cos u}{2}}$

6. $-2\sin\left(\frac{u + v}{2}\right)\sin\left(\frac{u - v}{2}\right)$

7.
$$\sin 2x - \sin x = 0$$
$$2\sin x \cos x - \sin x = 0$$
$$\sin x(2\cos x - 1) = 0$$
$$\sin x = 0 \quad \text{or} \quad 2\cos x - 1 = 0$$
$$x = n\pi \qquad \cos x = \frac{1}{2}$$
$$x = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi$$

8.
$$\sin 2x \sin x = \cos x$$
$$2\sin x \cos x \sin x - \cos x = 0$$
$$\cos x\left(2\sin^2 x - 1\right) = 0$$
$$\cos x = 0 \quad \text{or} \quad 2\sin^2 x - 1 = 0$$
$$x = \frac{\pi}{2} + 2n\pi \qquad \sin^2 x = \frac{1}{2}$$
$$\sin x = \pm\frac{\sqrt{2}}{2}$$
$$x = \frac{\pi}{4} + \frac{n\pi}{2}$$

9.
$$\cos 2x - \cos x = 0$$
$$\cos 2x = \cos x$$
$$\cos^2 x - \sin^2 x = \cos x$$
$$\cos^2 x - \left(1 - \cos^2 x\right) - \cos x = 0$$
$$2\cos^2 x - \cos x - 1 = 0$$
$$(2\cos x + 1)(\cos x - 1) = 0$$
$$2\cos x + 1 = 0 \quad \text{or} \quad \cos x - 1 = 0$$
$$\cos x = -\frac{1}{2} \qquad \cos x = 1$$
$$x = \frac{2n\pi}{3} \qquad x = 0$$

10.
$$\cos 2x + \sin x = 0$$
$$1 - 2\sin^2 x + \sin x = 0$$
$$2\sin^2 x - \sin x - 1 = 0$$
$$(2\sin x + 1)(\sin x - 1) = 0$$
$$2\sin x + 1 = 0 \quad \text{or} \quad \sin x - 1 = 0$$
$$\sin x = -\frac{1}{2} \qquad \sin x = 1$$
$$x = \frac{7\pi}{6} + 2n\pi, \frac{11\pi}{6} + 2n\pi \qquad x = \frac{\pi}{2} + 2n\pi$$

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11.

$$\sin 4x = -2\sin 2x$$
$$\sin 4x + 2\sin 2x = 0$$
$$2\sin 2x\cos 2x + 2\sin 2x = 0$$
$$2\sin 2x(\cos 2x + 1) = 0$$

$2\sin 2x = 0$ or $\cos 2x + 1 = 0$

$\sin 2x = 0$ $\quad$ $\cos 2x = -1$

$2x = n\pi$ $\quad$ $2x = \pi + 2n\pi$

$x = \frac{n}{2}\pi$ $\quad$ $x = \frac{\pi}{2} + n\pi$

12.

$$(\sin 2x + \cos 2x)^2 = 1$$
$$\sin^2 2x + 2\sin 2x\cos 2x + \cos^2 2x = 1$$
$$2\sin 2x\cos 2x = 0$$
$$\sin 4x = 0$$
$$4x = n\pi$$
$$x = \frac{n\pi}{4}$$

13.

$$\tan 2x - \cot x = 0$$
$$\frac{2\tan x}{1 - \tan^2 x} = \cot x$$
$$2\tan x = \cot x(1 - \tan^2 x)$$
$$2\tan x = \cot x - \cot x\tan^2 x$$
$$2\tan x = \cot x - \tan x$$
$$3\tan x = \cot x$$
$$3\tan x - \cot x = 0$$
$$3\tan x - \frac{1}{\tan x} = 0$$
$$\frac{3\tan^2 x - 1}{\tan x} = 0$$
$$\frac{1}{\tan x}(3\tan^2 x - 1) = 0$$
$$\cot x(3\tan^2 x - 1) = 0$$

$\cot x = 0$ or $3\tan^2 x - 1 = 0$

$x = \frac{\pi}{2} + n\pi$ $\quad$ $\tan^2 x = \frac{1}{3}$

$\tan x = \pm\frac{\sqrt{3}}{3}$

$x = \frac{\pi}{6} + n\pi, \frac{5\pi}{6} + n\pi$

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14.
$$\tan 2x - 2\cos x = 0$$
$$\frac{2\tan x}{1-\tan^2 x} = 2\cos x$$
$$2\tan x = 2\cos x\left(1-\tan^2 x\right)$$
$$2\tan x = 2\cos x - 2\cos x\tan^2 x$$
$$2\tan x = 2\cos x - 2\cos x\frac{\sin^2 x}{\cos^2 x}$$
$$2\tan x = 2\cos x - 2\frac{\sin^2 x}{\cos x}$$
$$\tan x = \cos x - \frac{\sin^2 x}{\cos x}$$
$$\frac{\sin x}{\cos x} = \cos x - \frac{\sin^2 x}{\cos x}$$
$$\frac{\sin x}{\cos x} + \frac{\sin^2 x}{\cos x} - \cos x = 0$$
$$\frac{\sin x + \sin^2 x - \cos^2 x}{\cos x} = 0$$
$$\frac{1}{\cos x}\left[\sin x + \sin^2 x - \left(1-\sin^2 x\right)\right] = 0$$
$$\sec x\left[2\sin^2 x + \sin x - 1\right] = 0$$
$$\sec x(2\sin x - 1)(\sin x + 1) = 0$$

$\sec x = 0$ or $2\sin x - 1 = 0$ or $\sin x + 1 = 0$

No solution; $\sin x = \frac{1}{2}$, $x = \frac{\pi}{6}, \frac{5\pi}{6}$; $\sin x = -1$, $x = \frac{3\pi}{2}$

Also, values for which $\cos x = 0$ need to be checked.

$\frac{\pi}{2}, \frac{3\pi}{2}$ are solutions.

$$x = \frac{\pi}{6} + 2n\pi, \frac{\pi}{2} + n\pi, \frac{5\pi}{6} + 2n\pi$$

15. $6\sin x\cos x = 3(2\sin x\cos x)$
$= 3\sin 2x$

16. $\sin x\cos x = \frac{1}{2}(2\sin x\cos x)$
$= \frac{1}{2}\sin 2x$

17. $6\cos^2 x - 3 = 3\left(2\cos^2 x - 1\right)$
$= 3\cos 2x$

18. $\cos^2 x - \frac{1}{2} = \frac{1}{2}\left[2\left(\cos^2 x - \frac{1}{2}\right)\right]$
$= \frac{1}{2}\left(2\cos^2 x - 1\right)$
$= \frac{1}{2}\cos 2x$

19. $4 - 8\sin^2 x = 4\left(1 - 2\sin^2 x\right)$
$= 4\cos 2x$

20. $10\sin^2 x - 5 = 5\left(2\sin^2 x - 1\right)$
$= -5\left(1 - 2\sin^2 x\right)$
$= -5\cos 2x$

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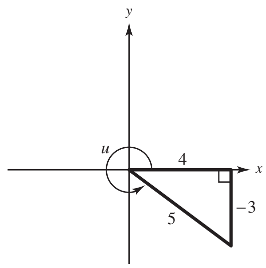
21. $\sin u = -\frac{3}{5}, \frac{3\pi}{2} < u < 2\pi$

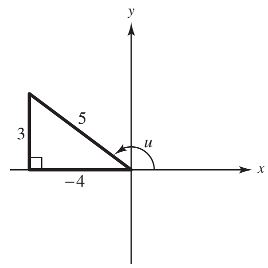


$$\sin 2u = 2\sin u\cos u = 2\left(-\frac{3}{5}\right)\left(\frac{4}{5}\right) = -\frac{24}{25}$$

$$\cos 2u = \cos^2 u - \sin^2 u = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

$$\tan 2u = \frac{2\tan u}{1 - \tan^2 u} = \frac{2\left(-\frac{3}{4}\right)}{1 - \frac{9}{16}} = -\frac{3}{2}\left(\frac{16}{7}\right) = -\frac{24}{7}$$

22. $\cos u = -\frac{4}{5}, \frac{\pi}{2} < u < \pi$

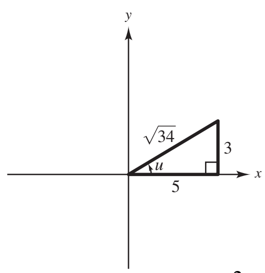


$$\sin 2u = 2\sin u\cos u = 2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right) = -\frac{24}{25}$$

$$\cos 2u = \cos^2 u - \sin^2 u = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

$$\tan 2u = \frac{2\tan u}{1 - \tan^2 u} = \frac{2\left(-\frac{3}{4}\right)}{1 - \frac{9}{16}} = -\frac{3}{2}\left(\frac{16}{7}\right) = -\frac{24}{7}$$

23. $\tan u = \frac{3}{5}, 0 < u < \frac{\pi}{2}$

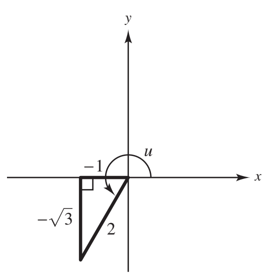


$$\sin 2u = 2\sin u\cos u = 2\left(\frac{3}{\sqrt{34}}\right)\left(\frac{5}{\sqrt{34}}\right) = \frac{15}{17}$$

$$\cos 2u = \cos^2 u - \sin^2 u = \frac{25}{34} - \frac{9}{34} = \frac{8}{17}$$

$$\tan 2u = \frac{2\tan u}{1 - \tan^2 u} = \frac{2\left(\frac{3}{5}\right)}{1 - \frac{9}{25}} = \frac{6}{5}\left(\frac{25}{16}\right) = \frac{15}{8}$$

24. $\sec u = -2, \pi < u < \frac{3\pi}{2}$



$$\sin 2u = 2\sin u\cos u = 2\left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$$

$$\cos 2u = \cos^2 u - \sin^2 u = \left(-\frac{1}{2}\right)^2 - \left(-\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} - \frac{3}{4} = -\frac{1}{2}$$

$$\tan 2u = \frac{2\tan u}{1 - \tan^2 u} = \frac{2\left(\frac{\sqrt{3}}{1}\right)}{1 - \left(\frac{\sqrt{3}}{1}\right)^2} = \frac{2\sqrt{3}}{-2} = -\sqrt{3}$$

25.
$$\begin{aligned}
\cos 4x &= \cos(2x + 2x)\\
&= \cos 2x\cos 2x - \sin 2x\sin 2x\\
&= \cos^2 2x - \sin^2 2x\\
&= \cos^2 2x - \left(1 - \cos^2 2x\right)\\
&= 2\cos^2 2x - 1\\
&= 2(\cos 2x)^2 - 1\\
&= 2\left(2\cos^2 x - 1\right)^2 - 1\\
&= 2\left(4\cos^4 x - 4\cos x + 1\right) - 1\\
&= 8\cos^4 x - 8\cos x + 1
\end{aligned}$$

26.
$$\begin{aligned}
\tan 3x &= \tan(2x + x)\\
&= \frac{\tan 2x + \tan x}{1 - \tan 2x\tan x}\\
&= \frac{\frac{2\tan x}{1 - \tan^2 x} + \tan x}{1 - \left(\frac{2\tan x}{1 - \tan^2 x}\right)(\tan x)}\\
&= \frac{\frac{2\tan x + \tan x - \tan^3 x}{1 - \tan^2 x}}{\frac{1 - \tan^2 x - 2\tan^2 x}{1 - \tan^2 x}}\\
&= \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}
\end{aligned}$$

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27. $\cos^4 x = (\cos^2 x)(\cos^2 x) = \left(\frac{1 + \cos 2x}{2}\right)\left(\frac{1 + \cos 2x}{2}\right) = \frac{1 + 2\cos 2x + \cos^2 2x}{4}$

$$= \frac{1 + 2\cos 2x + \frac{1 + \cos 4x}{2}}{4}$$

$$= \frac{2 + 4\cos 2x + 1 + \cos 4x}{8}$$

$$= \frac{3 + 4\cos 2x + \cos 4x}{8}$$

$$= \frac{1}{8}(3 + 4\cos 2x + \cos 4x)$$

28. $\sin^8 x = (\sin^4 x)(\sin^4 x) = (\sin^2 x)^2 (\sin^2 x)^2$

$$= \left(\frac{1 - \cos 2x}{2}\right)^2\left(\frac{1 - \cos 2x}{2}\right)^2$$

$$= \left(\frac{1 - 2\cos 2x + \cos^2 2x}{4}\right)\left(\frac{1 - 2\cos 2x + \cos^2 2x}{4}\right)$$

$$= \frac{1 - 2\cos 2x + \cos^2 2x - 2\cos 2x + 4\cos^2 2x - 2\cos^3 2x + \cos^2 2x - 2\cos^3 2x + \cos^4 2x}{16}$$

$$= \frac{1 - 4\cos 2x + 6\cos^2 2x - 4\cos^3 2x + \cos^4 2x}{16}$$

$$= \frac{1 - 4\cos 2x + 6\cos^2 2x - 4\cos^3 2x + (\cos^2 2x)^2}{16}$$

$$= \frac{1 - 4\cos 2x + 6\left(\frac{1 + \cos 4x}{2}\right) - 4\cos^3 2x + \left(\frac{1 + \cos 4x}{2}\right)^2}{16}$$

$$= \frac{1 - 4\cos 2x + 3 + 3\cos 4x - 4\cos^3 2x + \left(\frac{1 + 2\cos 4x + \cos^2 4x}{4}\right)}{16}$$

$$= \frac{4 - 16\cos 2x + 12 + 12\cos 4x - 16\cos^3 2x + 1 + 2\cos 4x + \cos^2 4x}{64}$$

$$= \frac{17 - 16\cos 2x + 14\cos 4x - 16\cos^3 2x + \left(\frac{1 + \cos 8x}{2}\right)}{64}$$

$$= \frac{34 - 32\cos 2x + 28\cos 4x - 32\cos^3 2x + 1 + \cos 8x}{128}$$

$$= \frac{35 - 32\cos 2x + 28\cos 4x - 32\cos^3 2x + \cos 8x}{128}$$

$$= \frac{35 - 32\cos 2x + 28\cos 4x - 32\cos^2 2x\cos 2x + \cos 8x}{128}$$

$$= \frac{35 - 32\cos 2x + 28\cos 4x - 32\left(\frac{1 + \cos 4x}{2}\right)\cos 2x + \cos 8x}{128}$$

$$= \frac{35 - 32\cos 2x + 28\cos 4x - 16\cos 2x - 16\cos 4x\cos 2x + \cos 8x}{128}$$

$$= \frac{35 - 48\cos 2x + 28\cos 4x - 16\cos 4x\cos 2x + \cos 8x}{128}$$

$$= \frac{1}{128}(35 - 48\cos 2x + 28\cos 4x + \cos 8x - 16\cos 2x\cos 4x)$$

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29. $$\begin{aligned}\sin^4 2x &= \left(\sin^2 2x\right)^2\\ &= \left(\frac{1 - \cos 4x}{2}\right)^2\\ &= \frac{1}{4}\left(1 - 2\cos 4x + \cos^2 4x\right)\\ &= \frac{1}{4}\left(1 - 2\cos 4x + \frac{1 + \cos 8x}{2}\right)\\ &= \frac{1}{4} - \frac{1}{2}\cos 4x + \frac{1}{8} + \frac{1}{8}\cos 8x\\ &= \frac{3}{8} - \frac{1}{2}\cos 4x + \frac{1}{8}\cos 8x\\ &= \frac{1}{8}\left(3 - 4\cos 4x + \cos 8x\right)\end{aligned}$$

30. $$\begin{aligned}\cos^4 2x &= \left(\cos^2 2x\right)^2\\ &= \left(\frac{1 + \cos 4x}{2}\right)^2\\ &= \frac{1}{4}\left(1 + 2\cos 4x + \cos^2 4x\right)\\ &= \frac{1}{4}\left(1 + 2\cos 4x + \frac{1 + \cos 8x}{2}\right)\\ &= \frac{1}{4} + \frac{1}{2}\cos 4x + \frac{1}{8} + \frac{1}{8}\cos 8x\\ &= \frac{3}{8} + \frac{1}{2}\cos 4x + \frac{1}{8}\cos 8x\\ &= \frac{1}{8}\left(3 + 4\cos 4x + \cos 8x\right)\end{aligned}$$

31. $$\begin{aligned}\tan^4 2x &= \left(\tan^2 2x\right)^2\\ &= \left(\frac{1 - \cos 4x}{1 + \cos 4x}\right)^2\\ &= \frac{1 - 2\cos 4x + \cos^2 4x}{1 + 2\cos 4x + \cos^2 4x}\\ &= \frac{1 - 2\cos 4x + \frac{1 + \cos 8x}{2}}{1 + 2\cos 4x + \frac{1 + \cos 8x}{2}}\\ &= \frac{\frac{1}{2}\left(2 - 4\cos 4x + 1 + \cos 8x\right)}{\frac{1}{2}\left(2 + 4\cos 4x + 1 + \cos 8x\right)}\\ &= \frac{3 - 4\cos 4x + \cos 8x}{3 + 4\cos 4x + \cos 8x}\end{aligned}$$

32. $$\begin{aligned}\tan^2 2x\cos^4 2x &= \left(\frac{1 - \cos 4x}{1 + \cos 4x}\right)\left(\cos^2 2x\right)^2\\ &= \left(\frac{1 - \cos 4x}{1 + \cos 4x}\right)\left(\frac{1 + \cos 4x}{2}\right)^2\\ &= \frac{(1 - \cos 4x)(1 + \cos 4x)(1 + \cos 4x)}{4(1 + \cos 4x)}\\ &= \frac{(1 - \cos 4x)(1 + \cos 4x)}{4}\\ &= \frac{1}{4}\left(1 - \cos^2 4x\right)\\ &= \frac{1}{4}\left(1 - \frac{1 + \cos 8x}{2}\right)\\ &= \frac{1}{4} - \frac{1}{8} - \frac{1}{8}\cos 8x\\ &= \frac{1}{8} - \frac{1}{8}\cos 8x\\ &= \frac{1}{8}\left(1 - \cos 8x\right)\end{aligned}$$

33. $$\begin{aligned}\sin^2 2x\cos^2 2x &= \left(\frac{1 - \cos 4x}{2}\right)\left(\frac{1 + \cos 4x}{2}\right)\\ &= \frac{1}{4}\left(1 - \cos^2 4x\right)\\ &= \frac{1}{4}\left(1 - \frac{1 + \cos 8x}{2}\right)\\ &= \frac{1}{4} - \frac{1}{8} - \frac{1}{8}\cos 8x\\ &= \frac{1}{8} - \frac{1}{8}\cos 8x\\ &= \frac{1}{8}\left(1 - \cos 8x\right)\end{aligned}$$

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34. $\sin^4 x \cos^2 x = \sin^2 x \sin^2 x \cos^2 x$

$$= \left(\frac{1 - \cos 2x}{2}\right)\left(\frac{1 - \cos 2x}{2}\right)\left(\frac{1 + \cos 2x}{2}\right)$$

$$= \frac{1}{8}(1 - \cos 2x)(1 - \cos^2 2x)$$

$$= \frac{1}{8}(1 - \cos 2x - \cos^2 2x + \cos^3 2x)$$

$$= \frac{1}{8}\left[1 - \cos 2x - \left(\frac{1 + \cos 4x}{2}\right) + \cos 2x\left(\frac{1 + \cos 4x}{2}\right)\right]$$

$$= \frac{1}{16}[2 - 2\cos 2x - 1 - \cos 4x + \cos 2x + \cos 2x \cos 4x]$$

$$= \frac{1}{16}[1 - \cos 2x - \cos 4x + \cos 2x \cos 4x]$$

35. $\sin 75° = \sin\left(\frac{1}{2} \cdot 150°\right) = \sqrt{\frac{1 - \cos 150°}{2}} = \sqrt{\frac{1 + \left(\sqrt{3}/2\right)}{2}}$

$$= \frac{1}{2}\sqrt{2 + \sqrt{3}}$$

$\cos 75° = \cos\left(\frac{1}{2} \cdot 150°\right) = \sqrt{\frac{1 + \cos 150°}{2}} = \sqrt{\frac{1 - \left(\sqrt{3}/2\right)}{2}}$

$$= \frac{1}{2}\sqrt{2 - \sqrt{3}}$$

$\tan 75° = \tan\left(\frac{1}{2} \cdot 150°\right) = \frac{\sin 150°}{1 + \cos 150°} = \frac{1/2}{1 - \left(\sqrt{3}/2\right)}$

$$= \frac{1}{2 - \sqrt{3}} \cdot \frac{2 + \sqrt{3}}{2 + \sqrt{3}} = \frac{2 + \sqrt{3}}{4 - 3} = 2 + \sqrt{3}$$

36. $\sin 165° = \sin\left(\frac{1}{2} \cdot 330°\right) = \sqrt{\frac{1 - \cos 330°}{2}} = \sqrt{\frac{1 - \left(\sqrt{3}/2\right)}{2}}$

$$= \frac{1}{2}\sqrt{2 - \sqrt{3}}$$

$\cos 165° = \cos\left(\frac{1}{2} \cdot 330°\right) = -\sqrt{\frac{1 + \cos 330°}{2}} = -\sqrt{\frac{1 + \left(\sqrt{3}/2\right)}{2}}$

$$= -\frac{1}{2}\sqrt{2 + \sqrt{3}}$$

$\tan 165° = \tan\left(\frac{1}{2} \cdot 330°\right) = \frac{\sin 330°}{1 + \cos 330°} = \frac{-1/2}{1 + \left(\sqrt{3}/2\right)}$

$$= \frac{-1}{2 + \sqrt{3}} \cdot \frac{2 - \sqrt{3}}{2 - \sqrt{3}} = \frac{-2 + \sqrt{3}}{4 - 3} = -2 + \sqrt{3}$$

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37. $\sin 112° 30' = \sin\left(\frac{1}{2} \cdot 225°\right) = \sqrt{\frac{1 - \cos 225°}{2}} = \sqrt{\frac{1 - \left(-\sqrt{2}/2\right)}{2}} = \frac{1}{2}\sqrt{2 + \sqrt{2}}$

$\cos 112° 30' = \cos\left(\frac{1}{2} \cdot 225°\right) = -\sqrt{\frac{1 + \cos 225°}{2}} = -\sqrt{\frac{1 + \left(-\sqrt{2}/2\right)}{2}} = \frac{1}{2} - \sqrt{2 - 2}$

$\tan 112° 30' = \tan\left(\frac{1}{2} \cdot 225°\right) = \frac{\sin 225°}{1 + \cos 225°} = \frac{-\sqrt{2}/2}{1 + \left(-\sqrt{2}/2\right)} = \frac{-\sqrt{2}}{2 - \sqrt{2}} \cdot \frac{2 + \sqrt{2}}{2 + \sqrt{2}} = \frac{-2\sqrt{2} - 2}{2} = -1 - \sqrt{2}$

38. $\sin 67° 30' = \sin\left(\frac{1}{2} \cdot 135°\right) = \sqrt{\frac{1 - \cos 135°}{2}} = \sqrt{\frac{1 + \left(\sqrt{2}/2\right)}{2}} = \frac{1}{2}\sqrt{2 + \sqrt{2}}$

$\cos 67° 30' = \cos\left(\frac{1}{2} \cdot 135°\right) = \sqrt{\frac{1 + \cos 135°}{2}} = \sqrt{\frac{1 - \left(\sqrt{2}/2\right)}{2}} = \frac{1}{2}\sqrt{2 - \sqrt{2}}$

$\tan 67° 30' = \tan\left(\frac{1}{2} \cdot 135°\right) = \frac{\sin 135°}{1 + \cos 135°} = \frac{\sqrt{2}/2}{1 - \left(\sqrt{2}/2\right)} = 1 + \sqrt{2}$

39. $\sin \frac{\pi}{8} = \sin\left[\frac{1}{2}\left(\frac{\pi}{4}\right)\right] = \sqrt{\frac{1 - \cos \frac{\pi}{4}}{2}} = \frac{1}{2}\sqrt{2 - \sqrt{2}}$

$\cos \frac{\pi}{8} = \cos\left[\frac{1}{2}\left(\frac{\pi}{4}\right)\right] = \sqrt{\frac{1 + \cos \frac{\pi}{4}}{2}} = \frac{1}{2}\sqrt{2 + \sqrt{2}}$

$\tan \frac{\pi}{8} = \tan\left[\frac{1}{2}\left(\frac{\pi}{4}\right)\right] = \frac{\sin \frac{\pi}{4}}{1 + \cos \frac{\pi}{4}} = \frac{\frac{\sqrt{2}}{2}}{1 + \frac{\sqrt{2}}{2}} = \sqrt{2} - 1$

40. $\sin \frac{7\pi}{12} = \sin\left[\frac{1}{2}\left(\frac{7\pi}{6}\right)\right] = \sqrt{\frac{1 - \cos \frac{7\pi}{6}}{2}} = \sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} = \frac{1}{2}\sqrt{2 + \sqrt{3}}$

$\cos \frac{7\pi}{12} = \cos\left[\frac{1}{2}\left(\frac{7\pi}{6}\right)\right] = -\sqrt{\frac{1 + \cos \frac{7\pi}{6}}{2}} = -\sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = -\frac{1}{2}\sqrt{2 - \sqrt{3}}$

$\tan \frac{7\pi}{12} = \tan\left[\frac{1}{2}\left(\frac{7\pi}{6}\right)\right] = \frac{\sin \frac{7\pi}{6}}{1 + \cos \frac{7\pi}{6}} = \frac{-\frac{1}{2}}{1 - \frac{\sqrt{3}}{2}} = -2 - \sqrt{3}$

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41. $\cos u = \frac{7}{25}, 0 < u < \frac{\pi}{2}$

(a) Because u is in Quadrant I, $\frac{u}{2}$ is also in Quadrant I.

(b) $$\sin\frac{u}{2} = \sqrt{\frac{1 - \cos u}{2}} = \sqrt{\frac{1 - \frac{7}{25}}{2}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

$$\cos\frac{u}{2} = \sqrt{\frac{1 + \cos u}{2}} = \sqrt{\frac{1 + \frac{7}{25}}{2}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$\tan\frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{1 - \frac{7}{25}}{\frac{24}{25}} = \frac{3}{4}$$

42. $\sin u = \frac{5}{13}, \frac{\pi}{2} < u < \pi \Rightarrow \cos u = -\frac{12}{13}$

(a) Because u is in Quadrant II, $\frac{u}{2}$ is in Quadrant I.

(b) $$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1 - \cos u}{2}} = \sqrt{\frac{1 + \frac{12}{13}}{2}} = \frac{5\sqrt{26}}{26}$$

$$\cos\left(\frac{u}{2}\right) = \sqrt{\frac{1 + \cos u}{2}} = \sqrt{\frac{1 - \frac{12}{13}}{2}} = \frac{\sqrt{26}}{26}$$

$$\tan\left(\frac{u}{2}\right) = \frac{\sin u}{1 + \cos u} = \frac{\frac{5}{13}}{1 - \frac{12}{13}} = 5$$

43. $\tan u = -\frac{5}{12}, \frac{3\pi}{2} < u < 2\pi$

(a) Because u is in Quadrant IV, $\frac{u}{2}$ is in Quadrant II.

(b) $$\sin\frac{u}{2} = \sqrt{\frac{1 - \cos u}{2}} = \sqrt{\frac{1 - \frac{12}{13}}{2}} = \sqrt{\frac{1}{26}} = \frac{\sqrt{26}}{26}$$

$$\cos\frac{u}{2} = -\sqrt{\frac{1 + \cos u}{2}} = -\sqrt{\frac{1 + \frac{12}{13}}{2}} = -\sqrt{\frac{25}{26}} = -\frac{5\sqrt{26}}{26}$$

$$\tan\frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{1 - \frac{12}{13}}{\left(-\frac{5}{13}\right)} = -\frac{1}{5}$$

44. $\cot u = 3, \pi < u < \frac{3\pi}{2}$

(a) Because u is in Quadrant III, $\frac{u}{2}$ is in Quadrant II.

(b) $$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1 - \cos u}{2}} = \sqrt{\frac{1 + \frac{3}{\sqrt{10}}}{2}} = \sqrt{\frac{10 + 3\sqrt{10}}{20}} = \frac{1}{2}\sqrt{\frac{10 + 3\sqrt{10}}{5}}$$

$$\cos\left(\frac{u}{2}\right) = -\sqrt{\frac{1 + \cos u}{2}} = -\sqrt{\frac{1 - \frac{3}{\sqrt{10}}}{2}} = -\sqrt{\frac{10 - 3\sqrt{10}}{20}} = -\frac{1}{2}\sqrt{\frac{10 - 3\sqrt{10}}{5}}$$

$$\tan\left(\frac{u}{2}\right) = \frac{1 - \cos u}{\sin u} = \frac{1 + \frac{3}{\sqrt{10}}}{-\frac{1}{\sqrt{10}}} = -\sqrt{10} - 3$$

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45. $\sin\frac{x}{2} + \cos x = 0$

$$\pm\sqrt{\frac{1-\cos x}{2}} = -\cos x$$

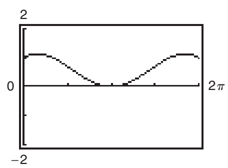
$$\frac{1-\cos x}{2} = \cos^2 x$$

$$0 = 2\cos^2 x + \cos x - 1$$

$$= (2\cos x - 1)(\cos x + 1)$$

$\cos x = \frac{1}{2}$ or $\cos x = -1$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$ $\quad x = \pi$



By checking these values in the original equation, $x = \pi/3$ and $x = 5\pi/3$ are extraneous, and $x = \pi$ is the only solution.

46. $h(x) = \sin\frac{x}{2} + \cos x - 1$

$$\sin\frac{x}{2} + \cos x - 1 = 0$$

$$\pm\sqrt{\frac{1-\cos x}{2}} = 1 - \cos x$$

$$\frac{1-\cos x}{2} = 1 - 2\cos x + \cos^2 x$$

$$1 - \cos x = 2 - 4\cos x + 2\cos^2 x$$

$$2\cos^2 x - 3\cos x + 1 = 0$$

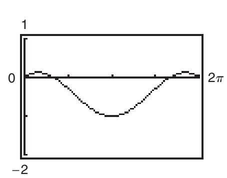
$$(2\cos x - 1)(\cos x - 1) = 0$$

$2\cos x - 1 = 0$ or $\cos x - 1 = 0$

$\cos x = \frac{1}{2}$ $\quad \cos x = 1$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$ $\quad x = 0$

$0, \frac{\pi}{3},$ and $\frac{5\pi}{3}$ are all solutions to the equation.



47. $\cos\frac{x}{2} - \sin x = 0$

$$\pm\sqrt{\frac{1+\cos x}{2}} = \sin x$$

$$\frac{1+\cos x}{2} = \sin^2 x$$

$$1 + \cos x = 2\sin^2 x$$

$$1 + \cos x = 2 - 2\cos^2 x$$

$$2\cos^2 x + \cos x - 1 = 0$$

$$(2\cos x - 1)(\cos x + 1) = 0$$

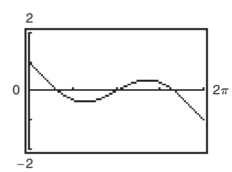
$2\cos x - 1 = 0$ or $\cos x + 1 = 0$

$\cos x = \frac{1}{2}$ $\quad \cos x = -1$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$ $\quad x = \pi$

$x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

$\pi/3, \pi,$ and $5\pi/3$ are all solutions to the equation.



48. $g(x) = \tan\frac{x}{2} - \sin x$

$$\tan\frac{x}{2} - \sin x = 0$$

$$\frac{1-\cos x}{\sin x} = \sin x$$

$$1 - \cos x = \sin^2 x$$

$$1 - \cos x = 1 - \cos^2 x$$

$$\cos^2 x - \cos x = 0$$

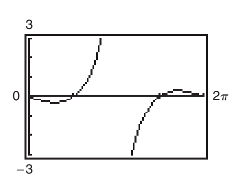
$$\cos x(\cos x - 1) = 0$$

$\cos x = 0$ or $\cos x - 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad \cos x = 1$

$x = 0$

$0, \frac{\pi}{2},$ and $\frac{3\pi}{2}$ are all solutions to the equation.



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49. $\sin 5\theta \sin 3\theta = \frac{1}{2}\left[\cos(5\theta - 3\theta) - \cos(5\theta + 3\theta)\right] = \frac{1}{2}(\cos 2\theta - \cos 8\theta)$

50. $7\cos(-5\beta)\sin 3\beta = 7 \cdot \frac{1}{2}\left[\sin(-5\beta + 3\beta) - \sin(-5\beta - 3\beta)\right] = \frac{7}{2}(\sin(-2\beta) - \sin(-8\beta))$

51. $\cos 2\theta \cos 4\theta = \frac{1}{2}\left[\cos(2\theta - 4\theta) + \cos(2\theta + 4\theta)\right] = \frac{1}{2}\left[\cos(-2\theta) + \cos 6\theta\right]$

52. $\sin(x + y)\cos(x - y) = \frac{1}{2}(\sin 2x + \sin 2y)$

53. $$\sin 5\theta - \sin 3\theta = 2\cos\left(\frac{5\theta + 3\theta}{2}\right)\sin\left(\frac{5\theta - 3\theta}{2}\right)$$
$$= 2\cos 4\theta \sin\theta$$

54. $$\sin 3\theta + \sin\theta = 2\sin\left(\frac{3\theta + \theta}{2}\right)\cos\left(\frac{3\theta - \theta}{2}\right)$$
$$= 2\sin 2\theta\cos\theta$$

55. $$\cos 6x + \cos 2x = 2\cos\left(\frac{6x + 2x}{2}\right)\cos\left(\frac{6x - 2x}{2}\right)$$
$$= 2\cos 4x\cos 2x$$

56. $$\cos x + \cos 4x = 2\cos\left(\frac{x + 4x}{2}\right)\cos\left(\frac{x - 4x}{2}\right)$$
$$= 2\cos\left(\frac{5x}{2}\right)\cos\left(\frac{-3x}{2}\right)$$

57. $$\sin 75° + \sin 15° = 2\sin\left(\frac{75° + 15°}{2}\right)\cos\left(\frac{75° - 15°}{2}\right) = 2\sin 45°\cos 30° = 2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{6}}{2}$$

58. $$\cos 120° + \cos 60° = 2\cos\left(\frac{120° + 60°}{2}\right)\cos\left(\frac{120° - 60°}{2}\right) = 2\cos 90°\cos 30° = 2(0)\left(\frac{\sqrt{3}}{2}\right) = 0$$

59. $$\cos\frac{3\pi}{4} - \cos\frac{\pi}{4} = -2\sin\left(\frac{\frac{3\pi}{4} + \frac{\pi}{4}}{2}\right)\sin\left(\frac{\frac{3\pi}{4} - \frac{\pi}{4}}{2}\right) = -2\sin\frac{\pi}{2}\sin\frac{\pi}{4}$$

$$\cos\frac{3\pi}{4} - \cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = -\sqrt{2}$$

60. $$\sin\frac{5\pi}{4} - \sin\frac{3\pi}{4} = 2\cos\left(\frac{\frac{5\pi}{4} + \frac{3\pi}{4}}{2}\right)\sin\left(\frac{\frac{5\pi}{4} - \frac{3\pi}{4}}{2}\right) = 2\cos\pi\sin\frac{\pi}{4}$$

$$\sin\frac{5\pi}{4} - \sin\frac{3\pi}{4} = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = -\sqrt{2}$$

61. $$\sin 6x + \sin 2x = 0$$
$$2\sin\left(\frac{6x + 2x}{2}\right)\cos\left(\frac{6x - 2x}{2}\right) = 0$$
$$2(\sin 4x)\cos 2x = 0$$

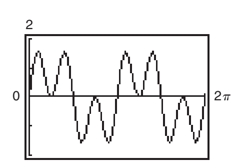
$\sin 4x = 0$ or $\cos 2x = 0$

$4x = n\pi$ $\qquad 2x = \frac{\pi}{2} + n\pi$

$x = \frac{n\pi}{4}$ $\qquad x = \frac{\pi}{4} + \frac{n\pi}{2}$

In the interval $[0, 2\pi)$

$x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}.$



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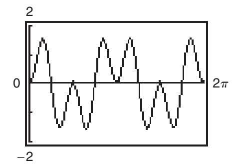
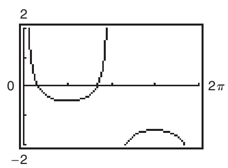
62.
$$h(x) = \cos 2x - \cos 6x$$
$$\cos 2x - \cos 6x = 0$$
$$-2\sin 4x \sin(-2x) = 0$$
$$2\sin 4x \sin 2x = 0$$

$$\sin 4x = 0 \qquad \text{or} \qquad \sin 2x = 0$$
$$4x = n\pi \qquad\qquad 2x = n\pi$$
$$x = \frac{n\pi}{4} \qquad\qquad x = \frac{n\pi}{2}$$
$$x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4} \qquad\qquad x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$$

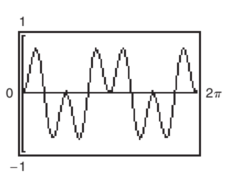


63.
$$\frac{\cos 2x}{\sin 3x - \sin x} - 1 = 0$$
$$\frac{\cos 2x}{\sin 3x - \sin x} = 1$$
$$\frac{\cos 2x}{2\cos 2x \sin x} = 1$$
$$2\sin x = 1$$
$$\sin x = \frac{1}{2}$$
$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$



64.
$$f(x) = \sin^2 3x - \sin^2 x$$
$$\sin^2 3x - \sin^2 x = 0$$
$$(\sin 3x + \sin x)(\sin 3x - \sin x) = 0$$
$$(2\sin 2x \cos x)(2\cos 2x \sin x) = 0$$

$$\sin 2x = 0 \Rightarrow x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2} \quad \text{or}$$
$$\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2} \quad \text{or}$$
$$\cos 2x = 0 \Rightarrow x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \quad \text{or}$$
$$\sin x = 0 \Rightarrow x = 0, \pi$$



65.
$$\csc 2\theta = \frac{1}{\sin 2\theta}$$
$$= \frac{1}{2\sin\theta\cos\theta}$$
$$= \frac{1}{\sin\theta} \cdot \frac{1}{2\cos\theta}$$
$$= \frac{\csc\theta}{2\cos\theta}$$

66.
$$\cos^4 x - \sin^4 x = (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$$
$$= (\cos 2x)(1)$$
$$= \cos 2x$$

67.
$$(\sin x + \cos x)^2 = \sin^2 x + 2\sin x\cos x + \cos^2 x$$
$$= (\sin^2 x + \cos^2 x) + 2\sin x\cos x$$
$$= 1 + \sin 2x$$

68.
$$\tan\frac{u}{2} = \frac{1 - \cos u}{\sin u}$$
$$= \frac{1}{\sin u} - \frac{\cos u}{\sin u}$$
$$= \csc u - \cot u$$

69.
$$\frac{\sin x \pm \sin y}{\cos x + \cos y} = \frac{2\sin\left(\frac{x \pm y}{2}\right)\cos\left(\frac{x \mp y}{2}\right)}{2\cos\left(\frac{x + y}{2}\right)\cos\left(\frac{x - y}{2}\right)}$$
$$= \tan\left(\frac{x \pm y}{2}\right)$$

70.
$$\cos\left(\frac{\pi}{3} + x\right) + \cos\left(\frac{\pi}{3} - x\right) = 2\cos\left(\frac{\frac{\pi}{3} + x + \frac{\pi}{3} - x}{2}\right)\cos\left(\frac{\frac{\pi}{3} + x - \left(\frac{\pi}{3} - x\right)}{2}\right)$$
$$= 2\cos\left(\frac{\pi}{3}\right)\cos(x)$$
$$= 2\left(\frac{1}{2}\right)\cos x = \cos x$$

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71. (a)

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}} = \frac{1}{M}$$

$$\left(\pm\sqrt{\frac{1-\cos\theta}{2}}\right)^2 = \left(\frac{1}{M}\right)^2$$

$$\frac{1-\cos\theta}{2} = \frac{1}{M^2}$$

$$M^2(1-\cos\theta) = 2$$

$$1-\cos\theta = \frac{2}{M^2}$$

$$-\cos\theta = \frac{2}{M^2} - 1$$

$$\cos\theta = 1 - \frac{2}{M^2}$$

$$\cos\theta = \frac{M^2-2}{M^2}$$

(b) When $M = 2$, $\cos\theta = \dfrac{2^2-2}{2^2} = \dfrac{1}{2}$. So, $\theta = \dfrac{\pi}{3}$.

(c) When $M = 4.5$, $\cos\theta = \dfrac{(4.5)^2 - 2}{(4.5)^2}$

$$\cos\theta \approx 0.901235.$$

So, $\theta \approx 0.4482$ radian.

(d) When $M = 2$, $\dfrac{\text{speed of object}}{\text{speed of sound}} = M$

$$\frac{\text{speed of object}}{760 \text{ mph}} = 2$$

speed of object = 1520 mph.

When $M = 4.5$, $\dfrac{\text{speed of object}}{\text{speed of sound}} = M$

$$\frac{\text{speed of object}}{760 \text{ mph}} = 4.5$$

speed of object = 3420 mph.

72. $\dfrac{1}{32}(75)^2 \sin 2\theta = 130$

$$\sin 2\theta = \frac{130(32)}{75^2}$$

$$\theta = \frac{1}{2}\sin^{-1}\left(\frac{130(32)}{75^2}\right)$$

$$\theta \approx 23.85°$$

73. $\dfrac{x}{2} = 2r\sin^2\dfrac{\theta}{2} = 2r\left(\dfrac{1-\cos\theta}{2}\right)$

$$= r(1-\cos\theta)$$

So, $x = 2r(1-\cos\theta)$.

74. (a) Using the graph, $\sin 2u \approx 1$ and $2\sin u\cos u \approx 2(0.7)(0.7) \approx 1$.

Because $1 = 1$, $\sin 2u = 2\sin u\cos u$.

(b) Using the graph, $\cos 2u \approx 0$ and $\cos^2 u - \sin^2 u \approx (0.7)^2 - (0.7)^2 = 0$.

Because $0 = 0$, $\cos 2u = \cos^2 u - \sin^2 u$.

75. True. Using the double angle formula and that sine is an odd function and cosine is an even function,

$$\begin{aligned}\sin(-2x) &= \sin[2(-x)]\\ &= 2\sin(-x)\cos(-x)\\ &= 2(-\sin x)\cos x\\ &= -2\sin x\cos x.\end{aligned}$$

76. False. If $90° < u < 180°$,

$\dfrac{u}{2}$ is in the first quadrant and

$$\sin\frac{u}{2} = \sqrt{\frac{1-\cos u}{2}}.$$

77. Because ϕ and θ are complementary angles, $\sin\phi = \cos\theta$ and $\cos\phi = \sin\theta$.

(a)
$$\begin{aligned}\sin(\phi-\theta) &= \sin\phi\cos\theta - \sin\theta\cos\phi\\ &= (\cos\theta)(\cos\theta) - (\sin\theta)(\sin\theta)\\ &= \cos^2\theta - \sin^2\theta\\ &= \cos 2\theta\end{aligned}$$

(b)
$$\begin{aligned}\cos(\phi-\theta) &= \cos\phi\cos\theta + \sin\phi\sin\theta\\ &= (\sin\theta)(\cos\theta) + (\cos\theta)(\sin\theta)\\ &= 2\sin\theta\cos\theta\\ &= \sin 2\theta\end{aligned}$$

Review Exercises for Chapter 5

1. $\cot x$

2. $\sec x$

3. $\cos x$

4. $\sqrt{\cot^2 x + 1} = \sqrt{\csc^2 x} = |\csc x|$

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5. $\cos\theta = -\frac{2}{5}$, $\tan\theta > 0$, θ is in Quadrant III.

$$\sec\theta = \frac{1}{\cos\theta} = -\frac{5}{2}$$

$$\sin\theta = -\sqrt{1-\cos^2\theta} = -\sqrt{1-\frac{4}{25}} = -\sqrt{\frac{21}{25}} = -\frac{\sqrt{21}}{5}$$

$$\csc\theta = \frac{1}{\sin\theta} = -\frac{5}{\sqrt{21}} = -\frac{5\sqrt{21}}{21}$$

$$\tan\theta = \frac{\sin}{\cos\theta} = \frac{-\frac{\sqrt{21}}{5}}{-\frac{2}{5}} = \frac{\sqrt{21}}{2}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{2}{\sqrt{21}} = \frac{2\sqrt{21}}{21}$$

6. $\cot x = -\frac{2}{3}$, $\cos x < 0$, x is in Quadrant II.

$$\tan x = \frac{1}{\cot x} = -\frac{3}{2}$$

$$\csc x = \sqrt{1+\cot^2 x} = \sqrt{1+\frac{4}{9}} = \sqrt{\frac{13}{9}} = \frac{\sqrt{13}}{3}$$

$$\sin x = \frac{1}{\csc x} = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

$$\cos x = -\sqrt{1-\sin^2 x} = -\sqrt{1-\frac{9}{13}} = -\sqrt{\frac{4}{13}} = -\frac{2}{\sqrt{13}} = -\frac{2\sqrt{13}}{13}$$

$$\sec x = \frac{1}{\cos x} = -\frac{\sqrt{13}}{2}$$

7. $$\frac{1}{\cot^2 x + 1} = \frac{1}{\csc^2 x} = \sin^2 x$$

8. $$\frac{\tan\theta}{1-\cos^2\theta} = \frac{\frac{\sin\theta}{\cos\theta}}{\sin^2\theta} = \frac{1}{\sin\theta\cos\theta} = \csc\theta\sec\theta$$

9. $$\tan^2 x(\csc^2 x - 1) = \tan^2 x(\cot^2 x) = \tan^2 x\left(\frac{1}{\tan^2 x}\right) = 1$$

10. $$\cot^2 x(\sin^2 x) = \frac{\cos^2 x}{\sin^2 x}\sin^2 x = \cos^2 x$$

11. $$\frac{\cot\left(\frac{\pi}{2}-u\right)}{\cos u} = \frac{\tan u}{\cos u} = \tan u\sec u$$

12. $$\frac{\sec^2(-\theta)}{\csc^2\theta} = \frac{\sec^2\theta}{\csc^2\theta} = \frac{1/\cos^2\theta}{1/\sin^2\theta} = \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$$

13. $$\cos^2 x + \cos^2 x\cot^2 x = \cos^2 x(1+\cot^2 x) = \cos^2 x(\csc^2 x) = \cos^2 x\left(\frac{1}{\sin^2 x}\right) = \frac{\cos^2 x}{\sin^2 x} = \cot^2 x$$

14. $$(\tan x + 1)^2\cos x = (\tan^2 x + 2\tan x + 1)\cos x = (\sec^2 x + 2\tan x)\cos x = \sec^2 x\cos x + 2\left(\frac{\sin x}{\cos x}\right)\cos x = \sec x + 2\sin x$$

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15. $\dfrac{1}{\csc\theta+1}-\dfrac{1}{\csc\theta-1}=\dfrac{(\csc\theta-1)-(\csc\theta+1)}{(\csc\theta+1)(\csc\theta-1)}$

$=\dfrac{-2}{\csc^2\theta-1}$

$=\dfrac{-2}{\cot^2\theta}$

$=-2\tan^2\theta$

16. $\dfrac{\tan^2 x}{1+\sec x}=\dfrac{\sec^2 x-1}{1+\sec x}$

$=\dfrac{(\sec x+1)(\sec x-1)}{\sec x+1}$

$=\sec x-1$

17. Let $x = 5\sin\theta$, then

$$\sqrt{25-x^2}=\sqrt{25-(5\sin\theta)^2}=\sqrt{25-25\sin^2\theta}=\sqrt{25(1-\sin^2\theta)}=\sqrt{25\cos^2\theta}=5\cos\theta.$$

18. Let $x = 4\sec\theta$, then

$$\sqrt{x^2-16}=\sqrt{(4\sec\theta)^2-16}=\sqrt{16\sec^2\theta-16}=\sqrt{16(\sec^2\theta-1)}=\sqrt{16\tan^2\theta}=4\tan\theta.$$

19. $\cos x(\tan^2 x+1)=\cos x\sec^2 x$

$=\dfrac{1}{\sec x}\sec^2 x$

$=\sec x$

20. $\sec^2 x\cot x-\cot x=\cot x(\sec^2 x-1)$

$=\cot x\tan^2 x$

$=\left(\dfrac{1}{\tan x}\right)\tan^2 x=\tan x$

21. $\sin\left(\dfrac{\pi}{2}-\theta\right)\tan\theta=\cos\theta\tan\theta$

$=\cos\theta\left(\dfrac{\sin\theta}{\cos\theta}\right)$

$=\sin\theta$

22. $\cot\left(\dfrac{\pi}{2}-\theta\right)\csc\theta=\tan\theta\csc\theta$

$=\left(\dfrac{\sin\theta}{\cos\theta}\right)\left(\dfrac{1}{\sin\theta}\right)$

$=\dfrac{1}{\cos\theta}$

$=\sec\theta$

23. $\dfrac{1}{\tan\theta\csc\theta}=\dfrac{1}{\dfrac{\sin\theta}{\cos\theta}\cdot\dfrac{1}{\sin\theta}}=\cos\theta$

24. $\dfrac{1}{\tan x\csc x\sin x}=\dfrac{1}{(\tan x)\left(\dfrac{1}{\sin x}\right)(\sin x)}=\dfrac{1}{\tan x}$

$=\cot x$

25. $\sin^5 x\cos^2 x=\sin^4 x\cos^2 x\sin x$

$=(1-\cos^2 x)^2\cos^2 x\sin x$

$=(1-2\cos^2 x+\cos^4 x)\cos^2 x\sin x$

$=(\cos^2 x-2\cos^4 x+\cos^6 x)\sin x$

26. $\cos^3 x\sin^2 x=\cos x\cos^2 x\sin^2 x$

$=\cos x(1-\sin^2 x)\sin^2 x$

$=\cos x(\sin^2 x-\sin^4 x)$

$=(\sin^2 x-\sin^4 x)\cos x$

27. $\sin x=\sqrt{3}-\sin x$

$\sin x=\dfrac{\sqrt{3}}{2}$

$x=\dfrac{\pi}{3}+2\pi n,\ \dfrac{2\pi}{3}+2\pi n$

28. $4\cos\theta=1+2\cos\theta$

$2\cos\theta=1$

$\cos\theta=\dfrac{1}{2}$

$\theta=\dfrac{\pi}{3}+2n\pi$ or $\dfrac{5\pi}{3}+2n\pi$

29. $3\sqrt{3}\tan u=3$

$\tan u=\dfrac{1}{\sqrt{3}}$

$u=\dfrac{\pi}{6}+n\pi$

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30. $\frac{1}{2}\sec x - 1 = 0$

$$\frac{1}{2}\sec x = 1$$
$$\sec x = 2$$
$$\cos x = \frac{1}{2}$$
$$x = \frac{\pi}{3} + 2n\pi \quad \text{or} \quad \frac{5\pi}{3} + 2n\pi$$

31. $3\csc^2 x = 4$

$$\csc^2 x = \frac{4}{3}$$
$$\sin x = \pm\frac{\sqrt{3}}{2}$$
$$x = \frac{\pi}{3} + 2\pi n, \frac{2\pi}{3} + 2\pi n, \frac{4\pi}{3} + 2\pi n, \frac{5\pi}{3} + 2\pi n$$

These can be combined as:

$$x = \frac{\pi}{3} + n\pi \quad \text{or} \quad x = \frac{2\pi}{3} + n\pi$$

32. $4\tan^2 u - 1 = \tan^2 u$

$$3\tan^2 u - 1 = 0$$
$$\tan^2 u = \frac{1}{3}$$
$$\tan u = \pm\frac{1}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}$$
$$u = \frac{\pi}{6} + n\pi \quad \text{or} \quad \frac{5\pi}{6} + n\pi$$

33. $\sin^3 x = \sin x$

$$\sin^3 x - \sin x = 0$$
$$\sin x(\sin^2 x - 1) = 0$$
$$\sin x = 0 \Rightarrow x = 0, \pi$$
$$\sin^2 x = 1$$
$$\sin x = \pm 1 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$$

34. $2\cos^2 x + 3\cos x = 0$

$$\cos x(2\cos x + 3) = 0$$
$$\cos x = 0 \quad \text{or} \quad 2\cos x + 3 = 0$$
$$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad 2\cos x = -3$$
$$\cos x = -\frac{3}{2}$$

No solution

35. $\cos^2 x + \sin x = 1$

$$1 - \sin^2 x + \sin x - 1 = 0$$
$$-\sin x(\sin x - 1) = 0$$
$$\sin x = 0 \qquad \sin x - 1 = 0$$
$$x = 0, \pi \qquad \sin x = 1$$
$$x = \frac{\pi}{2}$$

36. $\sin^2 x + 2\cos x = 2$

$$1 - \cos^2 x + 2\cos x = 2$$
$$0 = \cos^2 x - 2\cos x + 1$$
$$0 = (\cos x - 1)^2$$
$$\cos x - 1 = 0$$
$$\cos x = 1$$
$$x = 0$$

37. $2\sin 2x - \sqrt{2} = 0$

$$\sin 2x = \frac{\sqrt{2}}{2}$$
$$2x = \frac{\pi}{4} + 2\pi n, \frac{3\pi}{4} + 2\pi n$$
$$x = \frac{\pi}{8} + \pi n, \frac{3\pi}{8} + \pi n$$
$$x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$$

38. $2\cos\frac{x}{2} + 1 = 0$

$$\cos\frac{x}{2} = -\frac{1}{2}$$
$$\frac{x}{2} = \frac{2\pi}{3}$$
$$x = \frac{4\pi}{3}$$

39. $3\tan^2\left(\frac{x}{3}\right) - 1 = 0$

$$\tan^2\left(\frac{x}{3}\right) = \frac{1}{3}$$
$$\tan\frac{x}{3} = \pm\sqrt{\frac{1}{3}}$$
$$\tan\frac{x}{3} = \pm\frac{\sqrt{3}}{3}$$
$$\frac{x}{3} = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$$
$$x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$

$\frac{5\pi}{2}$ and $\frac{7\pi}{2}$ are greater than 2π, so they are not solutions. The solution is $x = \frac{\pi}{2}$.

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40. $\sqrt{3}\tan 3x = 0$

$\tan 3x = 0$

$3x = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi$

$x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$

41. $\cos 4x(\cos x - 1) = 0$

$\cos 4x = 0$ $\qquad$ $\cos x - 1 = 0$

$4x = \frac{\pi}{2} + 2\pi n, \frac{3\pi}{2} + 2\pi n$ $\qquad$ $\cos x = 1$

$x = \frac{\pi}{8} + \frac{\pi}{2}n, \frac{3\pi}{8} + \frac{\pi}{2}n$ $\qquad$ $x = 0$

$x = 0, \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$

42. $3\csc^2 5x = -4$

$\csc^2 5x = -\frac{4}{3}$

$\csc 5x = \pm\sqrt{-\frac{4}{3}}$

No real solution

43. $\tan^2 x - 2\tan x = 0$

$\tan x(\tan x - 2) = 0$

$\tan x = 0$ or $\tan x - 2 = 0$

$x = n\pi$ $\qquad$ $\tan x = 2$

$x = \arctan 2 + n\pi$

44. $2\tan^2 x - 3\tan x = -1$

$2\tan^2 x - 3\tan x + 1 = 0$

$(2\tan x - 1)(\tan x - 1) = 0$

$2\tan x - 1 = 0$ or $\tan x - 1 = 0$

$2\tan x = 1$ $\qquad$ $\tan x = 1$

$\tan x = \frac{1}{2}$ $\qquad$ $x = \frac{\pi}{4} + n\pi$

$x = \arctan\left(\frac{1}{2}\right) + n\pi$

45. $\tan^2\theta + \tan\theta - 6 = 0$

$(\tan\theta + 3)(\tan\theta - 2) = 0$

$\tan\theta + 3 = 0$ or $\tan\theta - 2 = 0$

$\tan\theta = -3$ $\qquad$ $\tan\theta = 2$

$\theta = \arctan(-3) + n\pi$ $\qquad$ $\theta = \arctan 2 + n\pi$

46. $\sec^2 x + 6\tan x + 4 = 0$

$1 + \tan^2 x + 6\tan x + 4 = 0$

$\tan^2 x + 6\tan x + 5 = 0$

$(\tan x + 5)(\tan x + 1) = 0$

$\tan x + 5 = 0$ or $\tan x + 1 = 0$

$\tan x = -5$ $\qquad$ $\tan x = -1$

$x = \arctan(-5) + n\pi$ $\qquad$ $x = \frac{3\pi}{4} + n\pi$

47. $\sin 75° = \sin(120° - 45°)$

$= \sin 120°\cos 45° - \cos 120°\sin 45°$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$

$\cos 75° = \cos(120° - 45°)$

$= \cos 120°\cos 45° + \sin 120°\sin 45°$

$= \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{4}\left(\sqrt{3} - 1\right)$

$\tan 75° = \tan(120° - 45°) = \frac{\tan 120° - \tan 45°}{1 + \tan 120°\tan 45°}$

$= \frac{-\sqrt{3} - 1}{1 + \left(-\sqrt{3}\right)(1)} = \frac{-\sqrt{3} - 1}{1 - \sqrt{3}}$

$= \frac{-\sqrt{3} - 1}{1 - \sqrt{3}} \cdot \frac{1 + \sqrt{3}}{1 + \sqrt{3}}$

$= \frac{-4 - 2\sqrt{3}}{-2} = 2 + \sqrt{3}$

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48. $\sin(375°) = \sin(135° + 240°)$

$= \sin 135° \cos 240° + \cos 135° \sin 240°$

$= \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{2}}{4}\left(\sqrt{3} - 1\right)$

$\cos(375°) = \cos(135° + 240°)$

$= \cos 135° \cos 240° - \sin 135° \sin 240°$

$= \left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{2}}{4}\left(1 + \sqrt{3}\right)$

$\tan(375°) = \tan(135° + 240°)$

$= \frac{\tan 135° + \tan 240°}{1 - \tan 135° \tan 240°}$

$= \frac{-1 + \sqrt{3}}{1 - (-1)\left(\sqrt{3}\right)}$

$= \frac{-1 + \sqrt{3}}{1 + \sqrt{3}} \cdot \frac{1 - \sqrt{3}}{1 - \sqrt{3}} = \frac{-4 + 2\sqrt{3}}{1 - 3} = 2 - \sqrt{3}$

49. $\sin \frac{25\pi}{12} = \sin\left(\frac{11\pi}{6} + \frac{\pi}{4}\right) = \sin \frac{11\pi}{6} \cos \frac{\pi}{4} + \cos \frac{11\pi}{6} \sin \frac{\pi}{4}$

$= \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{4}\left(\sqrt{3} - 1\right)$

$\cos \frac{25\pi}{12} = \cos\left(\frac{11\pi}{6} + \frac{\pi}{4}\right) = \cos \frac{11\pi}{6} \cos \frac{\pi}{4} - \sin \frac{11\pi}{6} \sin \frac{\pi}{4}$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$

$\tan \frac{25\pi}{12} = \tan\left(\frac{11\pi}{6} + \frac{\pi}{4}\right) = \frac{\tan \frac{11\pi}{6} + \tan \frac{\pi}{4}}{1 - \tan \frac{11\pi}{6} \tan \frac{\pi}{4}}$

$= \frac{\left(-\frac{\sqrt{3}}{3}\right) + 1}{1 - \left(-\frac{\sqrt{3}}{3}\right)(1)} = 2 - \sqrt{3}$

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50. $\sin\left(\frac{19\pi}{12}\right) = \sin\left(\frac{11\pi}{6} - \frac{\pi}{4}\right)$

$= \sin\frac{11\pi}{6}\cos\frac{\pi}{4} - \cos\frac{11\pi}{6}\sin\frac{\pi}{4}$

$= -\frac{1}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$

$= -\frac{\sqrt{2}}{4}\left(1 + \sqrt{3}\right) = -\frac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$

$\cos\left(\frac{19\pi}{12}\right) = \cos\left(\frac{11\pi}{6} - \frac{\pi}{4}\right)$

$= \cos\frac{11\pi}{6}\cos\frac{\pi}{4} + \sin\frac{11\pi}{6}\sin\frac{\pi}{4}$

$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \left(-\frac{1}{2}\right)\frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{4}\left(\sqrt{3} - 1\right)$

$\tan\left(\frac{19\pi}{12}\right) = \tan\left(\frac{11\pi}{6} - \frac{\pi}{4}\right)$

$= \dfrac{\tan\frac{11\pi}{6} - \tan\frac{\pi}{4}}{1 + \tan\frac{11\pi}{6}\tan\frac{\pi}{4}}$

$= \dfrac{-\frac{\sqrt{3}}{3} - 1}{1 + \left(-\frac{\sqrt{3}}{3}\right)(1)} = \frac{-\sqrt{3} - 3}{3 - \sqrt{3}}\cdot\frac{3 + \sqrt{3}}{3 + \sqrt{3}}$

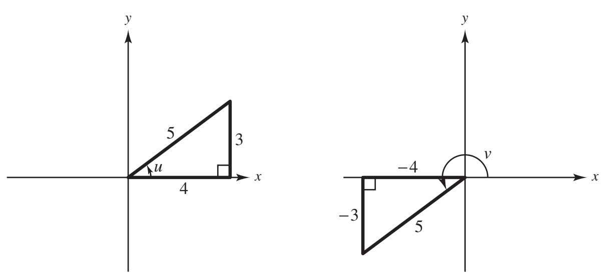
$= \dfrac{-\left(12 + 6\sqrt{3}\right)}{6} = -2 - \sqrt{3}$

51. $\sin 60^\circ \cos 45^\circ - \cos 60^\circ \sin 45^\circ = \sin\left(60^\circ - 45^\circ\right)$

$= \sin 15^\circ$

52. $\dfrac{\tan 68^\circ - \tan 115^\circ}{1 + \tan 68^\circ \tan 115^\circ} = \tan\left(68^\circ - 115^\circ\right)$

$= \tan\left(-47^\circ\right)$



Figures for Exercises 53–56

53. $\sin(u + v) = \sin u \cos v + \cos u \sin v = \frac{3}{5}\left(-\frac{4}{5}\right) + \frac{4}{5}\left(-\frac{3}{5}\right) = -\frac{24}{25}$

54. $\tan(u + v) = \dfrac{\tan u + \tan v}{1 - \tan u \tan v} = \dfrac{\frac{3}{4} + \frac{3}{4}}{1 - \frac{3}{4}\left(\frac{3}{4}\right)} = \dfrac{\frac{3}{2}}{\frac{7}{16}} = \frac{3}{2}\left(\frac{16}{7}\right) = \frac{24}{7}$

55. $\cos(u - v) = \cos u \cos v + \sin u \sin v = \frac{4}{5}\left(-\frac{4}{5}\right) + \frac{3}{5}\left(-\frac{3}{5}\right) = -1$

56. $\sin(u - v) = \sin u \cos v - \cos u \sin v = \frac{3}{5}\left(-\frac{4}{5}\right) - \frac{4}{5}\left(-\frac{3}{5}\right) = 0$

57. $\cos\left(x + \frac{\pi}{2}\right) = \cos x \cos\frac{\pi}{2} - \sin x \sin\frac{\pi}{2} = \cos x(0) - \sin x(1) = -\sin x$

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58. $\tan\left(x - \frac{\pi}{2}\right) = -\tan\left(\frac{\pi}{2} - x\right) = -\cot x$

59. $\tan(\pi - x) = \frac{\tan \pi - \tan x}{1 - \tan \pi \tan x} = -\tan x$

60. $\sin(x - \pi) = \sin x \cos \pi - \cos x \sin \pi$

$= \sin x(-1) - \cos x(0)$

$= -\sin x$

61. $\sin\left(x + \frac{\pi}{4}\right) - \sin\left(x - \frac{\pi}{4}\right) = 1$

$2 \cos x \sin \frac{\pi}{4} = 1$

$\cos x = \frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}, \frac{7\pi}{4}$

62. $\cos\left(x + \frac{\pi}{6}\right) - \cos\left(x - \frac{\pi}{6}\right) = 1$

$\left(\cos x \cos \frac{\pi}{6} - \sin x \sin \frac{\pi}{6}\right) - \left(\cos x \cos \frac{\pi}{6} + \sin x \sin \frac{\pi}{6}\right) = 1$

$-2 \sin x \sin \frac{\pi}{6} = 1$

$-2 \sin x\left(\frac{1}{2}\right) = 1$

$\sin x = -1$

$x = \frac{3\pi}{2}$

63. $\sin u = -\frac{4}{5}, \pi < u < \frac{3\pi}{2}$

$\cos u = -\sqrt{1 - \sin^2 u} = \frac{-3}{5}$

$\tan u = \frac{\sin u}{\cos u} = \frac{4}{3}$

$\sin 2u = 2 \sin u \cos u = 2\left(-\frac{4}{5}\right)\left(-\frac{3}{5}\right) = \frac{24}{25}$

$\cos 2u = \cos^2 u - \sin^2 u = \left(-\frac{3}{5}\right)^2 - \left(-\frac{4}{5}\right)^2 = -\frac{7}{25}$

$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u} = \frac{2\left(\frac{4}{3}\right)}{1 - \left(\frac{4}{3}\right)^2} = -\frac{24}{7}$

64. $\cos u = -\frac{2}{\sqrt{5}}, \frac{\pi}{2} < u < \pi \Rightarrow \sin u = \frac{1}{\sqrt{5}}$ and

$\tan u = -\frac{1}{2}$

$\sin 2u = 2 \sin u \cos u = 2\left(\frac{1}{\sqrt{5}}\right)\left(-\frac{2}{\sqrt{5}}\right) = -\frac{4}{5}$

$\cos 2u = \cos^2 u - \sin^2 u = \left(-\frac{2}{\sqrt{5}}\right)^2 - \left(\frac{1}{\sqrt{5}}\right)^2 = \frac{3}{5}$

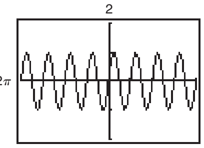
$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u} = \frac{2\left(-\frac{1}{2}\right)}{1 - \left(-\frac{1}{2}\right)^2} = \frac{-1}{\frac{3}{4}} = -\frac{4}{3}$

65. $\sin 4x = 2 \sin 2x \cos 2x$

$= 2\left[2 \sin x \cos x\left(\cos^2 x - \sin^2 x\right)\right]$

$= 4 \sin x \cos x\left(2 \cos^2 x - 1\right)$

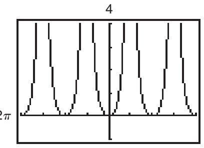
$= 8 \cos^3 x \sin x - 4 \cos x \sin x$



66. $\frac{1 - \cos 2x}{1 + \cos 2x} = \frac{1 - \left(1 - 2 \sin^2 x\right)}{1 + \left(2 \cos x^2 - 1\right)}$

$= \frac{2 \sin^2 x}{2 \cos^2 x}$

$= \tan^2 x$



67. $\tan^2 3x = \frac{\sin^2 3x}{\cos^2 3x} = \frac{\frac{1 - \cos 6x}{2}}{\frac{1 + \cos 6x}{2}} = \frac{1 - \cos 6x}{1 + \cos 6x}$

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68. $\sin^2 x \cos^2 x = \left(\dfrac{1 - \cos 2x}{2}\right)\left(\dfrac{1 + \cos 2x}{2}\right)$

$$= \frac{1 - \cos^2 2x}{4}$$

$$= \frac{1 - \left(\dfrac{1 + \cos 4x}{2}\right)}{4}$$

$$= \frac{1 - \cos 4x}{8}$$

69. $\sin(-75°) = -\sqrt{\dfrac{1 - \cos 150°}{2}} = -\sqrt{\dfrac{1 - \left(-\dfrac{\sqrt{3}}{2}\right)}{2}} = -\dfrac{\sqrt{2 + \sqrt{3}}}{2} = -\dfrac{1}{2}\sqrt{2 + \sqrt{3}}$

$\cos(-75°) = -\sqrt{\dfrac{1 + \cos 150°}{2}} = \sqrt{\dfrac{1 + \left(-\dfrac{\sqrt{3}}{2}\right)}{2}} = \dfrac{\sqrt{2 - \sqrt{3}}}{2} = \dfrac{1}{2}\sqrt{2 - \sqrt{3}}$

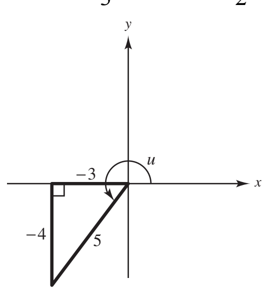
$\tan(-75°) = -\left(\dfrac{1 - \cos 150°}{\sin 150°}\right) = -\left(\dfrac{1 - \left(-\dfrac{\sqrt{3}}{2}\right)}{\dfrac{1}{2}}\right) = -\left(2 + \sqrt{3}\right) = -2 - \sqrt{3}$

70. $\sin\left(\dfrac{5\pi}{12}\right) = \sqrt{\dfrac{1 - \cos\dfrac{5\pi}{6}}{2}} = \sqrt{\dfrac{1 - \left(-\dfrac{\sqrt{3}}{2}\right)}{2}} = \dfrac{\sqrt{2 + \sqrt{3}}}{2} = \dfrac{1}{2}\sqrt{2 + \sqrt{3}}$

$\cos\left(\dfrac{5\pi}{12}\right) = \sqrt{\dfrac{1 + \cos\dfrac{5\pi}{6}}{2}} = \sqrt{\dfrac{1 + \left(-\dfrac{\sqrt{3}}{2}\right)}{2}} = \dfrac{\sqrt{2 - \sqrt{3}}}{2} = \dfrac{1}{2}\sqrt{2 - \sqrt{3}}$

$\tan\left(\dfrac{5\pi}{12}\right) = \dfrac{1 - \cos\dfrac{5\pi}{6}}{\sin\dfrac{5\pi}{6}} = \dfrac{1 - \left(-\dfrac{\sqrt{3}}{2}\right)}{\dfrac{1}{2}} = 2 + \sqrt{3}$

71. $\tan u = \dfrac{4}{3}, \pi < u < \dfrac{3\pi}{2}$



(a) Because u is in Quadrant III, $\dfrac{u}{2}$ is in Quadrant II.

(b) $\sin\dfrac{u}{2} = \sqrt{\dfrac{1 - \cos u}{2}} = \sqrt{\dfrac{1 - \left(-\dfrac{3}{5}\right)}{2}} = \sqrt{\dfrac{4}{5}}$

$$= \frac{2\sqrt{5}}{5}$$

$\cos\dfrac{u}{2} = -\sqrt{\dfrac{1 + \cos u}{2}} = -\sqrt{\dfrac{1 + \left(-\dfrac{3}{5}\right)}{2}} = -\sqrt{\dfrac{1}{5}}$

$$= -\frac{\sqrt{5}}{5}$$

$\tan\dfrac{u}{2} = \dfrac{1 - \cos u}{\sin u} = \dfrac{1 - \left(-\dfrac{3}{5}\right)}{\left(-\dfrac{4}{5}\right)} = -2$

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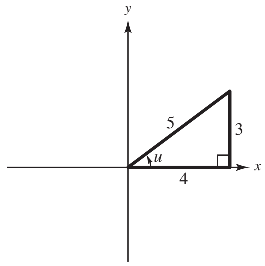
72. $\sin u = \frac{3}{5}, 0 < u < \frac{\pi}{2}$



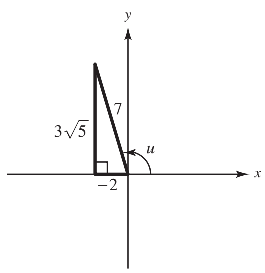
(a) Because u is in Quadrant I, $\frac{u}{2}$ is in Quadrant I.

(b) $\sin\frac{u}{2} = \sqrt{\frac{1 - \cos u}{2}} = \sqrt{\frac{1 - \left(\frac{4}{5}\right)}{2}} = \sqrt{\frac{1}{10}} = \frac{\sqrt{10}}{10}$

$\cos\frac{u}{2} = \sqrt{\frac{1 + \cos u}{2}} = \sqrt{\frac{1 + \left(\frac{4}{5}\right)}{2}} = \sqrt{\frac{9}{10}} = \frac{3\sqrt{10}}{10}$

$\tan\frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{1 - \left(\frac{4}{5}\right)}{\frac{3}{5}} = \frac{1}{3}$

73. $\cos u = -\frac{2}{7}, \frac{\pi}{2} < u < \pi$



(a) Because u is in Quadrant II, $\frac{u}{2}$ is in Quadrant I.

(b) $\sin\frac{u}{2} = \sqrt{\frac{1 - \cos u}{2}} = \sqrt{\frac{1 - \left(-\frac{2}{7}\right)}{2}} = \sqrt{\frac{9}{14}}$

$= \frac{3\sqrt{14}}{14}$

$\cos\frac{u}{2} = \sqrt{\frac{1 + \cos u}{2}} = \sqrt{\frac{1 + \left(-\frac{2}{7}\right)}{2}} = \sqrt{\frac{5}{14}}$

$= \frac{\sqrt{70}}{14}$

$\tan\frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{1 - \left(-\frac{2}{7}\right)}{\frac{3\sqrt{5}}{7}} = \frac{3\sqrt{5}}{5}$

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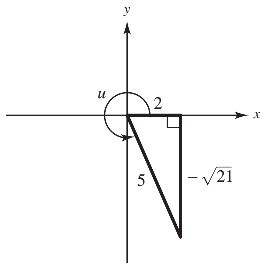
74. $\tan u = -\frac{\sqrt{21}}{2}, \frac{3\pi}{2} < u < 2\pi$



(a) Because u is in Quadrant IV, $\frac{u}{2}$ is in Quadrant II.

(b) $$\sin\frac{u}{2} = \sqrt{\frac{1-\cos u}{2}} = \sqrt{\frac{1-\left(\frac{2}{5}\right)}{2}} = \sqrt{\frac{3}{10}} = \frac{\sqrt{30}}{10}$$

$$\cos\frac{u}{2} = -\sqrt{\frac{1+\cos u}{2}} = -\sqrt{\frac{1+\left(\frac{2}{5}\right)}{2}} = -\sqrt{\frac{7}{10}} = -\frac{\sqrt{70}}{10}$$

$$\tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{1-\left(\frac{2}{5}\right)}{\left(-\frac{\sqrt{21}}{5}\right)} = -\frac{3}{\sqrt{21}} = -\frac{3\sqrt{21}}{21} = -\frac{\sqrt{21}}{7}$$

75. $\cos 4\theta \sin 6\theta = \frac{1}{2}\left[\sin(4\theta + 6\theta) - \sin(4\theta - 6\theta)\right] = \frac{1}{2}\left[\sin 10\theta - \sin(-2\theta)\right]$

76. $2\sin 7\theta \cos 3\theta = 2 \cdot \frac{1}{2}\left[\sin(7\theta + 3\theta) + \sin(7\theta - 3\theta)\right] = \sin 10\theta + \sin 4\theta$

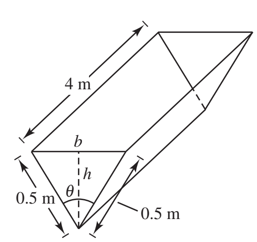
77. $\cos 6\theta + \cos 5\theta = 2\cos\left(\frac{6\theta + 5\theta}{2}\right)\cos\left(\frac{6\theta - 5\theta}{2}\right) = 2\cos\frac{11\theta}{2}\cos\frac{\theta}{2}$

78. $$\begin{aligned}\sin 3x - \sin x &= 2\cos\left(\frac{3x + x}{2}\right)\sin\left(\frac{3x - x}{2}\right)\\ &= 2\cos 2x \sin x\end{aligned}$$

79. $$\begin{aligned} r &= \frac{1}{32}v_0^2 \sin 2\theta\\ \text{range} &= 100 \text{ feet}\\ v_0 &= 80 \text{ feet per second}\\ r &= \frac{1}{32}(80)^2 \sin 2\theta = 100\\ \sin 2\theta &= 0.5\\ 2\theta &= 30^\circ\\ \theta &= 15^\circ \text{ or } \frac{\pi}{12}\end{aligned}$$

80. Volume V of the trough will be the area A of the isosceles triangle times the length l of the trough.

$V = A \cdot l$



Not drawn to scale

(a) $$A = \frac{1}{2}bh$$

$$\cos\frac{\theta}{2} = \frac{h}{0.5} \Rightarrow h = 0.5\cos\frac{\theta}{2}$$

$$\sin\frac{\theta}{2} = \frac{\frac{b}{2}}{0.5} \Rightarrow \frac{b}{2} = 0.5\sin\frac{\theta}{2}$$

$$A = 0.5\sin\frac{\theta}{2}\,0.5\cos\frac{\theta}{2} = (0.5)^2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 0.25\sin\frac{\theta}{2}\cos\frac{\theta}{2} \text{ square meters}$$

$$V = (0.25)(4)\sin\frac{\theta}{2}\cos\frac{\theta}{2} \text{ cubic meters} = \sin\frac{\theta}{2}\cos\frac{\theta}{2} \text{ cubic meters}$$

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(b) $V = \sin\frac{\theta}{2}\cos\frac{\theta}{2} = \frac{1}{2}\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right) = \frac{1}{2}\sin\theta$ cubic meters

Volume is maximum when $\theta = \frac{\pi}{2}$.

81. False. If $\frac{\pi}{2} < \theta < \pi$, then $\frac{\pi}{4} < \frac{\theta}{2} < \frac{\pi}{2}$, and $\frac{\theta}{2}$ is in Quadrant I. $\cos\frac{\theta}{2} > 0$

82. True. $\cot x \sin^2 x = \left(\frac{\cos x}{\sin x}\right)\sin^2 x = \cos x \sin x$.

83. True. $4\sin(-x)\cos(-x) = 4(-\sin x)\cos x$

$$= -4\sin x\cos x$$
$$= -2(2\sin x\cos x)$$
$$= -2\sin 2x$$

84. True. It can be verified using a product-to-sum formula.

$$4\sin 45^\circ \cos 15^\circ = 4\cdot\frac{1}{2}[\sin 60^\circ + \sin 30^\circ]$$
$$= 2\left[\frac{\sqrt{3}}{2} + \frac{1}{2}\right] = \sqrt{3} + 1$$

85. Yes. *Sample Answer*. When the domain is all real numbers, the solutions of $\sin x = \frac{1}{2}$ are $x = \frac{\pi}{6} + 2n\pi$ and $x = \frac{5\pi}{6} + 2n\pi$, so there are infinitely many solutions.

Problem Solving for Chapter 5

1. $\sin\theta = \pm\sqrt{1 - \cos^2\theta}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \pm\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$$
$$\csc\theta = \frac{1}{\sin\theta} = \pm\frac{1}{\sqrt{1-\cos^2\theta}}$$
$$\sec\theta = \frac{1}{\cos\theta}$$
$$\cot\theta = \frac{1}{\tan\theta} = \pm\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$$

You also have the following relationships:

$$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$$
$$\tan\theta = \frac{\cos[(\pi/2) - \theta]}{\cos\theta}$$
$$\csc\theta = \frac{1}{\cos[(\pi/2) - \theta]}$$
$$\sec\theta = \frac{1}{\cos\theta}$$
$$\cot\theta = \frac{\cos\theta}{\cos[(\pi/2) - \theta]}$$

2. $\cos\left[\frac{(2n+1)\pi}{2}\right] = \cos\left(\frac{2n\pi + \pi}{2}\right)$

$$= \cos\left(n\pi + \frac{\pi}{2}\right)$$
$$= \cos n\pi\cos\frac{\pi}{2} - \sin n\pi\sin\frac{\pi}{2}$$
$$= (\pm 1)(0) - (0)(1)$$
$$= 0$$

So, $\cos\left[\frac{(2n+1)\pi}{2}\right] = 0$ for all integers n.

3. $\sin\left[\frac{(12n+1)\pi}{6}\right] = \sin\left[\frac{1}{6}(12n\pi + \pi)\right]$

$$= \sin\left(2n\pi + \frac{\pi}{6}\right)$$
$$= \sin\frac{\pi}{6} = \frac{1}{2}$$

So, $\sin\left[\frac{(12n+1)\pi}{6}\right] = \frac{1}{2}$ for all integers n.

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4. $p(t) = \dfrac{1}{4\pi}\left[p_1(t) + 30p_2(t) + p_3(t) + p_5(t) + 30p_6(t)\right]$

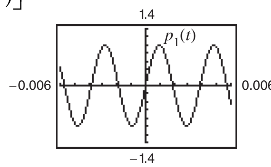
(a) $p_1(t) = \sin(524\pi t)$

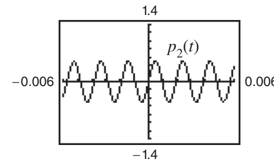
$p_2(t) = \dfrac{1}{2}\sin(1048\pi t)$

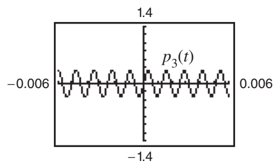
$p_3(t) = \dfrac{1}{3}\sin(1572\pi t)$

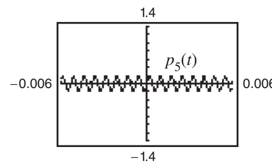
$p_5(t) = \dfrac{1}{5}\sin(2620\pi t)$

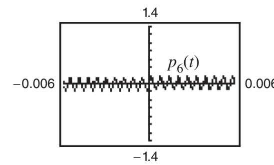
$p_6(t) = \dfrac{1}{6}\sin(3144\pi t)$







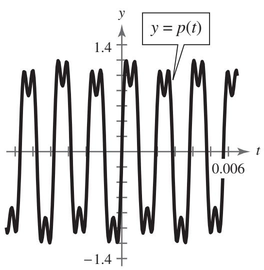




The graph of

$$p(t) = \frac{1}{4\pi}\left[\sin(524\pi t) + 15\sin(1048\pi t) + \frac{1}{3}\sin(1572\pi t) + \frac{1}{5}\sin(2620\pi t) + 5\sin(3144\pi t)\right]$$

yields the graph shown in the text below.

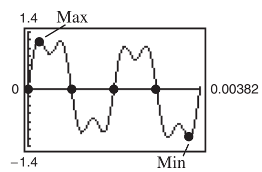


(b) Function Period

Function	Period
$p_1(t)$	$\dfrac{2\pi}{524\pi} = \dfrac{1}{262} \approx 0.0038$
$p_2(t)$	$\dfrac{2\pi}{1048\pi} = \dfrac{1}{524} \approx 0.0019$
$p_3(t)$	$\dfrac{2\pi}{1572\pi} = \dfrac{1}{786} \approx 0.0013$
$p_5(t)$	$\dfrac{2\pi}{2620\pi} = \dfrac{1}{1310} \approx 0.0008$
$p_6(t)$	$\dfrac{2\pi}{3144\pi} = \dfrac{1}{1572} \approx 0.0006$

The graph of p appears to be periodic with a period of $\dfrac{1}{262} \approx 0.0038$.

(c)



Over one cycle, $0 \le t < \dfrac{1}{262}$, you have five t-intercepts:

$t = 0, t \approx 0.00096, t \approx 0.00191, t \approx 0.00285,$

$t \approx 0.00382$

(d) The absolute maximum value of p over one cycle is $p \approx 1.1952$, and the absolute minimum value of p over one cycle is $p \approx -1.1952$.

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5. From the figure, it appears that $u + v = w$. Assume that u, v, and w are all in Quadrant I.

From the figure:

$$\tan u = \frac{s}{3s} = \frac{1}{3}$$

$$\tan v = \frac{s}{2s} = \frac{1}{2}$$

$$\tan w = \frac{s}{s} = 1$$

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} = \frac{1/3 + 1/2}{1 - (1/3)(1/2)} = \frac{5/6}{1 - (1/6)} = 1 = \tan w.$$

So, $\tan(u + v) = \tan w$. Because u, v, and w are all in Quadrant I, you have

$\arctan[\tan(u + v)] = \arctan[\tan w]u + v = w.$

6. $y = -\dfrac{16}{v_0^2 \cos^2 \theta}x^2 + (\tan \theta)x + h_0$

Let $h_0 = 0$ and take half of the horizontal distance:

$$\frac{1}{2}\left(\frac{1}{32}v_0^2 \sin 2\theta\right) = \frac{1}{64}v_0^2(2 \sin \theta \cos \theta) = \frac{1}{32}v_0^2 \sin \theta \cos \theta$$

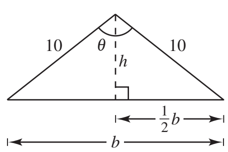
Substitute this expression for x in the model.

$$y = -\frac{16}{v_0^2 \cos^2 \theta}\left(\frac{1}{32}v_0^2 \sin \theta \cos \theta\right)^2 + \left(\frac{\sin \theta}{\cos \theta}\right)\left(\frac{1}{32}v_0^2 \sin \theta \cos \theta\right)$$

$$= -\frac{1}{64}v_0^2 \sin^2 \theta + \frac{1}{32}v_0^2 \sin^2 \theta$$

$$= \frac{1}{64}v_0^2 \sin^2 \theta$$

7. (a)



$$\sin \frac{\theta}{2} = \frac{\frac{1}{2}b}{10} \quad \text{and} \quad \cos \frac{\theta}{2} = \frac{h}{10}$$

$$b = 20 \sin \frac{\theta}{2} \qquad h = 10 \cos \frac{\theta}{2}$$

$$A = \frac{1}{2}bh$$

$$= \frac{1}{2}\left(20 \sin \frac{\theta}{2}\right)\left(10 \cos \frac{\theta}{2}\right)$$

$$= 100 \sin \frac{\theta}{2} \cos \frac{\theta}{2}$$

(b) $A = 50\left(2 \sin \dfrac{\theta}{2} \cos \dfrac{\theta}{2}\right)$

$$= 50 \sin\left(2\left(\frac{\theta}{2}\right)\right)$$

$$= 50 \sin \theta$$

Because $\sin \dfrac{\pi}{2} = 1$ is a maximum, $\theta = \dfrac{\pi}{2}$. So, the area is a maximum at $A = 50 \sin \dfrac{\pi}{2} = 50$ square meters.

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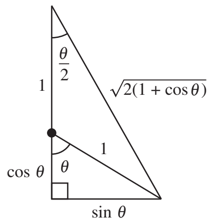
8. 

The hypotenuse of the larger right triangle is:

$$\sqrt{\sin^2\theta + (1+\cos\theta)^2} = \sqrt{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}$$
$$= \sqrt{2 + 2\cos\theta}$$
$$= \sqrt{2(1+\cos\theta)}$$

$$\sin\left(\frac{\theta}{2}\right) = \frac{\sin\theta}{\sqrt{2(1+\cos\theta)}} = \frac{\sin\theta}{\sqrt{2(1+\cos\theta)}}\cdot\frac{\sqrt{1-\cos\theta}}{\sqrt{1-\cos\theta}} = \frac{\sin\theta\sqrt{1-\cos\theta}}{\sqrt{2(1-\cos^2\theta)}} = \frac{\sin\theta\sqrt{1-\cos\theta}}{\sqrt{2}\sin\theta} = \sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \frac{1+\cos\theta}{\sqrt{2(1+\cos\theta)}} = \frac{\sqrt{(1+\cos\theta)^2}}{\sqrt{2(1+\cos\theta)}} = \sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{\sin\theta}{1+\cos\theta}$$

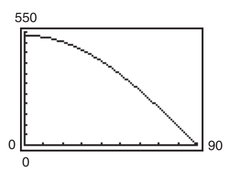
9. $F = \dfrac{0.6W\sin(\theta + 90^\circ)}{\sin 12^\circ}$

(a) $F = \dfrac{0.6W(\sin\theta\cos 90^\circ + \cos\theta\sin 90^\circ)}{\sin 12^\circ}$

$= \dfrac{0.6W[(\sin\theta)(0) + (\cos\theta)(1)]}{\sin 12^\circ}$

$= \dfrac{0.6W\cos\theta}{\sin 12^\circ}$

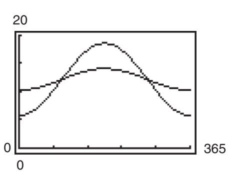
(b) Let $y_1 = \dfrac{0.6(185)\cos x}{\sin 12^\circ}$.



(c) The force is maximum (533.88 pounds) when $\theta = 0^\circ$.
The force is minimum (0 pounds) when $\theta = 90^\circ$.

10. Seward: $D = 12.2 - 6.4\cos\left[\dfrac{\pi(t+0.2)}{182.6}\right]$

New Orleans: $D = 12.2 - 1.9\cos\left[\dfrac{\pi(t+0.2)}{182.6}\right]$

(a) 

(b) The graphs intersect when $t \approx 91$ and when $t \approx 274$. These values correspond to April 1 and October 1, the spring equinox and the fall equinox.

(c) Seward has the greater variation in the number of daylight hours. This is determined by the amplitudes, 6.4 and 1.9.

(d) Period: $\dfrac{2\pi}{\pi/182.6} = 365.2$ days

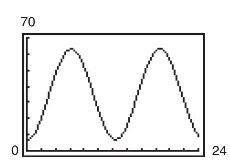
11. $d = 35 - 28\cos\dfrac{\pi}{6.2}t$ when $t = 0$ corresponds to 12:00 A.M.

(a) The high tides occur when $\cos\dfrac{\pi}{6.2}t = -1$. Solving yields $t = 6.2$ or $t = 18.6$.

These t-values correspond to 6:12 A.M. and 6:36 P.M.

The low tide occurs when $\cos\dfrac{\pi}{6.2}t = 1$. Solving yields $t = 0$ and $t = 12.4$ which corresponds to 12:00 A.M. and 12:24 P.M.

(b) The water depth is never 3.5 feet. At low tide, the depth is $d = 35 - 28 = 7$ feet.

(c) 

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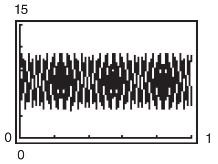
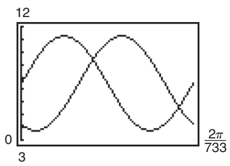
12. $h_1 = 3.75 \sin 733t + 7.5$

$h_2 = 3.75 \sin 733\left(t + \frac{4\pi}{3}\right) + 7.5$

(a)



(b) The period for h_1 and h_2 is $\frac{2\pi}{733} \approx 0.0086$.



The graphs intersect twice per cycle.

There are $\frac{1}{2\pi/733} \approx 116.66$ cycles in the interval $[0, 1]$, so the graphs intersect approximately 233.3 times.

13. (a) $$n = \frac{\sin\left(\frac{\theta}{2} + \frac{\alpha}{2}\right)}{\sin\frac{\theta}{2}}$$

$$= \frac{\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\alpha}{2}\right) + \cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\alpha}{2}\right)}{\sin\left(\frac{\theta}{2}\right)}$$

$$= \cos\left(\frac{\alpha}{2}\right) + \cot\left(\frac{\theta}{2}\right)\sin\left(\frac{\alpha}{2}\right)$$

For $\alpha = 60°$, $n = \cos 30° + \cot\left(\frac{\theta}{2}\right)\sin 30°$

$$n = \frac{\sqrt{3}}{2} + \frac{1}{2}\cot\left(\frac{\theta}{2}\right).$$

(b) For glass, $n = 1.50$.

$$1.50 = \frac{\sqrt{3}}{2} + \frac{1}{2}\cot\left(\frac{\theta}{2}\right)$$

$$2\left(1.50 - \frac{\sqrt{3}}{2}\right) = \cot\left(\frac{\theta}{2}\right)$$

$$\frac{1}{3 - \sqrt{3}} = \tan\left(\frac{\theta}{2}\right)$$

$$\theta = 2\tan^{-1}\left(\frac{1}{3 - \sqrt{3}}\right)$$

$$\theta \approx 76.5°$$

14. (a) $$\sin(u + v + w) = \sin[(u + v) + w]$$
$$= \sin(u + v)\cos w + \cos(u + v)\sin w$$
$$= [\sin u \cos v + \cos u \sin v]\cos w + [\cos u \cos v - \sin u \sin v]\sin w$$
$$= \sin u \cos v \cos w + \cos u \sin v \cos w + \cos u \cos v \sin w - \sin u \sin v \sin w$$

(b) $$\tan(u + v + w) = \tan[(u + v) + w]$$
$$= \frac{\tan(u + v) + \tan w}{1 - \tan(u + v)\tan w}$$
$$= \frac{\left[\frac{\tan u + \tan v}{1 - \tan u \tan v}\right] + \tan w}{1 - \left[\frac{\tan u + \tan v}{1 - \tan u \tan v}\right]\tan w} \cdot \frac{(1 - \tan u \tan v)}{(1 - \tan u \tan v)}$$
$$= \frac{\tan u + \tan v + (1 - \tan u \tan v)\tan w}{(1 - \tan u \tan v) - (\tan u + \tan v)\tan w}$$
$$= \frac{\tan u + \tan v + \tan w - \tan u \tan v \tan w}{1 - \tan u \tan v - \tan u \tan w - \tan v \tan w}$$

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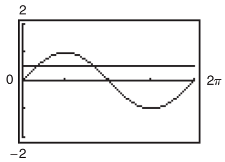
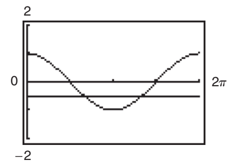
15. (a) Let $y_1 = \sin x$ and $y_2 = 0.5$.



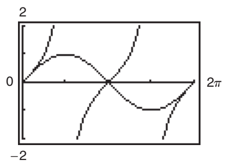
$\sin x \geq 0.5$ on the interval $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$.

(b) Let $y_1 = \cos x$ and $y_2 = -0.5$.



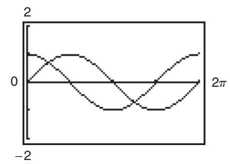
$\cos x \leq -0.5$ on the interval $\left[\frac{2\pi}{3}, \frac{4\pi}{3}\right]$.

(c) Let $y_1 = \tan x$ and $y_2 = \sin x$.



$\tan x < \sin x$ on the intervals $\left(\frac{\pi}{2}, \pi\right)$ and $\left(\frac{3\pi}{2}, 2\pi\right)$.

(d) Let $y_1 = \cos x$ and $y_2 = \sin x$.



$\cos x \geq \sin x$ on the intervals $\left[0, \frac{\pi}{4}\right]$ and $\left[\frac{5\pi}{4}, 2\pi\right)$.

16. (a)
$$\begin{aligned} f(x) &= \sin^4 x + \cos^4 x \\ &= \left(\sin^2 x\right)^2 + \left(\cos^2 x\right)^2 \\ &= \left(\frac{1 - \cos 2x}{2}\right)^2 + \left(\frac{1 + \cos 2x}{2}\right)^2 \\ &= \frac{1}{4}\left[\left(1 - 2\cos 2x + \cos^2 2x\right) + \left(1 + 2\cos 2x + \cos^2 2x\right)\right] \\ &= \frac{1}{4}\left(2 + 2\cos^2 2x\right) \\ &= \frac{1}{2}\left(1 + \cos^2 2x\right) \\ &= \frac{1}{2}\left(1 + \frac{\cos 4x}{2}\right) \\ &= \frac{1}{4}\left(3 + \cos 4x\right) \end{aligned}$$

(b)
$$\begin{aligned} f(x) &= \sin^4 x + \cos^4 x \\ &= \left(\sin^2 x\right)^2 + \cos^4 x \\ &= \left(1 - \cos^2 x\right)^2 + \cos^4 x \\ &= 1 - 2\cos^2 2x + \cos^4 x + \cos^4 x \\ &= 2\cos^4 x - 2\cos^2 x + 1 \end{aligned}$$

(c)
$$\begin{aligned} f(x) &= \sin^4 x + \cos^4 x \\ &= \sin^4 x + 2\sin^2 x\cos^2 x + \cos^4 x - 2\sin^2 x\cos^2 x \\ &= \left(\sin^2 x + \cos^2 x\right)^2 - 2\sin^2 x\cos^2 x \\ &= 1 - 2\sin^2 x\cos^2 x \end{aligned}$$

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(d) $$\begin{aligned} f(x) &= 1 - 2\sin^2 x \cos^2 x \\ &= 1 - 2\left(\frac{1 - \cos 2x}{2}\right)\left(\frac{1 + \cos 2x}{2}\right) \\ &= 1 - \frac{1}{2}\left(1 - \cos^2 2x\right) \\ &= \frac{1}{2} + \frac{1}{2}\cos^2 2x \\ &= \frac{1}{2} + \frac{1}{2}\left(1 - \sin^2 2x\right) \\ &= 1 - \frac{1}{2}\sin^2 2x \end{aligned}$$

(e) No; there is often more than one way to rewrite a trigonometric expression, so your result and your friend's result could both be correct.

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Practice Test for Chapter 5

1. Find the value of the other five trigonometric functions, given $\tan x = \frac{4}{11}$, $\sec x < 0$.

2. Simplify $\dfrac{\sec^2 x + \csc^2 x}{\csc^2 x(1 + \tan^2 x)}$.

3. Rewrite as a single logarithm and simplify $\ln|\tan\theta| - \ln|\cot\theta|$.

4. True or false:

$$\cos\left(\frac{\pi}{2} - x\right) = \frac{1}{\csc x}$$

5. Factor and simplify: $\sin^4 x + (\sin^2 x)\cos^2 x$

6. Multiply and simplify: $(\csc x + 1)(\csc x - 1)$

7. Rationalize the denominator and simplify:

$$\frac{\cos^2 x}{1 - \sin x}$$

8. Verify:

$$\frac{1 + \cos\theta}{\sin\theta} + \frac{\sin\theta}{1 + \cos\theta} = 2\csc\theta$$

9. Verify:

$$\tan^4 x + 2\tan^2 x + 1 = \sec^4 x$$

10. Use the sum or difference formulas to determine:

(a) $\sin 105°$

(b) $\tan 15°$

11. Simplify: $(\sin 42°)\cos 38° - (\cos 42°)\sin 38°$

12. Verify $\tan\left(\theta + \dfrac{\pi}{4}\right) = \dfrac{1 + \tan\theta}{1 - \tan\theta}$.

13. Write $\sin(\arcsin x - \arccos x)$ as an algebraic expression in x.

14. Use the double-angle formulas to determine:

(a) $\cos 120°$

(b) $\tan 300°$

15. Use the half-angle formulas to determine:

(a) $\sin 22.5°$

(b) $\tan \dfrac{\pi}{12}$

16. Given $\sin\theta = 4/5$, θ lies in Quadrant II, find $\cos(\theta/2)$.

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17. Use the power-reducing identities to write $(\sin^2 x)\cos^2 x$ in terms of the first power of cosine.

18. Rewrite as a sum: $6(\sin 5\theta)\cos 2\theta$.

19. Rewrite as a product: $\sin(x + \pi) + \sin(x - \pi)$.

20. Verify $\dfrac{\sin 9x + \sin 5x}{\cos 9x - \cos 5x} = -\cot 2x$.

21. Verify:

$(\cos u)\sin v = \frac{1}{2}\left[\sin(u + v) - \sin(u - v)\right]$.

22. Find all solutions in the interval $[0, 2\pi)$:

$4\sin^2 x = 1$

23. Find all solutions in the interval $[0, 2\pi)$:

$\tan^2 \theta + \left(\sqrt{3} - 1\right)\tan \theta - \sqrt{3} = 0$

24. Find all solutions in the interval $[0, 2\pi)$:

$\sin 2x = \cos x$

25. Use the quadratic formula to find all solutions in the interval $[0, 2\pi)$:

$\tan^2 x - 6\tan x + 4 = 0$

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