CHAPTER 4
Trigonometry

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CHAPTER 4
Trigonometry

Section 4.1 Radian and Degree Measure

1. coterminal

2. radian

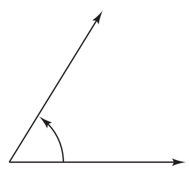
3. complementary; supplementary

4. degree

5. linear; angular

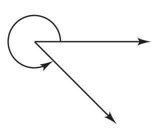
6. $A = \frac{1}{2}r^2\theta$

7.



The angle shown is approximately 1 radian.

8.

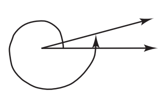


The angle shown is approximately 5.5 radians.

9.

The angle shown is approximately -3 radians.

10.



The angle shown is approximately 6.5 radians.

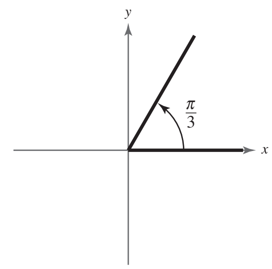
11. (a) Because $0 < \frac{\pi}{4} < \frac{\pi}{2}, \frac{\pi}{4}$ lies in Quadrant I.

(b) Because $-\frac{5\pi}{4}$ is coterminal with $\frac{3\pi}{4}$ and $\frac{\pi}{2} < \frac{3\pi}{4} < \pi, -\frac{5\pi}{4}$ lies in Quadrant II.

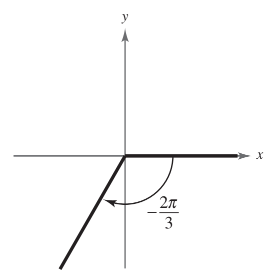
12. (a) Because $-\frac{\pi}{6}$ is coterminal with $\frac{11\pi}{6}$ and $\frac{3\pi}{2} < \frac{11\pi}{6} < 2\pi, -\frac{\pi}{6}$ lies in Quadrant IV.

(b) Because $\pi < \frac{11\pi}{9} < \frac{3\pi}{2}, \frac{11\pi}{9}$ lies in Quadrant III.

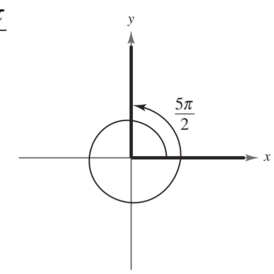
13. (a) $\frac{\pi}{3}$



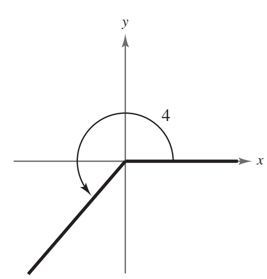
(b) $-\frac{2\pi}{3}$



14. (a) $\frac{5\pi}{2}$



(b) 4



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15. *Sample answers*:

(a) $\frac{\pi}{6} + 2\pi = \frac{13\pi}{6}$

$\frac{\pi}{6} - 2\pi = -\frac{11\pi}{6}$

(b) $-\frac{5\pi}{6} + 2\pi = \frac{7\pi}{6}$

$-\frac{5\pi}{6} - 2\pi = -\frac{17\pi}{6}$

16. *Sample answers:*

(a) $\frac{2\pi}{3} + 2\pi = \frac{8\pi}{3}$

$\frac{2\pi}{3} - 2\pi = -\frac{4\pi}{3}$

(b) $-\frac{9\pi}{4} + 2\pi = -\frac{\pi}{4}$

$-\frac{9\pi}{4} + 4\pi = \frac{7\pi}{4}$

17. (a) Complement: $\frac{\pi}{2} - \frac{\pi}{12} = \frac{5\pi}{12}$

Supplement: $\pi - \frac{\pi}{12} = \frac{11\pi}{12}$

(b) Complement: Not possible, $\frac{11\pi}{12}$ is greater than $\frac{\pi}{2}$.

Supplement: $\pi - \frac{11\pi}{12} = \frac{\pi}{12}$

18. (a) Complement: $\frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$

Supplement: $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$

(b) Complement: $\frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

Supplement: $\pi - \frac{\pi}{4} = \frac{3\pi}{4}$

19. (a) Complement: $\frac{\pi}{2} - 1 \approx 0.57$

Supplement: $\pi - 1 \approx 2.14$

(b) Complement: Not possible, 2 is greater than $\frac{\pi}{2}$.

Supplement: $\pi - 2 \approx 1.14$

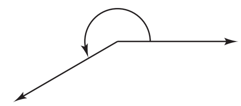
20. (a) Complement: Not possible, 3 is greater than $\frac{\pi}{2}$.

Supplement: $\pi - 3 \approx 0.14$

(b) Complement: $\frac{\pi}{2} - 1.5 \approx 0.07$

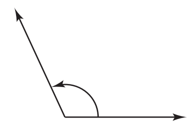
Supplement: $\pi - 1.5 \approx 1.64$

21.



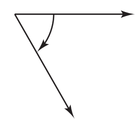
The angle shown is approximately $210°$.

22.



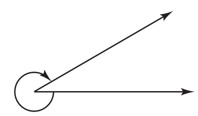
The angle shown is approximately $120°$.

23.



The angle shown is approximately $-60°$.

24.



The angle shown is approximately $-330°$.

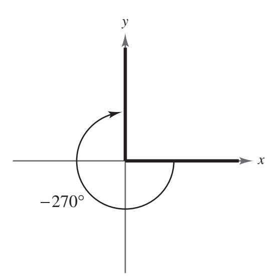
25. (a) Because $90° < 130° < 180°$, $130°$ lies in Quadrant II.

(b) Because $-8.3°$ is coterminal with $351.7°$ and $270° < 351.7° < 360°$, $-8.3°$ lies in Quadrant IV.

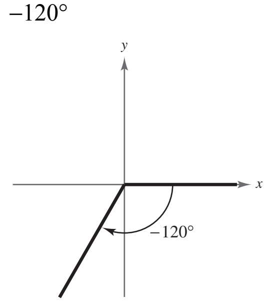
26. (a) Because $-132° 50'$ is coterminal with $227° 10'$ and $180° < 227° 10' < 270°$, $-132° 50'$ lies in Quadrant III.

(b) Because $0° < 3.4° < 90°$, $3.4°$ lies in Quadrant I.

27. (a) $-270°$



(b) $-120°$



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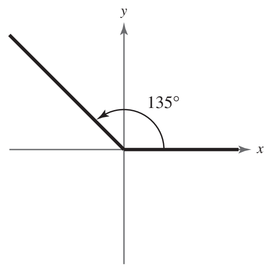
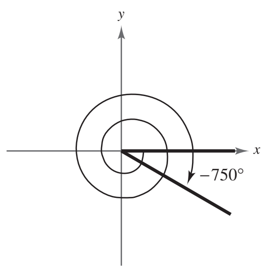
28. (a) $135°$



(b) $-750°$



29. (a) Coterminal angles for $120°$

$120° + 360° = 480°$

$120° - 360° = -240°$

(b) Coterminal angles for $-210°$

$-210° + 360° = 150°$

$-210° - 360° = -570°$

30. (a) Coterminal angles for $45°$

$45° + 360° = 405°$

$45° - 360° = -315°$

(b) Coterminal angles for $-420°$

$-420° + 720° = 300°$

$-420° + 360° = -60°$

31. (a) Complement: $90° - 18° = 72°$

Supplement: $180° - 18° = 162°$

(b) Complement: $90° - 85° = 5°$

Supplement: $180° - 85° = 95°$

32. (a) Complement: $90° - 46° = 44°$

Supplement: $180° - 46° = 134°$

(b) Complement: Not possible. $93°$ is greater than $90°$.

Supplement: $180° - 93° = 87°$

33. (a) Complement: $90° - 24° = 66°$

Supplement: $180° - 24° = 156°$

(b) Complement: Not possible. $126°$ is greater than $90°$.

Supplement: $180° - 126° = 54°$

34. (a) Complement: Not possible, $130°$ is greater than $90°$.

Supplement: $180° - 130° = 50°$

(b) Complement: Not possible, $170°$ is greater than $90°$.

Supplement: $180° - 170° = 10°$

35. (a) $120° = 120°\left(\dfrac{\pi}{180°}\right) = -\dfrac{2\pi}{3}$

(b) $-20° = -20\left(\dfrac{\pi}{180°}\right) = -\dfrac{\pi}{9}$

36. (a) $-60° = -60\left(\dfrac{\pi}{180°}\right) = -\dfrac{\pi}{3}$

(b) $144° = 144°\left(\dfrac{\pi}{180°}\right) = \dfrac{4\pi}{5}$

37. (a) $\dfrac{3\pi}{2} = \dfrac{3\pi}{2}\left(\dfrac{180°}{\pi}\right) = 270°$

(b) $-\dfrac{7\pi}{6} = -\dfrac{7\pi}{6}\left(\dfrac{180°}{\pi}\right) = -210°$

38. (a) $-\dfrac{7\pi}{12} = -\dfrac{7\pi}{12}\left(\dfrac{180°}{\pi}\right) = -105°$

(b) $\dfrac{5\pi}{4} = \dfrac{5\pi}{4}\left(\dfrac{180°}{\pi}\right) = 225°$

39. $45° = 45\left(\dfrac{\pi}{180°}\right) \approx 0.785$ radian

40. $-48.27° = -48.27°\left(\dfrac{\pi}{180°}\right) \approx -0.842$ radian

41. $-0.54° = -0.54°\left(\dfrac{\pi}{180°}\right) \approx -0.009$ radian

42. $345° = 345°\left(\dfrac{\pi}{180°}\right) \approx 6.021$ radians

43. $\dfrac{5\pi}{11} = \dfrac{5\pi}{11}\left(\dfrac{180°}{\pi}\right) \approx 81.818°$

44. $\dfrac{15\pi}{8} = \dfrac{15\pi}{8}\left(\dfrac{180°}{\pi}\right) = 337.5°$

45. $-4.2\pi = -4.2\pi\left(\dfrac{180°}{\pi}\right) = -756°$

46. $-0.57 = -0.57\left(\dfrac{180°}{\pi}\right) \approx -32.659°$

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47. (a) $54°45' = 54° + \left(\frac{45}{60}\right)^{\circ} = 54.75°$

(b) $-128°30' = -128° - \left(\frac{30}{60}\right)^{\circ} = -128.5°$

48. (a) $-135°\,10'\,36'' = 135° + \left(\frac{10}{60}\right)^{\circ} + \left(\frac{36}{3600}\right)^{\circ}$

$\approx 135° + 0.1667° + 0.01° \approx 135.177°$

(b) $-408°\,16'\,20'' = -\left(408° + \left(\frac{16}{60}\right)^{\circ} + \left(\frac{20}{3600}\right)^{\circ}\right)$

$\approx -(408° + 0.2667° + 0.0056°)$

$\approx -408.272°$

49. (a) $240.6° = 240° + 0.6(60)' = 240°36'$

(b) $-145.8° = -[145° + 0.8(60')] = -145°48'$

50. (a) $345.12° = 345° + (0.12)(60')$

$= 345° + 7.2'$

$= 345° + 7' + 0.2(60'')$

$= 345°\,7'\,12''$

(b) $-3.58° = -(3° + (0.58)(60'))$

$= -(3° + 34' + 0.8(60''))$

$= -3°34'48''$

51. $r = 15$ inches, $\theta = 120°$

$s = r\theta$

$s = 15(120°)\left(\frac{\pi}{180°}\right) = 10\pi$ inches

≈ 31.42 inches

52. $r = 9$ feet, $\theta = 150°$

$s = r\theta$

$s = 3(150°)\left(\frac{\pi}{180°}\right) = \frac{5\pi}{2}$ meters

≈ 7.85 meters

53. $r = 80$ kilometers, $s = 150$ kilometers

$s = r\theta$

$150 = 80\theta$

$\theta = \frac{150}{80} = \frac{15}{8}$ radians

54. $r = 14$ feet, $s = 8$ feet

$s = r\theta$

$8 = 14\theta$

$\theta = \frac{8}{14} = \frac{4}{7}$ radian

55. $s = r\theta$

$28 = 7\theta$

$\theta = 4$ radians

56. $s = r\theta$

$60 = 75\theta$

$\theta = \frac{60}{75} = \frac{4}{5}$ radian

Because the angle represented is clockwise, this angle is $-\frac{4}{5}$ radian.

57. $r = 6$ inches, $\theta = \frac{\pi}{3}$

$A = \frac{1}{2}r^2\theta = \frac{1}{2}(6)^2\left(\frac{\pi}{3}\right) = 6\pi \text{ in.}^2 \approx 18.85 \text{ in.}^2$

58. $A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2}(2.5)^2(225)\left(\frac{\pi}{180}\right)$

≈ 12.27 square feet

59. The angle in degrees should be multiplied by $\frac{\pi}{180°}$.

$20° = (20°)\left(\frac{\pi \text{ rad}}{180°}\right) = \frac{\pi}{9}$ radians.

60. The central angle should be converted to radians first to be used in the calculations.

$s = r\theta$

$s = (6 \text{ mm})\left(72°\frac{\pi}{180°}\right)$

$= \frac{12\pi}{5}$ mm

≈ 7.54 mm

61. $\theta = 41°\,15'\,50'' - 32°\,47'\,9'' \approx 8.47806° \approx 0.14797$ radian

$s = r\theta \approx 4000(0.14782) \approx 592$ miles

62. $\theta = 47°\,37'\,18'' - 37°\,47'\,36'' \approx 9°\,49'\,42'' = 9.82833° \approx 0.17154$ radian

$r = 4000$ miles

$s = r\theta = 4000(0.17154) \approx 686$ miles

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63. $\theta = \frac{s}{r} = \frac{2.5}{6} = \frac{25}{60} = \frac{5}{12}$ radian $\approx 23.87°$

64. (a) Angular speed $= \frac{(5200)(2\pi) \text{ radians}}{1 \text{ minute}} = 10{,}400\pi$ radians per minute $\approx 32{,}672.56$ radians/minute

(b) Linear speed $= \frac{\left(\frac{7.25}{2} \text{ in.}\right)\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)(5200)(2\pi) \text{ feet}}{1 \text{ minute}} = \frac{9425\pi}{3}$ feet per minute ≈ 9869.84 feet/minute

65. (a) 4 rpm $= 4(2\pi)$ radians/minute $= 8\pi \approx 25$ radians/minute

(b) $r = 25$ ft

$\frac{r\theta}{t} = 200\pi$ ft/minute

Linear speed $\approx 25(25.13)$ ft/minute ≈ 628.3 ft/minute

66. (a) $\frac{10{,}000 \text{ rev.}}{1 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = \frac{500}{3} \frac{\text{rev}}{\text{sec}}$

$\frac{500}{3} \frac{\text{rev}}{\text{sec}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = \frac{1000\pi}{3} \frac{\text{rad}}{\text{sec}} \approx 1047.20 \frac{\text{rad}}{\text{sec}}$

(b) $V = \frac{s}{t} = \frac{r\theta}{t}$

$V = (0.06 \text{ m})\left(\frac{1000\pi \text{ rad}}{3 \text{ sec}}\right)$

$= 20\pi \frac{\text{m}}{\text{sec}} \approx 62.83 \frac{\text{m}}{\text{sec}}$

67. (a) Road speed (linear speed) $= \frac{\left(\frac{25}{2} \text{ in.}\right)\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)\left(\frac{1 \text{ mi}}{5280 \text{ ft}}\right)(480)(2\pi)}{1 \text{ minute}\left(\frac{1 \text{ hour}}{60 \text{ minutes}}\right)} \approx 35.70$ mi/h

(b) $\frac{55 \text{ mi}}{1 \text{ h}} \times \frac{5280 \text{ ft}}{1 \text{ mi}} \times \frac{12 \text{ in.}}{1 \text{ ft}} \times \frac{1 \text{ h}}{60 \text{ min}} = \frac{58{,}080 \text{ in.}}{1 \text{ min}}$

The circumference of the machine is $C = 2\pi\left(\frac{25}{2}\right) = 25\pi$ inches.

The number of revolutions per minute is

$r = 58{,}080/25\pi \approx 739.50$ revolutions/min.

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68. (a) Arc length of larger sprocket in feet:

$$s = r\theta$$

$$s = \frac{1}{3}(2\pi) = \frac{2\pi}{3} \text{ feet}$$

Therefore, the chain moves $\frac{2\pi}{3}$ feet as does the smaller rear sprocket.

Thus, the angle θ of the smaller sprocket is

$$\theta = \frac{s}{r} = \frac{\frac{2\pi}{3}\text{ ft}}{\frac{2}{12}\text{ ft}} = 4\pi \quad \left(r = 2 \text{ inches} = \frac{2}{12} \text{ feet}\right)$$

and the arc length of the tire in feet is:

$$s = \theta r$$

$$s = (4\pi)\left(\frac{14}{12}\right) = \frac{14\pi}{3} \text{ feet}$$

$$\text{Speed} = \frac{s}{t} = \frac{\frac{14\pi}{3}}{1 \text{ sec}} = \frac{14\pi}{3} \text{ feet per second}$$

$$\frac{14\pi \text{ feet}}{3 \text{ seconds}} \times \frac{3600 \text{ seconds}}{1 \text{ hour}} \times \frac{1 \text{ mile}}{5280 \text{ feet}} \approx 10 \text{ miles per hour}$$

(b) Since the arc length of the tire is $\frac{14\pi}{3}$ feet and the cyclist is pedaling at a rate of one revolution per second, we have:

$$\text{Distance} = \left(\frac{14\pi}{3}\frac{\text{feet}}{\text{revolutions}}\right)\left(\frac{1 \text{ mile}}{5280 \text{ feet}}\right)(n \text{ revolutions})$$

$$= \frac{7\pi}{7920}n \text{ miles}$$

(c) $$\text{Distance} = \text{Rate} \cdot \text{Time}$$

$$= \left(\frac{14\pi}{3} \text{ feet/second}\right)\left(\frac{1 \text{ mile}}{5280 \text{ feet}}\right)(t \text{ seconds})$$

$$= \frac{7\pi}{7920}t \text{ miles}$$

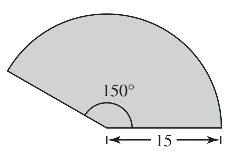
(d) The functions are both linear.

69. $A = \frac{1}{2}r^2\theta$

$= \frac{1}{2}(15)^2(150°)\left(\frac{\pi}{180°}\right)$

$= 93.75\pi \text{ m}^2$

$\approx 294.52 \text{ m}^2$



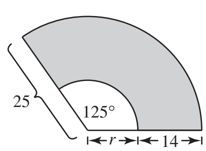
70. $A = \frac{1}{2}\theta\left(R^2 - r^2\right)$

$R = 25$

$r = 25 - 14 = 11$

$A = \frac{1}{2}\left(\frac{125}{180}\right)^{\circ}\pi \cdot \left(25^2 - 11^2\right) \approx 175\pi$

$\approx 549.8 \text{ in.}^2$



71. False. $\frac{180°}{\pi}$ is in degree measure.

72. False. A measurement of 4π radians corresponds to two complete revolutions from the initial to the terminal side of an angle.

73. True. If α and β are coterminal angles, then $\alpha = \beta + n(360°)$ or $\alpha = \beta + n(2\pi)$, where n is an integer. The difference between α and β is $\alpha - \beta = n(360°)$, or $\alpha - \beta = n(2\pi)$ if expressed in radians.

74. False. The terminal side of $-1260°$ lies on the negative x-axis.

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75. Since the arc length s is given by $s = r\theta$, if the central angle θ is fixed while the radius r increases, then s increases in proportion to r.

76. Angles B and C are coterminal with Angle A, since they have the same initial and terminal side as Angle A.

77. The speed increases. The linear speed is proportional to the radius.

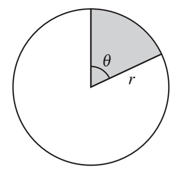
78. $1 \text{ radian} = \left(\dfrac{180°}{\pi}\right) \approx 57.3°$, so one radian is much larger than one degree.

79. $\text{Area of circle} = \pi r^2$

$$\frac{\text{Area of sector}}{\text{Area of circle}} = \frac{\text{Measure of central angle of sector}}{\text{Measure of central angle of circle}}$$

$$\frac{\text{Area of sector}}{\pi r^2} = \frac{\theta}{2\pi}$$

$$\text{Area of sector} = \left(\pi r^2\right)\left(\frac{\theta}{2\pi}\right) = \frac{1}{2}r^2\theta$$



Section 4.2 Trigonometric Functions: The Unit Circle

1. unit circle

2. periodic

3. period

4. odd; even

5. $x = \dfrac{12}{13}, y = \dfrac{5}{13}$

$\sin t = y = \dfrac{5}{13}$ $\quad\csc t = \dfrac{1}{y} = \dfrac{13}{5}$

$\cos t = x = \dfrac{12}{13}$ $\quad\sec t = \dfrac{1}{x} = \dfrac{13}{12}$

$\tan t = \dfrac{y}{x} = \dfrac{5}{12}$ $\quad\cot t = \dfrac{x}{y} = \dfrac{12}{5}$

6. $x = -\dfrac{8}{17}, y = \dfrac{15}{17}$

$\sin t = y = \dfrac{15}{17}$ $\quad\csc t = \dfrac{1}{y} = \dfrac{17}{15}$

$\cos t = x = -\dfrac{8}{17}$ $\quad\sec t = \dfrac{1}{x} = -\dfrac{17}{8}$

$\tan t = \dfrac{y}{x} = -\dfrac{15}{8}$ $\quad\cot t = \dfrac{x}{y} = -\dfrac{8}{15}$

7. $x = -\dfrac{4}{5}, y = -\dfrac{3}{5}$

$\sin t = y = -\dfrac{3}{5}$ $\quad\csc t = \dfrac{1}{y} = -\dfrac{5}{3}$

$\cos t = x = -\dfrac{4}{5}$ $\quad\sec t = \dfrac{1}{x} = -\dfrac{5}{4}$

$\tan t = \dfrac{y}{x} = \dfrac{3}{4}$ $\quad\cot t = \dfrac{x}{y} = \dfrac{4}{3}$

8. $x = \dfrac{12}{13}, y = -\dfrac{5}{13}$

$\sin t = y = -\dfrac{5}{13}$ $\quad\csc t = \dfrac{1}{y} = -\dfrac{13}{5}$

$\cos t = x = \dfrac{12}{13}$ $\quad\sec t = \dfrac{1}{x} = \dfrac{13}{12}$

$\tan t = \dfrac{y}{x} = -\dfrac{5}{12}$ $\quad\cot t = \dfrac{x}{y} = -\dfrac{12}{5}$

9. $t = \dfrac{\pi}{2}$ corresponds to the point $(x, y) = (0, 1)$.

10. $t = \dfrac{\pi}{4}$ corresponds to $\left(\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right)$.

11. $t = \dfrac{5\pi}{6}$ corresponds to the point $(x, y) = \left(-\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right)$.

12. $t = \dfrac{4\pi}{3}$ corresponds to the point

$(x, y) = \left(-\dfrac{1}{2}, -\dfrac{\sqrt{3}}{2}\right)$.

13. $t = \dfrac{\pi}{4}$ corresponds to the point $(x, y) = \left(\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right)$.

$\sin\dfrac{\pi}{4} = y = \dfrac{\sqrt{2}}{2}$

$\cos\dfrac{\pi}{4} = x = \dfrac{\sqrt{2}}{2}$

$\tan\dfrac{\pi}{4} = \dfrac{y}{x} = 1$

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14. $t = \frac{\pi}{3}$ corresponds to the point $(x, y) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

$$\sin\frac{\pi}{3} = y = \frac{\sqrt{3}}{2}$$
$$\cos\frac{\pi}{3} = x = \frac{1}{2}$$
$$\tan\frac{\pi}{3} = \frac{y}{x} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$$

15. $t = -\frac{\pi}{6}$ corresponds to $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$.

$$\sin -\frac{\pi}{6} = y = -\frac{1}{2}$$
$$\cos -\frac{\pi}{6} = x = \frac{\sqrt{3}}{2}$$
$$\tan -\frac{\pi}{6} = \frac{y}{x} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

16. $t = -\frac{\pi}{4}$ corresponds to the point

$(x, y) = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$.

$$\sin\left(-\frac{\pi}{4}\right) = y = -\frac{\sqrt{2}}{2}$$
$$\cos\left(-\frac{\pi}{4}\right) = x = \frac{\sqrt{2}}{2}$$
$$\tan\left(-\frac{\pi}{4}\right) = \frac{y}{x} = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$$

17. $t = -\frac{7\pi}{4}$ corresponds to the point

$(x, y) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

$$\sin\left(-\frac{7\pi}{4}\right) = y = \frac{\sqrt{2}}{2}$$
$$\cos\left(-\frac{7\pi}{4}\right) = x = \frac{\sqrt{2}}{2}$$
$$\tan\left(-\frac{7\pi}{4}\right) = \frac{y}{x} = 1$$

18. $t = -\frac{4\pi}{3}$ corresponds to the point $(x, y) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

$$\sin\left(-\frac{4\pi}{3}\right) = y = \frac{\sqrt{3}}{2}$$
$$\cos\left(-\frac{4\pi}{3}\right) = x = -\frac{1}{2}$$
$$\tan\left(-\frac{4\pi}{3}\right) = \frac{y}{x} = \frac{\sqrt{3}/2}{-1/2} = -\sqrt{3}$$

19. $t = \frac{11\pi}{6}$ corresponds to the point $(x, y) = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$.

$$\sin\frac{11\pi}{6} = y = -\frac{1}{2}$$
$$\cos\frac{11\pi}{6} = x = \frac{\sqrt{3}}{2}$$
$$\tan\frac{11\pi}{6} = \frac{y}{x} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

20. $t = \frac{5\pi}{3}$ corresponds to the point $(x, y) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

$$\sin\frac{5\pi}{3} = y = -\frac{\sqrt{3}}{2}$$
$$\cos\frac{5\pi}{3} = x = \frac{1}{2}$$
$$\tan\frac{5\pi}{3} = \frac{y}{x} = \frac{-\sqrt{3}/2}{1/2} = -\sqrt{3}$$

21. $t = -\frac{3\pi}{2}$ corresponds to the point $(x, y) = (0, 1)$.

$$\sin\left(-\frac{3\pi}{2}\right) = y = 1$$
$$\cos\left(-\frac{3\pi}{2}\right) = x = 0$$
$$\tan\left(-\frac{3\pi}{2}\right) = \frac{y}{x} \text{ is undefined.}$$

22. $t = -2\pi$ corresponds to the point $(x, y) = (1, 0)$.

$$\sin(-2\pi) = y = 0$$
$$\cos(-2\pi) = x = 1$$
$$\tan(-2\pi) = \frac{y}{x} = \frac{0}{1} = 0$$

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23. $t = \frac{2\pi}{3}$ corresponds to the point $(x, y) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

$\sin\frac{2\pi}{3} = y = \frac{\sqrt{3}}{2}$ $\qquad$ $\csc\frac{2\pi}{3} = \frac{1}{y} = \frac{2\sqrt{3}}{3}$

$\cos\frac{2\pi}{3} = x = -\frac{1}{2}$ $\qquad$ $\sec\frac{2\pi}{3} = \frac{1}{x} = -2$

$\tan\frac{2\pi}{3} = \frac{y}{x} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$ $\qquad$ $\cot\frac{2\pi}{3} = \frac{x}{y} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{\sqrt{3}}{3}$

24. $t = \frac{5\pi}{6}$ corresponds to the point $(x, y) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.

$\sin\frac{5\pi}{6} = y = \frac{1}{2}$ $\qquad$ $\csc\frac{5\pi}{6} = \frac{1}{y} = 2$

$\cos\frac{5\pi}{6} = x = -\frac{\sqrt{3}}{2}$ $\qquad$ $\sec\frac{5\pi}{6} = \frac{1}{x} = -\frac{2\sqrt{3}}{3}$

$\tan\frac{5\pi}{6} = \frac{y}{x} = \frac{1/2}{-\sqrt{3}/2} = -\frac{\sqrt{3}}{3}$ $\qquad$ $\cot\frac{5\pi}{6} = \frac{x}{y} = -\sqrt{3}$

25. $t = \frac{4\pi}{3}$ corresponds to the point $(x, y) = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

$\sin\frac{4\pi}{3} = y = -\frac{\sqrt{3}}{2}$ $\qquad$ $\csc\frac{4\pi}{3} = \frac{1}{y} = -\frac{2\sqrt{3}}{3}$

$\cos\frac{4\pi}{3} = x = -\frac{1}{2}$ $\qquad$ $\sec\frac{4\pi}{3} = \frac{1}{x} = -2$

$\tan\frac{4\pi}{3} = \frac{y}{x} = \sqrt{3}$ $\qquad$ $\cot\frac{4\pi}{3} = \frac{x}{y} = \frac{\sqrt{3}}{3}$

26. $t = \frac{7\pi}{4}$ corresponds to the point $(x, y) = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$.

$\sin\frac{7\pi}{4} = y = -\frac{\sqrt{2}}{2}$ $\qquad$ $\csc\frac{7\pi}{4} = \frac{1}{y} = -\sqrt{2}$

$\cos\frac{7\pi}{4} = x = \frac{\sqrt{2}}{2}$ $\qquad$ $\sec\frac{7\pi}{4} = \frac{1}{x} = \sqrt{2}$

$\tan\frac{7\pi}{4} = \frac{y}{x} = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$ $\qquad$ $\cot\frac{7\pi}{4} = \frac{x}{y} = -1$

27. $t = -\frac{5\pi}{3}$ corresponds to the point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

$\sin\left(-\frac{5\pi}{3}\right) = y = \frac{\sqrt{3}}{2}$ $\qquad$ $\csc\left(-\frac{5\pi}{3}\right) = \frac{1}{y} = \frac{2}{\sqrt{3}}$

$\cos\left(-\frac{5\pi}{3}\right) = x = \frac{1}{2}$ $\qquad$ $\sec\left(-\frac{5\pi}{3}\right) = \frac{1}{x} = 2$

$\tan\left(-\frac{5\pi}{3}\right) = \frac{y}{x} = \sqrt{3}$ $\qquad$ $\cot\left(-\frac{5\pi}{3}\right) = \frac{x}{y} = \frac{\sqrt{3}}{3}$

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28. $t = \frac{3\pi}{2}$ corresponds to the point $(x, y) = (0, 1)$.

$\sin \frac{3\pi}{2} = y = 1$ $\csc \frac{3\pi}{2} = \frac{1}{y} = 1$

$\cos \frac{3\pi}{2} = x = 0$ $\sec \frac{3\pi}{2} = \frac{1}{x}$ is undefined.

$\tan \frac{3\pi}{2} = \frac{y}{x}$ is undefined. $\cot \frac{3\pi}{2} = \frac{x}{y} = \frac{0}{1} = 0$

29. $t = -\frac{\pi}{2}$ corresponds to the point $(x, y) = (0, -1)$.

$\sin\left(-\frac{\pi}{2}\right) = y = -1$ $\csc\left(-\frac{\pi}{2}\right) = \frac{1}{y} = -1$

$\cos\left(-\frac{\pi}{2}\right) = x = 0$ $\sec\left(-\frac{\pi}{2}\right) = \frac{1}{x}$ is undefined.

$\tan\left(-\frac{\pi}{2}\right) = \frac{y}{x}$ is undefined. $\cot\left(-\frac{\pi}{2}\right) = \frac{x}{y} = 0$

30. $t = -\pi$ corresponds to the point $(x, y) = (-1, 0)$.

$\sin(-\pi) = y = 0$ $\csc(-\pi) = \frac{1}{y}$ is undefined.

$\cos(-\pi) = x = -1$ $\sec(-\pi) = \frac{1}{x} = -1$

$\tan(-\pi) = \frac{y}{x} = 0$ $\cot(-\pi) = \frac{x}{y}$ is undefined.

31. $\sin 4\pi = \sin 0 = 0$

32. $\cos 3\pi = \cos \pi = -1$

33. $\cos \frac{7\pi}{3} = \cos \frac{\pi}{3} = \frac{1}{2}$

34. $\sin \frac{9\pi}{4} = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

35. $\sin \frac{19\pi}{6} = \sin \frac{7\pi}{6} = -\frac{1}{2}$

36. $\sin\left(-\frac{8\pi}{3}\right) = \sin\left(-\frac{2\pi}{3}\right) = -\sin\left(\frac{2\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

37. $\sin t = \frac{1}{2}$

(a) $\sin(-t) = -\sin t = -\frac{1}{2}$

(b) $\csc(-t) = -\csc t = -2$

38. $\sin(-t) = \frac{3}{8}$

(a) $\sin t = -\sin(-t) = -\frac{3}{8}$

(b) $\csc t = \frac{1}{\sin t} = -\frac{8}{3}$

39. $\cos(-t) = -\frac{1}{5}$

(a) $\cos t = \cos(-t) = -\frac{1}{5}$

(b) $\sec(-t) = \frac{1}{\cos(-t)} = -5$

40. $\cos t = -\frac{3}{4}$

(a) $\cos(-t) = \cos t = -\frac{3}{4}$

(b) $\sec(-t) = \sec t = \frac{1}{\cos t} = -\frac{4}{3}$

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41. $\sin t = \frac{4}{5}$

(a) $\sin(\pi - t) = \sin t = \frac{4}{5}$

(b) $\sin(t + \pi) = -\sin t = -\frac{4}{5}$

42. $\cos t = \frac{4}{5}$

(a) $\cos(\pi - t) = -\cos t = -\frac{4}{5}$

(b) $\cos(t + \pi) = -\cos t = -\frac{4}{5}$

43. $\sin 0.6 \approx 0.5646$

44. $\cos(-2.8) \approx -0.9422$

45. $\tan\frac{\pi}{8} \approx 0.4142$

46. $\tan\left(\frac{5\pi}{7}\right) \approx -1.2540$

47. $\sec 3.1 = \frac{1}{\cos 3.1} \approx -1.0009$

48. $\cot(-1.1) = \frac{1}{\tan(-1.1)} \approx -0.5090$

49. $y(t) = \frac{1}{2}\cos 6t$

(a) $y(0) = \frac{1}{2}\cos 0 = 0.5$ foot

(b) $y\left(\frac{1}{4}\right) = \frac{1}{2}\cos\frac{3}{2} \approx 0.04$ foot

(c) $y\left(\frac{1}{2}\right) = \frac{1}{2}\cos 3 \approx -0.49$ foot

50. $y(t) = \frac{1}{2}e^{-t}\cos 6t$

(a)

t	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
y	0.5	0.0275	−0.3002	−0.0498	0.1766

(b) From the table feature of a graphing utility you estimate that $y \approx 0$ when $t \approx 0.26$ and $t = 0.78$ seconds.

(c) As t increases, the displacement oscillates but decreases in amplitude.

51. False. $\sin(-t) = -\sin t$ means the function is odd, not that the sine of a negative angle is a negative number.

For example: $\sin\left(-\frac{3\pi}{2}\right) = -\sin\left(\frac{3\pi}{2}\right) = -(-1) = 1.$

Even though the angle is negative, the sine value is positive.

52. False. The real number 0 corresponds to the point $(1, 0)$ on the unit circle.

53. True. $\tan a = \tan(a - 6\pi)$ because the period of the tangent function is π.

54. True.

$-\frac{7\pi}{2}$ is coterminal with $\frac{\pi}{2}$.

$\cos\left(-\frac{7\pi}{2}\right) = \cos\frac{\pi}{2} = 0$

$\cos\left(\pi + \frac{\pi}{2}\right) = \cos\left(\frac{3\pi}{2}\right) = 0$

So, $\cos\left(-\frac{7\pi}{2}\right) = \cos\left(\pi + \frac{\pi}{2}\right)$.

55. (a) The points have y-axis symmetry.

(b) $\sin t_1 = \sin(\pi - t_1)$ because they have the same y-value.

(c) $\cos(\pi - t_1) = -\cos t_1$ because the x-values have the opposite signs.

56. $\cos\theta = x = \cos(-\theta)$

$\sec\theta = \frac{1}{x} = \sec(-\theta)$

So, $\sec\theta$ and $\cos\theta$ are even.

$\sin\theta = y$

$\sin(-\theta) = -y = -\sin\theta$

$\csc\theta = \frac{1}{y}$

$\csc(-\theta) = -\frac{1}{y} = -\csc\theta$

So, $\sin\theta$ and $\csc\theta$ are odd.

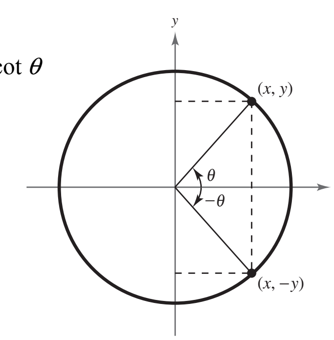
$\tan\theta = \frac{y}{x}$

$\tan(-\theta) = \frac{-y}{x} = -\tan\theta$

$\cot\theta = \frac{x}{y}$

$\cot(-\theta) = \frac{x}{-y} = -\cot\theta$

So, $\tan\theta$ and $\cot\theta$ are odd.



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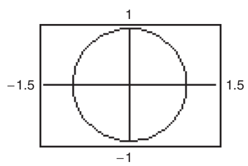
57. The calculator was in degree mode instead of radian mode. $\tan(\pi/2)$ is undefined.

58. $\sin(0.25) + \sin(0.75) \approx 0.2474 + 0.6816 = 0.9290$

$\sin 1 \approx 0.8415$

So, $\sin t_1 + \sin t_2 \neq \sin(t_1 + t_2)$.

59. (a)



Circle of radius 1 centered at $(0, 0)$

(b) The t-values represent the central angle in radians. The x- and y-values represent the location in the coordinate plane.

(c) $-1 \le x \le 1, -1 \le y \le 1$

60. (a) Yes: the functions $\sin t$ and $\cos t$ are the y and x coordinates, respectively, of the point. Because these coordinates are nonzero, the functions $\csc t = \frac{1}{y}$, $\sec t = \frac{1}{x}$, $\tan t = \frac{y}{x}$, and $\cot t = \frac{x}{y}$ all exist as well.

(b) The functions $\sin t = y$, $\cos t = x$, $\csc t = \frac{1}{y}$, and $\sec t = \frac{1}{x}$ are all negative because (x, y) is in the third quadrant, which means that both x and y (and their reciprocals) are negative. The functions $\tan t = \frac{y}{x}$ and $\cot t = \frac{x}{y}$ are both positive because x and y are both negative, and the ratio of two negative numbers is positive.

61. Let $h(t) = f(t)g(t) = \sin t \cos t$.

Then, $h(-t) = \sin(-t)\cos(-t)$

$= -\sin t \cos t$

$= -h(t)$.

So, $h(t)$ is odd.

62. Let $h(t) = f(t)g(t) = \sin t \tan t$.

Then, $h(-t) = \sin(-t)\tan(-t)$

$= (-\sin t)(-\tan t)$

$= \sin t \tan t$

$= h(t)$.

So, $h(t)$ is even.

Section 4.3 Right Triangle Trigonometry

1. (a) $\frac{\text{opposite}}{\text{hypotenuse}} = \sin\theta$ (v) (b) $\frac{\text{adjacent}}{\text{hypotenuse}} = \cos\theta$ (iv) (c) $\frac{\text{opposite}}{\text{adjacent}} = \tan\theta$ (vi)

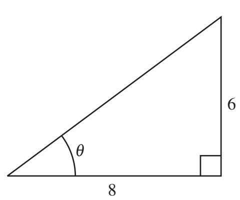
(d) $\frac{\text{hypotenuse}}{\text{opposite}} = \csc\theta$ (iii) (e) $\frac{\text{hypotenuse}}{\text{adjacent}} = \sec\theta$ (i) (f) $\frac{\text{adjacent}}{\text{opposite}} = \cot\theta$ (ii)

2. opposite, adjacent; hypotenuse

3. Complementary

4. elevation; depression

5. $\text{hyp} = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$



$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{6}{10} = \frac{3}{5}$ $\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{10}{6} = \frac{5}{3}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{8}{10} = \frac{4}{5}$ $\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{10}{8} = \frac{5}{4}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{6}{8} = \frac{3}{4}$ $\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{8}{6} = \frac{4}{3}$

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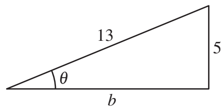
6. adj $= \sqrt{13^2 - 5^2} = \sqrt{169 - 25} = 12$

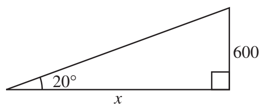


$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{5}{13}$ $\quad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{13}{5}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{12}{13}$ $\quad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{13}{12}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{5}{12}$ $\quad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{12}{5}$

7. adj $= \sqrt{41^2 - 9^2} = \sqrt{1681 - 81} = \sqrt{1600} = 40$

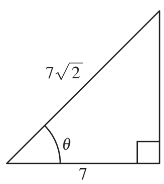


$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{9}{41}$ $\quad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{41}{9}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{40}{41}$ $\quad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{41}{40}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{9}{40}$ $\quad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{40}{9}$

8. opp $= \sqrt{\left(7\sqrt{2}\right)^2 - 7^2} = \sqrt{98 - 49} = \sqrt{49} = 7$

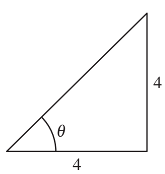


$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{7}{7\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\quad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{7\sqrt{2}}{7} = \sqrt{2}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{7}{7\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\quad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{7\sqrt{2}}{7} = \sqrt{2}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{7}{7} = 1$ $\quad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{7}{7} = 1$

9. hyp $= \sqrt{4^2 + 4^2} = \sqrt{32} = 4\sqrt{2}$

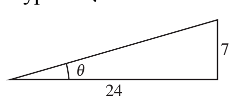


$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4}{4\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\quad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{4\sqrt{2}}{4} = \sqrt{2}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{4}{4\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\quad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{4\sqrt{2}}{4} = \sqrt{2}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{4} = 1$ $\quad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{4}{4} = 1$

10. hyp $= \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25$



$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{7}{25}$ $\quad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{25}{7}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{24}{25}$ $\quad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{25}{24}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{7}{24}$ $\quad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{24}{7}$

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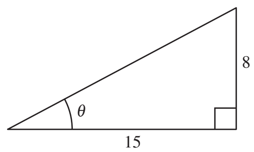
11.

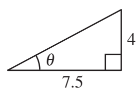


$$\text{hyp} = \sqrt{15^2 + 8^2} = \sqrt{289} = 17$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{8}{17} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{17}{8}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{15}{17} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{17}{15}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{8}{15} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{15}{8}$$



$$\text{hyp} = \sqrt{7.5^2 + 4^2} = \frac{17}{2}$$

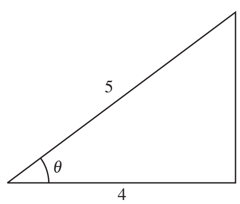
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4}{(17/2)} = \frac{8}{17} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{(17/2)}{4} = \frac{17}{8}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{7.5}{(17/2)} = \frac{15}{17} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{(17/2)}{7.5} = \frac{17}{15}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{7.5} = \frac{8}{15} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{7.5}{4} = \frac{15}{8}$$

The function values are the same because the triangles are similar, and corresponding sides are proportional.

12. $\text{opp} = \sqrt{5^2 - 4^2} = 3$

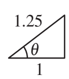


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{3}{5} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{5}{3}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{4}{5} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{5}{4}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{3}{4} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{4}{3}$$

$\text{opp} = \sqrt{1.25^2 - 1^2} = 0.75$



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{0.75}{1.25} = \frac{3}{5} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{1.25}{0.75} = \frac{5}{3}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{1}{1.25} = \frac{4}{5} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{1.25}{1} = \frac{5}{4}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{0.75}{1} = \frac{3}{4} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{1}{0.75} = \frac{4}{3}$$

The function values are the same since the triangles are similar and the corresponding sides are proportional.

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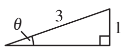
13. $\text{adj} = \sqrt{3^2 - 1^2} = \sqrt{8} = 2\sqrt{2}$

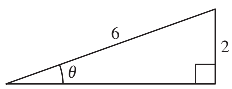


$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{1}{3}$ $\quad \csc\theta = \frac{\text{hyp}}{\text{opp}} = 3$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{2\sqrt{2}}{3}$ $\quad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$ $\quad \cot\theta = \frac{\text{adj}}{\text{opp}} = 2\sqrt{2}$

$\text{adj} = \sqrt{6^2 - 2^2} = \sqrt{32} = 4\sqrt{2}$



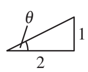
$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{2}{6} = \frac{1}{3}$ $\quad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{6}{2} = 3$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3}$ $\quad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{6}{4\sqrt{2}} = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{5\sqrt{2}}{4} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$ $\quad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$

The function values are the same since the triangles are similar and the corresponding sides are proportional.

14.

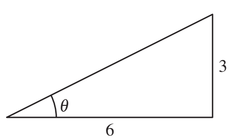


$\text{hyp} = \sqrt{1^2 + 2^2} = \sqrt{5}$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ $\quad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{5}}{1} = \sqrt{5}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ $\quad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{5}}{2}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{1}{2}$ $\quad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{2}{1} = 2$



$\text{hyp} = \sqrt{3^2 + 6^2} = 3\sqrt{5}$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{3}{3\sqrt{5}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ $\quad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{3\sqrt{5}}{3} = \sqrt{5}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{6}{3\sqrt{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ $\quad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{3\sqrt{5}}{6} = \frac{\sqrt{5}}{2}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{3}{6} = \frac{1}{2}$ $\quad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{6}{3} = 2$

The function values are the same because the triangles are similar, and corresponding sides are proportional

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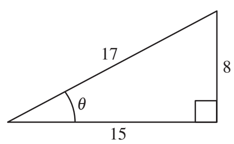
15. Given: $\cos\theta = \frac{15}{17} = \frac{\text{adj}}{\text{hyp}}$

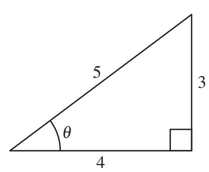
$$(\text{opp})^2 + 15^2 = 17^2$$
$$\text{opp} = \sqrt{289 - 225}$$
$$\text{opp} = \sqrt{64} = 8$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{8}{17}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{8}{15}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{17}{8}$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{17}{15}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{15}{8}$$



16. Given: $\sin\theta = \frac{3}{5} = \frac{\text{opp}}{\text{hyp}}$

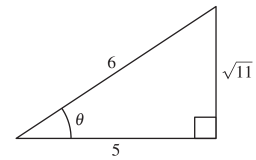
$$\text{adj}^2 + 3^2 = 5^2$$
$$\text{hyp} = \sqrt{25 - 9} = 4$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{4}{5}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{3}{4}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{5}{3}$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{5}{4}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{4}{3}$$



17. Given: $\sec\theta = \frac{6}{5} = \frac{\text{hyp}}{\text{adj}}$

$$(\text{opp})^2 + 5^2 = 6^2$$
$$\text{opp} = \sqrt{36 - 25} = \sqrt{11}$$

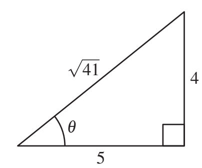
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{11}}{6}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{5}{6}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{11}}{5}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{6\sqrt{11}}{11}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{5\sqrt{11}}{11}$$



18. Given: $\tan\theta = \frac{4}{5} = \frac{\text{opp}}{\text{adj}}$

$$4^2 + 5^2 = (\text{hyp})^2$$
$$\text{hyp} = \sqrt{41}$$

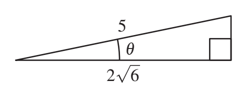
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4\sqrt{41}}{41}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{5\sqrt{41}}{41}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{41}}{4}$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{41}}{5}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{5}{4}$$



19. Given: $\sin\theta = \frac{1}{5} = \frac{\text{opp}}{\text{hyp}}$

$$1^2 + (\text{adj})^2 = 5^2$$
$$\text{adj} = \sqrt{24} = 2\sqrt{6}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{2\sqrt{6}}{5}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{6}}{12}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = 5$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{5\sqrt{6}}{12}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = 2\sqrt{6}$$



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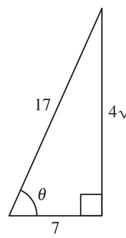
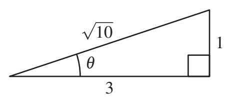
20. Given: $\sec\theta = \frac{17}{7} = \frac{\text{hyp}}{\text{adj}}$



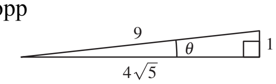
$$(\text{opp})^2 + 7^2 = 17^2$$
$$\text{opp} = \sqrt{240} = 4\sqrt{15}$$
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4\sqrt{15}}{17}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{7}{17}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4\sqrt{15}}{7}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{17\sqrt{15}}{60}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{7\sqrt{15}}{60}$$

21. Given: $\cot\theta = 3 = \frac{3}{1} = \frac{\text{adj}}{\text{opp}}$



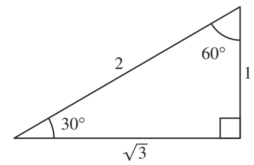
$$1^2 + 3^2 = (\text{hyp})^2$$
$$\text{hyp} = \sqrt{10}$$
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{10}}{10}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{3\sqrt{10}}{10}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{1}{3}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \sqrt{10}$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{10}}{3}$$

22. Given: $\csc\theta = 9 = \frac{9}{1} = \frac{\text{hyp}}{\text{opp}}$



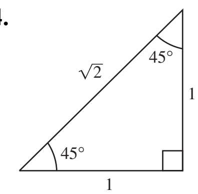
$$1^2 + (\text{adj})^2 = 9^2$$
$$\text{adj} = \sqrt{80} = 4\sqrt{5}$$
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{1}{9}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{4\sqrt{5}}{9}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{5}}{20}$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{9\sqrt{5}}{20}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = 4\sqrt{5}$$

23.



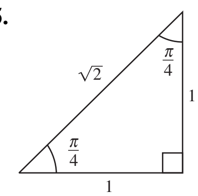
$$30° = 30°\left(\frac{\pi}{180°}\right) = \frac{\pi}{6} \text{ radian}$$
$$\tan 30° = \frac{\text{opp}}{\text{adj}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

24.



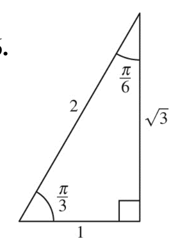
$$45° = 45°\left(\frac{\pi}{180°}\right) = \frac{\pi}{4} \text{ radian}$$
$$\cos 45° = \frac{\text{adj}}{\text{hyp}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

25.



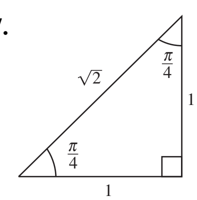
$$\frac{\pi}{4} = \frac{\pi}{4}\left(\frac{180°}{\pi}\right) = 45°$$
$$\sin\frac{\pi}{4} = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

26.



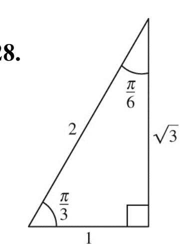
$$\frac{\pi}{3} = \frac{\pi}{3}\left(\frac{180°}{\pi}\right) = 60°$$
$$\tan\frac{\pi}{3} = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

27.



$$\frac{\pi}{4} = \frac{\pi}{4}\left(\frac{180°}{\pi}\right) = 45°$$
$$\sec\frac{\pi}{4} = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

28.



$$\frac{\pi}{6} = \frac{\pi}{6}\left(\frac{180°}{\pi}\right) = 30°$$
$$\csc\frac{\pi}{6} = \frac{\text{hyp}}{\text{opp}} = 2$$

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29. (a) $\sin 20° \approx 0.3420$

(b) $\cos 70° \approx 0.3420$

30. (a) $\tan 23.5° \approx 0.4348$

(b) $\cot 66.5° = \dfrac{1}{\tan 66.5°} \approx 0.4348$

31. (a) $\sin 14.21° \approx 0.2455$

(b) $\csc 14.21° = \dfrac{1}{\sin 14.21°} \approx 4.0737$

32. (a) $\cot 79.56° = \dfrac{1}{\tan 79.56°} \approx 0.1843$

(b) $\sec 79.56° = \dfrac{1}{\cos 79.56°} \approx 5.5186$

33. (a) $\cos 4° 50' 15'' = \cos\left(4 + \dfrac{50}{60} + \dfrac{15}{3600}\right)^{\circ} \approx 0.9964$

(b) $\sec 4° 50' 15'' = \dfrac{1}{\cos 4° 50' 15''} \approx 1.0036$

34. (a) $\sec 42° 12' = \sec 42.2° = \dfrac{1}{\cos 42.2°} \approx 1.3499$

(b) $\csc 48° 7' = \dfrac{1}{\sin\left(48 + \dfrac{7}{60}\right)^{\circ}} \approx 1.3432$

35. (a) $\cot 17° 15' = \cot\left(17 + \dfrac{15}{60}\right)^{\circ} = \dfrac{1}{\tan 17.25°} \approx 3.2205$

(b) $\tan 17° 15' = \tan\left(17 + \dfrac{15}{60}\right)^{\circ} = \tan 17.25° \approx 0.3105$

36. (a) $\sec 56° 8' 10'' = \sec\left(56 + \frac{8}{60} = \frac{10}{3600}\right)^{\circ} \approx 1.7946$

(b) $\cos 56° 8' 10'' = \cos\left(56 + \frac{8}{60} + \frac{10}{3600}\right)^{\circ} \approx 0.5572$

37. $\sin 60° = \dfrac{\sqrt{3}}{2}, \cos 60° = \dfrac{1}{2}$

(a) $\sin 30° = \cos 60° = \dfrac{1}{2}$

(b) $\cos 30° = \sin 60° = \dfrac{\sqrt{3}}{2}$

(c) $\tan 60° = \dfrac{\sin 60°}{\cos 60°} = \sqrt{3}$

(d) $\cot 60° = \dfrac{\cos 60°}{\sin 60°} = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

38. $\sin 30° = \dfrac{1}{2}, \tan 30° = \dfrac{\sqrt{3}}{3}$

(a) $\csc 30° = \dfrac{1}{\sin 30°} = 2$

(b) $\cot 60° = \tan(90° - 60°) = \tan 30° = \dfrac{\sqrt{3}}{3}$

(c) $\cos 30° = \dfrac{\sin 30°}{\tan 30°} = \dfrac{\frac{1}{2}}{\frac{\sqrt{3}}{3}} = \dfrac{3}{2\sqrt{3}} = \dfrac{\sqrt{3}}{2}$

(d) $\cot 30° = \dfrac{1}{\tan 30°} = \dfrac{3}{\sqrt{3}} = \dfrac{3\sqrt{3}}{3} = \sqrt{3}$

39. $\cos\theta = \dfrac{1}{3}$

(a)
$$\sin^2\theta + \cos^2\theta = 1$$
$$\sin^2\theta + \left(\frac{1}{3}\right)^2 = 1$$
$$\sin^2\theta = \frac{8}{9}$$
$$\sin\theta = \frac{2\sqrt{2}}{3}$$

(b) $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{2\sqrt{2}}{3}}{\frac{1}{3}} = 2\sqrt{2}$

(c) $\sec\theta = \dfrac{1}{\cos\theta} = 3$

(d) $\csc(90° - \theta) = \sec\theta = 3$

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40. $\sec\theta = 5$

(a) $\cos\theta = \dfrac{1}{\sec\theta} = \dfrac{1}{5}$

(b) $1 + \tan^2\theta = \sec^2\theta$

$1 + \tan^2\theta = 5^2$

$\tan^2\theta = 24$

$\tan\theta = 2\sqrt{6}$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{2\sqrt{6}} = \dfrac{\sqrt{6}}{12}$

(c) $\cot(90° - \theta) = \tan\theta = 2\sqrt{6}$

(d) $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \left(\dfrac{1}{5}\right)^2 = 1$

$\sin^2\theta = \dfrac{24}{25}$

$\sin\theta = \dfrac{2\sqrt{6}}{5}$

41. $\cot\alpha = 3$

(a) $\tan\alpha = \dfrac{1}{\cot\alpha} = \dfrac{1}{3}$

(b) $\csc^2\alpha = 1 + \cot^2\alpha$

$\csc^2\alpha = 1 + 3^2$

$\csc^2\alpha = 10$

$\csc\alpha = \sqrt{10}$

(c) $\cot(90° - \alpha) = \tan\alpha = \dfrac{1}{3}$

(d) $\csc\alpha = \sqrt{10}$

$\sin\alpha = \dfrac{1}{\csc\alpha} = \dfrac{1}{\sqrt{10}} = \dfrac{\sqrt{10}}{10}$

42. $\cos\beta = \dfrac{\sqrt{7}}{4}$

(a) $\sec\beta = \dfrac{1}{\cos\beta} = \dfrac{4\sqrt{7}}{7}$

(b) $\sin^2\beta + \cos^2\beta = 1$

$\sin^2\beta + \left(\dfrac{\sqrt{7}}{4}\right)^2 = 1$

$\sin^2\beta = \dfrac{9}{16}$

$\sin\beta = \dfrac{3}{4}$

(c) $\cot\beta = \dfrac{\cos\beta}{\sin\beta} = \dfrac{\frac{\sqrt{7}}{4}}{\frac{3}{4}} = \dfrac{\sqrt{7}}{3}$

(d) $\sin(90° - \beta) = \cos\beta = \dfrac{\sqrt{7}}{4}$

43. $\tan\theta\cot\theta = \tan\theta\left(\dfrac{1}{\tan\theta}\right) = 1$

44. $\cos\theta\sec\theta = \cos\theta\dfrac{1}{\cos\theta} = 1$

45. $\tan\alpha\cos\alpha = \left(\dfrac{\sin\alpha}{\cos\alpha}\right)\cos\alpha = \sin\alpha$

46. $\cot\alpha\sin\alpha = \dfrac{\cos\alpha}{\sin\alpha}\sin\alpha = \cos\alpha$

47. $(1 + \sin\theta)(1 - \sin\theta) = 1 - \sin^2\theta = \cos^2\theta$

48. $(1 + \cos\theta)(1 - \cos\theta) = 1 - \cos^2\theta$

$= (\sin^2\theta + \cos^2\theta) - \cos^2\theta$

$= \sin^2\theta$

49. $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = \sec^2\theta - \tan^2\theta$

$= (1 + \tan^2\theta) - \tan^2\theta$

$= 1$

50. $\sin^2\theta - \cos^2\theta = \sin^2\theta - (1 - \sin^2\theta)$

$= \sin^2\theta - 1 + \sin^2\theta$

$= 2\sin^2\theta - 1$

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51. $\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$= \dfrac{1}{\sin\theta\cos\theta}$

$= \dfrac{1}{\sin\theta}\cdot\dfrac{1}{\cos\theta}$

$= \csc\theta\sec\theta$

52. $\dfrac{\tan\beta + \cot\beta}{\tan\beta} = \dfrac{\tan\beta}{\tan\beta} + \dfrac{\cot\beta}{\tan\beta}$

$= 1 + \dfrac{\cot\beta}{\dfrac{1}{\cot\beta}}$

$= 1 + \cot^2\beta = \csc^2\beta$

53. (a) $\sin\theta = \dfrac{1}{2} \Rightarrow \theta = 30° = \dfrac{\pi}{6}$

(b) $\csc\theta = 2 \Rightarrow \theta = 30° = \dfrac{\pi}{6}$

54. (a) $\cos\theta = \dfrac{\sqrt{2}}{2} \Rightarrow \theta = 45° = \dfrac{\pi}{4}$

(b) $\tan\theta = 1 \Rightarrow \theta = 45° = \dfrac{\pi}{4}$

55. (a) $\sec\theta = 2 \Rightarrow \theta = 60° = \dfrac{\pi}{3}$

(b) $\cot\theta = 1 \Rightarrow \theta = 45° = \dfrac{\pi}{4}$

56. (a) $\tan\theta = \sqrt{3} \Rightarrow \theta = 60° = \dfrac{\pi}{3}$

(b) $\csc\theta = \sqrt{2} \Rightarrow \theta = 45° = \dfrac{\pi}{4}$

57. (a) $\csc\theta = \dfrac{2\sqrt{3}}{3} \Rightarrow \theta = 60° = \dfrac{\pi}{3}$

(b) $\sin\theta = \dfrac{\sqrt{2}}{2} \Rightarrow \theta = 45° = \dfrac{\pi}{4}$

58. (a) $\cot\theta = \dfrac{\sqrt{3}}{3}$

$\tan\theta = \dfrac{3}{\sqrt{3}} = \sqrt{3} \Rightarrow \theta = 60° = \dfrac{\pi}{3}$

(b) $\sec\theta = \sqrt{2}$

$\cos\theta = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2} \Rightarrow \theta = 45° = \dfrac{\pi}{4}$

59. $\cos 60° = \dfrac{x}{18}$

$x = 18\cos 60° = 18\left(\dfrac{1}{2}\right) = 9$

$\sin 60° = \dfrac{y}{18}$

$y = 18\sin 60° = 18\dfrac{\sqrt{3}}{2} = 9\sqrt{3}$

60. $\tan 30° = \dfrac{30}{x}$

$\dfrac{1}{\sqrt{3}} = \dfrac{30}{x}$

$x = 30\sqrt{3}$

$\sin 30° = \dfrac{30}{r}$

$r = \dfrac{30}{\sin 30°} = \dfrac{30}{\dfrac{1}{2}} = 60$

61. $\tan 60° = \dfrac{32}{x}$

$\sqrt{3} = \dfrac{32}{x}$

$\sqrt{3}x = 32$

$x = \dfrac{32}{\sqrt{3}} = \dfrac{32\sqrt{3}}{3}$

$\sin 60° = \dfrac{32}{r}$

$r = \dfrac{32}{\sin 60°}$

$r = \dfrac{32}{\dfrac{\sqrt{3}}{2}} = \dfrac{64\sqrt{3}}{3}$

62. $\tan 45° = \dfrac{20}{x}$

$x = \dfrac{20}{\tan 45°}$

$x = \dfrac{20}{1} = 20$

$\sin 45° = \dfrac{20}{r}$

$r = \dfrac{20}{\sin 45°} = \dfrac{20}{\dfrac{\sqrt{2}}{2}} = 20\sqrt{2}$

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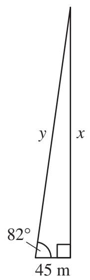
63.



$$\tan 82° = \frac{x}{45}$$
$$x = 45 \tan 82°$$

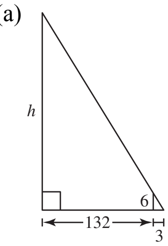
Height of the building:

$123 + 45 \tan 82° \approx 443.2$ meters

Distance between friends:

$$\cos 82° = \frac{45}{y} \Rightarrow y = \frac{45}{\cos 82°}$$
$$\approx 323.34 \text{ meters}$$

64. (a)



Not drawn to scale

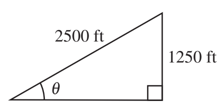
(b) $\tan \theta = \dfrac{6}{3} = \dfrac{h}{135}$

(c) $2(135) = h$

$h = 270$ feet

65. $\sin \theta = \dfrac{1250}{2500} = \dfrac{1}{2}$

$$\theta = 30° = \frac{\pi}{6}$$



66. $\tan \theta = \dfrac{\text{opp}}{\text{adj}}$

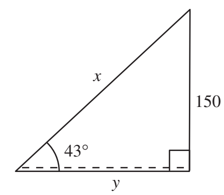
$$\tan 54° = \frac{w}{100}$$
$$w = 100 \tan 54° \approx 137.6 \text{ feet}$$

67. (a) $\sin 43° = \dfrac{150}{x}$

$$x = \frac{150}{\sin 43°} \approx 219.9 \text{ ft}$$

(b) $\tan 43° = \dfrac{150}{y}$

$$y = \frac{150}{\tan 43°} \approx 160.9 \text{ ft}$$



68. Let $h =$ the height of the mountain.

Let $x =$ the horizontal distance from where the $9°$ angle of elevation is sighted to the point at that level directly below the mountain peak.

Then $\tan 3.5° = \dfrac{h}{x + 13}$ and $\tan 9° = \dfrac{h}{x}$.

$$\tan 9° = \frac{h}{x} \Rightarrow x = \frac{h}{\tan 9°}$$

Substitute $x = \dfrac{h}{\tan 9°}$ into the expression for $\tan 3.5°$.

$$\tan 3.5° = \frac{h}{\frac{h}{\tan 9°} + 13}$$

$$\tan 3.5° = \frac{h \tan 9°}{h + 13 \tan 9°}$$

$$h \tan 3.5° + 13 \tan 9° \tan 3.5° = h \tan 9°$$

$$13 \tan 9° \tan 3.5° = h(\tan 9° - \tan 3.5°)$$

$$\frac{13 \tan 9° \tan 3.5°}{\tan 9° - \tan 3.5°} = h$$

$$1.2953 \approx h$$

The mountain is about 1.3 miles high

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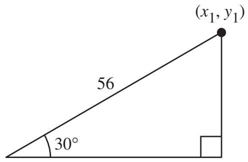
69.



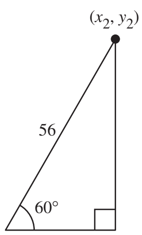
$\sin 30° = \dfrac{y_1}{56}$

$y_1 = (\sin 30°)(56) = \left(\dfrac{1}{2}\right)(56) = 28$

$\cos 30° = \dfrac{x_1}{56}$

$x_1 = \cos 30°(56) = \dfrac{\sqrt{3}}{2}(56) = 28\sqrt{3}$

$(x_1, y_1) = (28\sqrt{3}, 28)$



$\sin 60° = \dfrac{y_2}{56}$

$y_2 = \sin 60°(56) = \left(\dfrac{\sqrt{3}}{2}\right)(56) = 28\sqrt{3}$

$\cos 60° = \dfrac{x_2}{56}$

$x_2 = (\cos 60°)(56) = \left(\dfrac{1}{2}\right)(56) = 28$

$(x_2, y_2) = (28, 28\sqrt{3})$

70. $\tan 3° = \dfrac{x}{15}$

$x = 15 \tan 3°$

$d = 5 + 2x = 5 + 2(15 \tan 3°) \approx 6.57$ centimeters

71. $x \approx 9.397, y \approx 3.420$

$\sin 20° = \dfrac{y}{10} \approx 0.34$

$\cos 20° = \dfrac{x}{10} \approx 0.94$

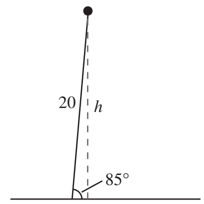
$\tan 20° = \dfrac{y}{x} \approx 0.36$

$\cot 20° = \dfrac{x}{y} \approx 2.75$

$\sec 20° = \dfrac{10}{x} \approx 1.06$

$\csc 20° = \dfrac{10}{y} \approx 2.92$

72. (a)



(b) $\sin 85° = \dfrac{h}{20}$

$h = 20 \sin 85° \approx 19.9$ meters

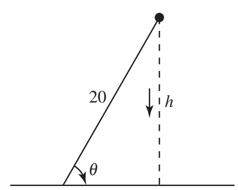
(c) The side of the triangle labeled h will become shorter.

(d)

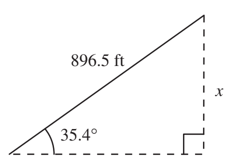
Angle, θ	Height (in meters)
80°	19.7
70°	18.8
60°	17.3
50°	15.3
40°	12.9
30°	10.0
20°	6.8
10°	3.5

(e) The height of the balloon decreases.

As $\theta \to 0°, h \to 0$.



73. (a)



$\sin 35.4° = \dfrac{x}{896.5}$

$x = 896.5 \sin 35.4° \approx 519.33$ feet

(b) Because the top of the incline is 1693.5 feet above sea level and the vertical rise of the inclined plane is 519.33 feet, the elevation of the lower end of the inclined plan is about

$1693.5 - 519.33 = 1174.17$ feet.

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(c) Ascent time:

$$\begin{aligned} d &= rt \\ 896.5 &= 300t \\ 3 &\approx t \end{aligned}$$

It takes about 3 minutes for the cars to get from the bottom to the top.

Vertical rate:

$$\begin{aligned} d &= rt \\ 519.33 &\approx r(3) \\ r &\approx 173.11 \text{ ft/min} \end{aligned}$$

74. $\cos 60° = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{1}{2}$

75. $\sin 60° \csc 60° = 1$

True.

$$\csc x = \frac{1}{\sin x} \Rightarrow \sin 60° \csc 60° = \sin 60°\left(\frac{1}{\sin 60°}\right) = 1$$

76. False.

Because $\sec 30° = \dfrac{1}{\cos 30°} = \dfrac{2\sqrt{3}}{3}$

and

$\sin 30° = \dfrac{1}{\csc 30°} = 2, \sec 30° \neq \csc 30°.$

77. False, $\dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2} = \sqrt{2} \neq 1$

78. True. $\cos 60° - \sin 30° = \dfrac{1}{2} - \dfrac{1}{2} = 0$

79. False, $\dfrac{\sin 60°}{\sin 30°} = \dfrac{\cos 30°}{\sin 30°} = \cot 30° \approx 1.7321$

$\sin 2° \approx 0.0349$

80. False.

$$\tan\left[(5°)^2\right] = \tan 25° \approx 0.466$$

$$\tan^2 5° = (\tan 5°)^2 \approx 0.008$$

81. Yes. Given $\tan\theta$, $\sec\theta$ can be found from the identity $1 + \tan^2\theta = \sec^2\theta$.

82. (a) Side y is opposite θ.

(b) Side y is adjacent to $90° - \theta$.

(c) Because side y is the side opposite angle θ and is the same side that is adjacent to the angle $90° - \theta$, $\sin\theta = \cos(90° - \theta)$.

83.

θ	0.1	0.2	0.3	0.4	0.5
$\sin\theta$	0.0998	0.1987	0.2955	0.3894	0.4794

(a) In the interval $(0, 0.5]$, $\theta > \sin\theta$.

(b) As $\theta \to 0$, $\sin\theta \to 0$, and $\dfrac{\theta}{\sin\theta} \to 1$.

84. (a)

θ	0°	18°	36°	54°	72°	90°
$\sin\theta$	0	0.3090	0.5878	0.8090	0.9511	1
$\cos\theta$	1	0.9511	0.8090	0.5878	0.3090	0

(b) On $0° \le \theta \le 90°$, $\sin\theta$ is an increasing function.

(c) On $0° \le \theta \le 90°$, $\cos\theta$ is a decreasing function.

(d) As the angle increases the length of the side opposite the angle increases relative to the length of the hypotenuse and the length of the side adjacent to the angle decreases relative to the length of the hypotenuse. So, the sine increases and the cosine decreases.

Section 4.4 Trigonometric Functions of Any Angle

1. $\dfrac{y}{r}$

2. $\csc\theta$

3. $\dfrac{y}{x}$

4. $\dfrac{r}{x}$

5. $\cos\theta$

6. $\cot\theta$

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7. zero; defined

8. reference

9. (a) $(x, y) = (4, 3)$

$r = \sqrt{16 + 9} = 5$

$\sin\theta = \frac{y}{r} = \frac{3}{5}$ $\quad\csc\theta = \frac{r}{y} = \frac{5}{3}$

$\cos\theta = \frac{x}{r} = \frac{4}{5}$ $\quad\sec\theta = \frac{r}{x} = \frac{5}{4}$

$\tan\theta = \frac{y}{x} = \frac{3}{4}$ $\quad\cot\theta = \frac{x}{y} = \frac{4}{3}$

(b) $(x, y) = (-8, 15)$

$r = \sqrt{64 + 225} = 17$

$\sin\theta = \frac{y}{r} = \frac{15}{17}$ $\quad\csc\theta = \frac{r}{y} = \frac{17}{15}$

$\cos\theta = \frac{x}{r} = -\frac{8}{17}$ $\quad\sec\theta = \frac{r}{x} = -\frac{17}{8}$

$\tan\theta = \frac{y}{x} = -\frac{15}{8}$ $\quad\cot\theta = \frac{x}{y} = -\frac{8}{15}$

10. (a) $x = -12, y = -5$

$r = \sqrt{(-12)^2 + (-5)^2} = 13$

$\sin\theta = \frac{y}{r} = -\frac{5}{13}$ $\quad\csc\theta = \frac{r}{y} = -\frac{13}{5}$

$\cos\theta = \frac{x}{r} = -\frac{12}{13}$ $\quad\sec\theta = \frac{r}{x} = -\frac{13}{12}$

$\tan\theta = \frac{y}{x} = \frac{-5}{-12} = \frac{5}{12}$ $\quad\cot\theta = \frac{x}{y} = \frac{-12}{-5} = \frac{12}{5}$

(b) $(x, y) = (1, -1)$

$r = \sqrt{1 + 1} = \sqrt{2}$

$\sin\theta = \frac{y}{r} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$ $\quad\csc\theta = \frac{r}{y} = -\sqrt{2}$

$\cos\theta = \frac{x}{r} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\quad\sec\theta = \frac{r}{x} = \sqrt{2}$

$\tan\theta = \frac{y}{x} = -1$ $\quad\cot\theta = \frac{x}{y} = -1$

11. (a) $(x, y) = (-\sqrt{3}, -1)$

$r = \sqrt{3 + 1} = 2$

$\sin\theta = \frac{y}{r} = -\frac{1}{2}$ $\quad\csc\theta = \frac{r}{y} = -2$

$\cos\theta = \frac{x}{r} = -\frac{\sqrt{3}}{2}$ $\quad\sec\theta = \frac{r}{x} = -\frac{2\sqrt{3}}{3}$

$\tan\theta = \frac{y}{x} = \frac{\sqrt{3}}{3}$ $\quad\cot\theta = \frac{x}{y} = \sqrt{3}$

(b) $(x, y) = (4, -1)$

$r = \sqrt{16 + 1} = \sqrt{17}$

$\sin\theta = \frac{y}{r} = -\frac{1}{\sqrt{17}} = -\frac{\sqrt{17}}{17}$ $\quad\csc\theta = \frac{r}{y} = -\sqrt{17}$

$\cos\theta = \frac{x}{r} = \frac{4}{\sqrt{17}} = \frac{4\sqrt{17}}{17}$ $\quad\sec\theta = \frac{r}{x} = \frac{\sqrt{17}}{4}$

$\tan\theta = \frac{y}{x} = -\frac{1}{4}$ $\quad\cot\theta = \frac{x}{y} = -4$

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12. (a) $x = 3, y = 1$

$r = \sqrt{3^2 + 1^2} = \sqrt{10}$

$\sin\theta = \frac{y}{r} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$

$\cos\theta = \frac{x}{r} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$

$\tan\theta = \frac{y}{x} = \frac{1}{3}$

$\csc\theta = \frac{r}{y} = \frac{\sqrt{10}}{1} = \sqrt{10}$

$\sec\theta = \frac{r}{x} = \frac{\sqrt{10}}{3}$

$\cot\theta = \frac{x}{y} = \frac{3}{1} = 3$

(b) $(x, y) = \left(-2, \sqrt{21}\right)$

$r = \sqrt{(-2)^2 + \left(\sqrt{21}\right)^2} = \sqrt{4 + 21} = \sqrt{25} = 5$

$\sin\theta = \frac{y}{r} = \frac{\sqrt{21}}{5}$ $\qquad\csc\theta = \frac{r}{y} = \frac{5}{\sqrt{21}} = \frac{5\sqrt{21}}{21}$

$\cos\theta = \frac{x}{r} = -\frac{2}{5}$ $\qquad\sec\theta = \frac{r}{x} = -\frac{5}{2}$

$\tan\theta = \frac{y}{x} = -\frac{\sqrt{21}}{2}$ $\qquad\cot\theta = \frac{x}{y} = -\frac{2\sqrt{21}}{21}$

13. $(x, y) = (5, 12)$

$r = \sqrt{25 + 144} = 13$

$\sin\theta = \frac{y}{r} = \frac{12}{13}$ $\qquad\csc\theta = \frac{r}{y} = \frac{13}{12}$

$\cos\theta = \frac{x}{r} = \frac{5}{13}$ $\qquad\sec\theta = \frac{r}{x} = \frac{13}{5}$

$\tan\theta = \frac{y}{x} = \frac{12}{5}$ $\qquad\cot\theta = \frac{x}{y} = \frac{5}{12}$

14. $x = 8, y = 15$

$r = \sqrt{8^2 + 15^2} = 17$

$\sin\theta = \frac{y}{r} = \frac{15}{17}$ $\qquad\csc\theta = \frac{r}{y} = \frac{17}{15}$

$\cos\theta = \frac{x}{r} = \frac{8}{17}$ $\qquad\sec\theta = \frac{r}{x} = \frac{17}{8}$

$\tan\theta = \frac{y}{x} = \frac{15}{8}$ $\qquad\cot\theta = \frac{x}{y} = \frac{8}{15}$

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15. $x = -5, y = -2$

$$r = \sqrt{(-5)^2 + (-2)^2} = \sqrt{29}$$

$$\sin\theta = \frac{y}{r} = \frac{-2}{\sqrt{29}} = -\frac{2\sqrt{29}}{29}$$

$$\cos\theta = \frac{x}{r} = \frac{-5}{\sqrt{29}} = -\frac{5\sqrt{29}}{29}$$

$$\tan\theta = \frac{y}{x} = \frac{-2}{-5} = \frac{2}{5}$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{29}}{-2} = -\frac{\sqrt{29}}{2}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{29}}{-5} = -\frac{\sqrt{29}}{5}$$

$$\cot\theta = \frac{x}{y} = \frac{-5}{-2} = \frac{5}{2}$$

16. $(x, y) = (-4, 10)$

$$r = \sqrt{16 + 100} = 2\sqrt{29}$$

$$\sin\theta = \frac{y}{r} = \frac{5\sqrt{29}}{29}$$

$$\cos\theta = \frac{x}{r} = -\frac{2\sqrt{29}}{29}$$

$$\tan\theta = \frac{y}{x} = -\frac{5}{2}$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{29}}{5}$$

$$\sec\theta = \frac{r}{x} = -\frac{\sqrt{29}}{2}$$

$$\cot\theta = \frac{x}{y} = -\frac{2}{5}$$

17. $(x, y) = (-5.4, 7.2)$

$$r = \sqrt{29.16 + 51.84} = 9$$

$$\sin\theta = \frac{y}{r} = \frac{7.2}{9} = \frac{4}{5} \qquad \csc\theta = \frac{r}{y} = \frac{9}{7.2} = \frac{5}{4}$$

$$\cos\theta = \frac{x}{r} = -\frac{5.4}{9} = -\frac{3}{5} \qquad \sec\theta = \frac{r}{x} = -\frac{9}{5.4} = -\frac{5}{3}$$

$$\tan\theta = \frac{y}{x} = -\frac{7.2}{5.4} = -\frac{4}{3} \qquad \tan\theta = \frac{x}{y} = -\frac{5.4}{7.2} = -\frac{3}{4}$$

18. $x = 3\frac{1}{2} = \frac{7}{2}, y = -2\sqrt{15}$

$$r = \sqrt{\left(\frac{7}{2}\right)^2 + \left(-2\sqrt{15}\right)^2} = \sqrt{\frac{49}{4} + 60} = \sqrt{\frac{289}{4}} = \frac{17}{2}$$

$$\sin\theta = \frac{y}{r} = \frac{-2\sqrt{15}}{\frac{7}{2}} = -\frac{4\sqrt{15}}{7} \qquad \csc\theta = \frac{r}{y} = -\frac{7}{4\sqrt{15}} = -\frac{7\sqrt{15}}{60}$$

$$\cos\theta = \frac{x}{r} = \frac{\frac{7}{2}}{\frac{17}{2}} = \frac{7}{17} \qquad \sec\theta = \frac{r}{x} = \frac{17}{7}$$

$$\tan\theta = \frac{y}{x} = \frac{-2\sqrt{15}}{\frac{7}{2}} = -\frac{4\sqrt{15}}{7} \qquad \cot\theta = \frac{x}{y} = \frac{7}{-4\sqrt{15}} = -\frac{7\sqrt{15}}{60}$$

19. $\sin\theta > 0 \Rightarrow \theta$ lies in Quadrant I or in Quadrant II.
$\cos\theta > 0 \Rightarrow \theta$ lies in Quadrant I or in Quadrant IV.
$\sin\theta > 0$ and $\cos\theta > 0 \Rightarrow \theta$ lies in Quadrant I.

20. $\sin\theta < 0 \Rightarrow \theta$ lies in Quadrant III or in Quadrant IV.
$\cos\theta < 0 \Rightarrow \theta$ lies in Quadrant II or in Quadrant III.
$\sin\theta < 0$ and $\cos\theta < 0 \Rightarrow \theta$ lies in Quadrant III.

21. $\sin\theta > 0 \Rightarrow \theta$ lies in Quadrant I or in Quadrant II.
$\cos\theta < 0 \Rightarrow \theta$ lies in Quadrant II or in Quadrant III.
$\sin\theta > 0$ and $\cos\theta < 0 \Rightarrow \theta$ lies in Quadrant II.

22. $\sec\theta > 0 \Rightarrow \theta$ lies in Quadrant I or in Quadrant IV.
$\cot\theta < 0 \Rightarrow \theta$ lies in Quadrant II or in Quadrant IV.
$\sec\theta > 0$ and $\cot\theta < 0 \Rightarrow \theta$ lies in Quadrant IV.

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23. $\tan\theta > 0$ and $\sin\theta > 0 \Rightarrow \theta$ is in Quadrant I $\Rightarrow x > 0$ and $y > 0$.

$\tan\theta = \frac{y}{x} = \frac{15}{8} \Rightarrow r = 17$

$\sin\theta = \frac{y}{r} = \frac{15}{7}$ $\qquad$ $\sec\theta = \frac{r}{x} = \frac{17}{8}$

$\cos\theta = \frac{x}{r} = \frac{8}{17}$ $\qquad$ $\cot\theta = \frac{x}{y} = \frac{8}{15}$

$\csc\theta = \frac{r}{y} = \frac{17}{15}$

24. $\cos\theta > 0$ and $\tan\theta < 0 \Rightarrow \theta$ is in Quadrant IV $\Rightarrow x > 0$ and $y < 0$.

$\cos\theta = \frac{x}{r} = \frac{8}{17} \Rightarrow y = -15$

$\sin\theta = \frac{y}{r} = \frac{-15}{17} = -\frac{15}{17}$ $\qquad$ $\sec\theta = \frac{17}{8}$

$\tan\theta = \frac{y}{x} = \frac{-15}{8} = -\frac{15}{8}$ $\qquad$ $\cot\theta = -\frac{8}{15}$

$\csc\theta = -\frac{17}{15}$

25. $\sin\theta = 0.6 = \frac{3}{5}$ and θ in Quadrant II

$x^2 + y^2 = r^2$

$\sin\theta = \frac{3}{5}$, $\qquad x = -\sqrt{r^2 - y^2}$

$x = -\sqrt{5^2 - 3^2} = -4$

$\cos\theta = \frac{x}{r} = -\frac{4}{5}$ $\qquad$ $\sec\theta = \frac{r}{x} = -\frac{5}{4}$

$\tan\theta = \frac{y}{x} = -\frac{3}{4}$ $\qquad$ $\cot\theta = \frac{x}{y} = -\frac{4}{3}$

$\csc\theta = \frac{r}{y} = \frac{5}{3}$

26. $\cos\theta = \frac{x}{r} = -0.8 = \frac{-4}{5} \Rightarrow y = -3$, θ in Quadrant III

$\sin\theta = \frac{y}{r} = -\frac{3}{5}$ $\qquad$ $\sec\theta = -\frac{5}{4}$

$\tan\theta = \frac{y}{x} = \frac{3}{4}$ $\qquad$ $\cot\theta = \frac{4}{3}$

$\csc\theta = -\frac{5}{3}$

27. $\cot\theta = \frac{x}{y} = -\frac{3}{1} = \frac{3}{-1}$

$\cos\theta > 0 \Rightarrow \theta$ is in Quadrant IV $\Rightarrow x$ is positive;

$x = 3, y = -1, r = \sqrt{10}$

$\sin\theta = \frac{y}{r} = -\frac{\sqrt{10}}{10}$ $\qquad$ $\csc\theta = \frac{r}{y} = -\sqrt{10}$

$\cos\theta = \frac{x}{r} = \frac{3\sqrt{10}}{10}$ $\qquad$ $\sec\theta = \frac{r}{x} = \frac{\sqrt{10}}{3}$

$\tan\theta = \frac{y}{x} = -\frac{1}{3}$

28. $\cot\theta < 0$ and $\cos\theta > 0 \Rightarrow \theta$ is in Quadrant IV

$\csc\theta = \frac{r}{y} = \frac{4}{1} \Rightarrow x = -\sqrt{15}$

$\cot\theta < 0 \Rightarrow x = -\sqrt{15}$

$\sin\theta = \frac{y}{r} = \frac{1}{4}$ $\qquad$ $\sec\theta = -\frac{4\sqrt{15}}{15}$

$\cos\theta = \frac{x}{r} = -\frac{\sqrt{15}}{4}$ $\qquad$ $\cot\theta = -\sqrt{15}$

$\tan\theta = \frac{y}{x} = -\frac{\sqrt{15}}{15}$

29. $\cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2} + \pi n$

$\csc\theta = 1 \Rightarrow \theta = \frac{\pi}{2} + 2\pi n$

$y = 1, x = 0, r = 1$

$\sin\theta = \frac{y}{r} = 1$

$\tan\theta = \frac{y}{x}$ is undefined

$\sec\theta$ is undefined

$\cot\theta = 0$.

30. $\sin\theta = 0 \Rightarrow \theta = 0 + \pi n$

$\sec\theta = -1 \Rightarrow \theta = \pi + 2\pi n$

$y = 0, x = -1, r = 1$

$\cos\theta = \frac{x}{r} = -1$

$\tan\theta = \frac{y}{x} = 0$

$\csc\theta$ is undefined

$\cot\theta$ is undefined.

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31. $\cot\theta$ is undefined,

$\frac{\pi}{2} \le \theta \le \frac{3\pi}{2} \Rightarrow y = 0 \Rightarrow \theta = \pi$

$\sin\theta = 0$ $\quad$ $\csc\theta$ is undefined.

$\cos\theta = -1$ $\quad$ $\sec\theta = -1$

$\tan\theta = 0$ $\quad$ $\cot\theta$ is undefined.

32. $\tan\theta$ is undefined $\Rightarrow \theta = n\pi + \frac{\pi}{2}$

$\pi \le \theta \le 2\pi \Rightarrow \theta = \frac{3\pi}{2}, x = 0, y = -r$

$\sin\theta = \frac{y}{r} = \frac{-r}{r} = -1$ $\quad$ $\csc\theta = \frac{r}{y} = -1$

$\cos\theta = \frac{x}{r} = \frac{0}{r} = 0$ $\quad$ $\sec\theta$ is undefined.

$\tan\theta$ is undefined. $\quad$ $\cot\theta = \frac{x}{y} = \frac{0}{y} = 0$

33. To find a point on the terminal side of θ, use any point on the line $y = -x$ that lies in Quadrant II. $(-1, 1)$ is one such point.

$x = -1, y = 1, r = \sqrt{2}$

$\sin\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\quad$ $\csc\theta = \sqrt{2}$

$\cos\theta = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$ $\quad$ $\sec\theta = -\sqrt{2}$

$\tan\theta = -1$ $\quad$ $\cot\theta = -1$

34. Let $x > 0$.

$\left(-x, -\frac{1}{3}x\right)$, Quadrant III

$r = \sqrt{x^2 + \frac{1}{9}x^2} = \frac{\sqrt{10}x}{3}$

$\sin\theta = \frac{y}{r} = \frac{\left(-\frac{1}{3}\right)x}{\frac{\sqrt{10}x}{3}} = -\frac{\sqrt{10}}{10}$ $\quad$ $\csc\theta = \frac{r}{y} = \frac{\frac{\sqrt{10}x}{3}}{\left(-\frac{1}{3}\right)x} = -\sqrt{10}$

$\cos\theta = \frac{x}{r} = \frac{-x}{\frac{\sqrt{10}x}{3}} = -\frac{3\sqrt{10}}{10}$ $\quad$ $\sec\theta = \frac{r}{x} = \frac{\frac{\sqrt{10}x}{3}}{-x} = -\frac{\sqrt{10}}{3}$

$\tan\theta = \frac{y}{x} = \frac{\left(-\frac{1}{3}\right)x}{-x} = \frac{1}{3}$ $\quad$ $\cot\theta = \frac{x}{y} = \frac{-x}{\left(-\frac{1}{3}\right)x} = 3$

35. To find a point on the terminal side of θ, use any point on the line $y = 2x$ that lies in Quadrant I. (1, 2) is one such point.

$x = 1, y = 2, y = \sqrt{5}$

$\sin\theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ $\quad$ $\csc\theta = \frac{\sqrt{5}}{2} = \frac{\sqrt{5}}{2}$

$\cos\theta = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ $\quad$ $\sec\theta = \frac{\sqrt{5}}{1} = \sqrt{5}$

$\tan\theta = \frac{2}{1} = 2$ $\quad$ $\cot\theta = \frac{1}{2}$

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36. Let $x > 0$.

$4x + 3y = 0 \Rightarrow y = -\frac{4}{3}x$

$\left(x, -\frac{4}{3}x\right)$, Quadrant IV

$r = \sqrt{x^2 + \frac{16}{9}x^2} = \frac{5}{3}x$

$\sin\theta = \frac{y}{r} = \frac{\left(-\frac{4}{3}\right)x}{\frac{5}{3}x} = -\frac{4}{5}$ $\qquad \csc\theta = -\frac{5}{4}$

$\cos\theta = \frac{x}{r} = \frac{x}{\frac{5}{3}x} = \frac{3}{5}$ $\qquad \sec\theta = \frac{5}{3}$

$\tan\theta = \frac{y}{x} = \frac{\left(-\frac{4}{3}\right)x}{x} = -\frac{4}{3}$ $\qquad \cot\theta = -\frac{3}{4}$

37. $(x, y) = (-1, 0), r = 1$

$\sin\pi = \frac{y}{r} = \frac{0}{1} = 0$

38. $(x, y) = (0, -1), r = 1$

$\csc\frac{3\pi}{2} = \frac{1}{-1} = -1$

39. $(x, y) = (0, -1), r = 1$

$\sec\frac{3\pi}{2} = \frac{r}{x} = \frac{1}{0} \Rightarrow$ undefined

40. $(x, y) = (-1, 0), r = 1$

$\sec\pi = \frac{r}{x} = \frac{1}{-1} = -1$

41. $(x, y) = (0, 1), r = 1$

$\sin\frac{\pi}{2} = \frac{y}{r} = \frac{1}{1} = 1$

42. $(x, y) = (-1, 0), r = 1$

$\cot\pi = \frac{-1}{0}$ undefined.

43. $(x, y) = (-1, 0), r = 1$

$\csc\pi = \frac{r}{y} = \frac{1}{0} \Rightarrow$ undefined

44. $(x, y) = (0, 1)$

$\cot\frac{\pi}{2} = \frac{x}{y} = \frac{0}{1} = 0$

45. $(x, y) = (0, 1)$

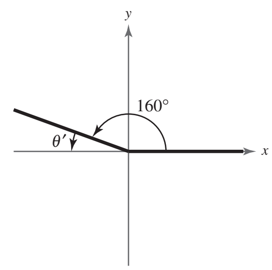
$\cot\frac{9\pi}{2} = \frac{x}{y} = \frac{0}{1} = 0$

46. $(x, y) = (0, -1)$

$\tan\left(-\frac{\pi}{2}\right) = \frac{y}{x} = \frac{-1}{0}$ is undefined.

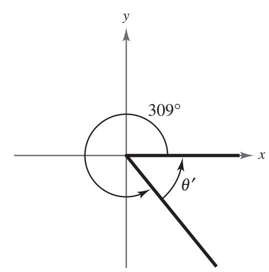
47. $\theta = 160°$

$\theta' = 180° - 160° = 20°$



48. $\theta = 309°$

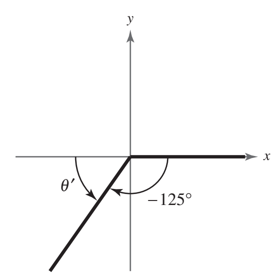
$\theta' = 360° - 309° = 51°$



49. $\theta = -125°$

$360° - 125° = 235°$ (coterminal angle)

$\theta' = 235° - 180° = 55°$



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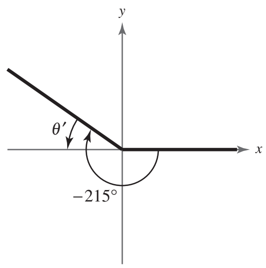
50. $\theta = -215°$

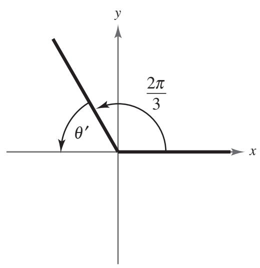
$360° - 215° = 145°$ (coterminal angle)

$\theta' = 180° - 145° = 35°$



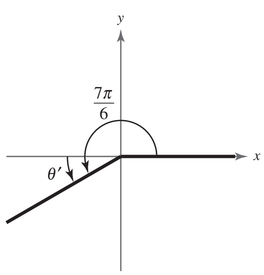
51. $\theta = \dfrac{2\pi}{3}$

$\theta' = \pi - \dfrac{2\pi}{3} = \dfrac{\pi}{3}$



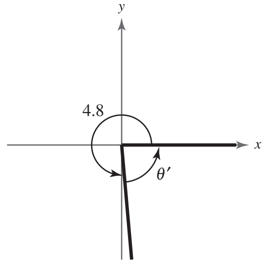
52. $\theta = \dfrac{7\pi}{6}$

$\theta' = \dfrac{7\pi}{6} - \pi = \dfrac{\pi}{6}$



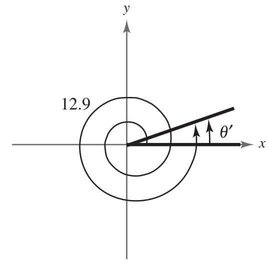
53. $\theta = 4.8$

$\theta' = 2\pi - 4.8 \approx 1.48$



54. $\theta = 12.9$

$\theta' = 12.9 - 4\pi \approx 0.3336$



55. $\theta = 225°, \theta' = 45°$, Quadrant III

$\sin 225° = -\sin 45° = -\dfrac{\sqrt{2}}{2}$

$\cos 225° = -\cos 45° = -\dfrac{\sqrt{2}}{2}$

$\tan 225° = \tan 45° = 1$

56. $\theta = 300°, \theta' = 360° - 300° = 60°$ in Quadrant IV.

$\sin 300° = -\sin 60° = -\dfrac{\sqrt{3}}{2}$

$\cos 300° = \cos 60° = \dfrac{1}{2}$

$\tan 300° = -\tan 60° = -\sqrt{3}$

57. $\theta = 750°, \theta' = 30°$, Quadrant I

$\sin 750° = \sin 30° = \dfrac{1}{2}$

$\cos 750° = \cos 30° = \dfrac{\sqrt{3}}{2}$

$\tan 750° = \tan 30° = \dfrac{\sqrt{3}}{3}$

58. $\theta = 675°, \theta' = 45°$, Quadrant IV

$\sin 675° = \sin 45° = -\dfrac{\sqrt{2}}{2}$

$\cos 675° = \cos 45° = \dfrac{\sqrt{2}}{2}$

$\tan 675° = \tan 45° = \sqrt{3}$

59. $\theta = -120°, \theta' = 60°$, Quadrant III

$\sin(-120°) = -\sin 60° = -\dfrac{\sqrt{3}}{2}$

$\cos(-120°) = -\cos 60° = -\dfrac{1}{2}$

$\tan(-120°) = \tan 60° = \sqrt{3}$

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60. $\theta = -570°, \theta' = 30°$ in Quadrant II.

$$\sin(-570°) = \sin 30° = \frac{1}{2}$$
$$\cos(-570°) = -\cos 30° = -\frac{\sqrt{3}}{2}$$
$$\tan(-570°) = -\tan 30° = -\frac{\sqrt{3}}{3}$$

61. $\theta = \frac{2\pi}{3}, \theta' = \frac{\pi}{3}$ in Quadrant II

$$\sin\frac{2\pi}{3} = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$
$$\cos\frac{2\pi}{3} = -\cos\frac{\pi}{3} = -\frac{1}{2}$$
$$\tan\frac{2\pi}{3} = -\tan\frac{\pi}{3} = -\sqrt{3}$$

62. $\theta = \frac{3\pi}{4}, \theta' = \frac{\pi}{4}$ in Quadrant II

$$\sin\frac{3\pi}{4} = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$
$$\cos\frac{3\pi}{4} = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$
$$\tan\frac{3\pi}{4} = -\tan\frac{\pi}{4} = -1$$

63. $\theta = -\frac{\pi}{6}, \theta' = \frac{\pi}{6}$, Quadrant IV

$$\sin\left(-\frac{\pi}{6}\right) = -\sin\frac{\pi}{6} = -\frac{1}{2}$$
$$\cos\left(-\frac{\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$
$$\tan\left(-\frac{\pi}{6}\right) = -\tan\frac{\pi}{6} = -\frac{\sqrt{3}}{3}$$

64. $\theta = -\frac{2\pi}{3}, \theta' = \frac{\pi}{3}$ in Quadrant III

$$\sin\left(-\frac{2\pi}{3}\right) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$
$$\cos\left(-\frac{2\pi}{3}\right) = -\cos\frac{\pi}{3} = -\frac{1}{2}$$
$$\tan\left(-\frac{2\pi}{3}\right) = \tan\frac{\pi}{3} = \sqrt{3}$$

65. $\theta = \frac{11\pi}{4}, \theta' = \frac{\pi}{4}$, Quadrant II

$$\sin\frac{11\pi}{4} = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$
$$\cos\frac{11\pi}{4} = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$
$$\tan\frac{11\pi}{4} = -\tan\frac{\pi}{4} = -1$$

66. $\theta = \frac{13\pi}{6}, \theta' = \frac{\pi}{6}$ in Quadrant I

$$\sin\frac{13\pi}{6} = \sin\frac{\pi}{6} = \frac{1}{2}$$
$$\cos\frac{13\pi}{6} = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$
$$\tan\frac{13\pi}{6} = \tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$$

67. $\theta = -\frac{17\pi}{6}, \theta' = \frac{\pi}{6}$ in Quadrant III

$$\sin\left(-\frac{17\pi}{6}\right) = -\sin\frac{\pi}{6} = -\frac{1}{2}$$
$$\cos\left(-\frac{17\pi}{6}\right) = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$$
$$\tan\left(-\frac{17\pi}{6}\right) = \tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$$

68. $\theta = -\frac{23\pi}{4}, \theta' = \frac{\pi}{4}$ in Quadrant I

$$\sin\left(-\frac{23\pi}{4}\right) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$
$$\cos\left(-\frac{23\pi}{4}\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$
$$\tan\left(-\frac{23\pi}{4}\right) = \tan\frac{\pi}{4} = 1$$

69. $\sin\theta = -\frac{3}{5}$

$$\sin^2\theta + \cos^2\theta = 1$$
$$\cos^2\theta = 1 - \sin^2\theta$$
$$\cos^2\theta = 1 - \left(-\frac{3}{5}\right)^2$$
$$\cos^2\theta = 1 - \frac{9}{25}$$
$$\cos^2\theta = \frac{16}{25}$$

$\cos\theta > 0$ in Quadrant IV.

$$\cos\theta = \frac{4}{5}$$

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70. $\cot\theta = -3$

$1 + \cot^2\theta = \csc^2\theta$

$1 + (-3)^2 = \csc^2\theta$

$10 = \csc^2\theta$

$\csc\theta > 0$ in Quadrant II.

$\csc\theta = \sqrt{10}$

71. $\tan\theta = \dfrac{3}{2}$

$\sec^2\theta = 1 + \tan^2\theta$

$\sec^2\theta = 1 + \left(\dfrac{3}{2}\right)^2$

$\sec^2\theta = 1 + \dfrac{9}{4}$

$\sec^2\theta = \dfrac{13}{4}$

$\sec\theta < 0$ in Quadrant III.

$\sec\theta = -\dfrac{\sqrt{13}}{2}$

72. $\csc\theta = -2$

$1 + \cot^2\theta = \csc^2\theta$

$\cot^2\theta = \csc^2\theta - 1$

$\cot^2\theta = (-2)^2 - 1$

$\cot^2\theta = 3$

$\cot\theta < 0$ in Quadrant IV.

$\cot\theta = -\sqrt{3}$

73. $\cos\theta = \dfrac{5}{8}$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \left(\dfrac{5}{8}\right)^2 = 1$

$\sin^2\theta = 1 - \dfrac{25}{64}$

$\sin^2\theta = \dfrac{39}{64}$

$\sin\theta > 0$ in Quadrant I.

$\sin\theta = \dfrac{\sqrt{39}}{8}$

$\csc\theta = \dfrac{1}{\sin\theta}$

$\csc\theta = \dfrac{8\sqrt{39}}{39}$

74. $\sec\theta = -\dfrac{9}{4}$

$1 + \tan^2\theta = \sec^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

$\tan^2\theta = \left(-\dfrac{9}{4}\right)^2 - 1$

$\tan^2\theta = \dfrac{65}{16}$

$\tan\theta > 0$ in Quadrant III.

$\tan\theta = \dfrac{\sqrt{65}}{4}$

$\cot\theta = \dfrac{1}{\tan\theta}$

$\cot\theta = \dfrac{4\sqrt{65}}{65}$

75. $\sin 10^\circ \approx 0.1736$

76. $\tan 304^\circ \approx -1.4826$

77. $\cos(-110^\circ) \approx -0.3420$

78. $\sin(-330^\circ) = 0.5$

79. $\cot 178^\circ \approx -28.6363$

80. $\sec 72^\circ = \dfrac{1}{\cos 72^\circ} \approx 3.2361$

81. $\csc 405^\circ = \dfrac{1}{\sin 405^\circ} \approx 1.4142$

82. $\cot(-560^\circ) = \dfrac{1}{\tan(-560^\circ)} \approx -2.7475$

83. $\tan\left(\dfrac{\pi}{9}\right) \approx 0.3640$

84. $\cos\dfrac{2\pi}{7} \approx 0.6235$

85. $\sec\dfrac{11\pi}{8} = \dfrac{1}{\cos\dfrac{11\pi}{8}} \approx -2.6131$

86. $\csc\dfrac{15\pi}{4} = \dfrac{1}{\sin\dfrac{15\pi}{4}} \approx -1.4142$

87. $\sin(-0.65) \approx -0.6052$

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88. $\cos 1.35 \approx 0.2190$

89. $\csc(-10) = \dfrac{1}{\sin(-10)} \approx 1.8382$

90. $\sec(-4.6) = \dfrac{1}{\cos(-4.6)} \approx -8.9164$

91. (a) $\sin\theta = \dfrac{1}{2} \Rightarrow$ reference angle is $30°$ or $\dfrac{\pi}{6}$ and θ is in Quadrant I or Quadrant II.

Values in degrees: $30°, 150°$

Values in radian: $\dfrac{\pi}{6}, \dfrac{5\pi}{6}$

(b) $\sin\theta = \dfrac{1}{2} \Rightarrow$ reference angle is $30°$ or $\dfrac{\pi}{6}$ and θ is in Quadrant III or Quadrant IV.

Values in degrees: $210°, 330°$

Values in radians: $\dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

92. (a) $\cos\theta = \dfrac{\sqrt{2}}{2} \Rightarrow$ reference angle is $45°$ or $\dfrac{\pi}{4}$ and θ is in Quadrant I or IV.

Values in degrees: $45°, 315°$

Values in radians: $\dfrac{\pi}{4}, \dfrac{7\pi}{4}$

(b) $\cos\theta = -\dfrac{\sqrt{2}}{2} \Rightarrow$ reference angle is $45°$ or $\dfrac{\pi}{4}$ and θ is in Quadrant II or III.

Values in degrees: $135°, 225°$

Values in radians: $\dfrac{3\pi}{4}, \dfrac{5\pi}{4}$

93. (a) $\cos\theta = \dfrac{1}{2} \Rightarrow$ reference angle is $60°$ or $\dfrac{\pi}{3}$ and θ is in Quadrant I or IV.

Values in degrees: $60°, 300°$

Values in radians: $\dfrac{\pi}{3}, \dfrac{5\pi}{3}$

(b) $\sec\theta = 2 \Rightarrow \cos\theta = \dfrac{1}{2} \Rightarrow$ reference angle is $60°$ or $\dfrac{\pi}{3}$ and θ is in Quadrant I or IV.

Values in degrees: $60°, 300°$

Values in radians: $\dfrac{\pi}{3}, \dfrac{5\pi}{3}$

94. (a) $\sin\theta = \dfrac{\sqrt{3}}{2} \Rightarrow$ reference angle is $60°$ or $\dfrac{\pi}{3}$ and θ is in Quadrant I or Quadrant II.

θ is in Quadrant I or Quadrant II.

Values in degrees: $60°, 120°$

Values in radians: $\dfrac{\pi}{3}, \dfrac{2\pi}{3}$

(b) $\csc\theta = \dfrac{2\sqrt{3}}{3} \Rightarrow \sin\theta = \dfrac{\sqrt{3}}{2}$ reference angle is $60°$ or $\dfrac{\pi}{3}$ and θ is in Quadrant I or Quadrant II.

Values in degrees: $60°, 120°$

Values in radians: $\dfrac{\pi}{3}, \dfrac{2\pi}{3}$

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95. (a) $\tan\theta = 1 \Rightarrow$ reference angle is $45°$ or $\frac{\pi}{4}$ and θ is in Quadrant I or Quadrant III.

Values in degrees: $45°, 225°$

Values in radians: $\frac{\pi}{4}, \frac{5\pi}{4}$

(b) $\cot\theta = -\sqrt{3} \Rightarrow$ reference angle is $30°$ or $\frac{\pi}{6}$ and θ is in Quadrant II or Quadrant IV.

Values in degrees: $150°, 330°$

Values in radians: $\frac{5\pi}{6}, \frac{11\pi}{6}$

96. (a) $\cot\theta = 0 \Rightarrow \tan\theta$ is undefined $\Rightarrow$ no reference angle, so the solutions are quadrants angles.

Values in degrees: $90°, 270°$

Values in radians: $\frac{\pi}{2}, \frac{3\pi}{2}$

(b) $\sec\theta = -\sqrt{2} \Rightarrow \cos\theta = -\frac{\sqrt{2}}{2}$ reference angle is $45°$ or $\frac{\pi}{4}$ and θ is in Quadrant II or Quadrant III.

Values in degrees: $135°, 225°$

Values in radians: $\frac{3\pi}{4}, \frac{5\pi}{4}$

97. $\sin\theta = \frac{6}{d} \Rightarrow d = \frac{6}{\sin\theta}$

(a) $\theta = 30°$

$$d = \frac{6}{\sin 30°} = \frac{6}{1/2} = 12 \text{ miles}$$

(b) $\theta = 90°$

$$d = \frac{6}{\sin 90°} = \frac{6}{1} = 6 \text{ miles}$$

(c) $\theta = 120°$

$$d = \frac{6}{\sin 120°} = \frac{6}{\sqrt{3}/2} \approx 6.9 \text{ miles}$$

98. $y(t) = 2\cos 6t$

(a) $y(0) = 2\cos 0 = 2$ centimeters

(b) $y\left(\frac{1}{4}\right) = 2\cos\left(\frac{3}{2}\right) \approx 0.14$ centimeter

(c) $y\left(\frac{1}{2}\right) = 2\cos 3 \approx -1.98$ centimeters

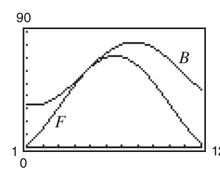
99. (a) Boston: $B \approx 24.593\sin(0.495t - 2.262) + 57.387$

Fairbanks:
$F \approx 39.071\sin(0.448t - 1.366) + 32.204$

(b)

Month	*Boston, B*	*Fairbanks, F*
February	33.9°	14.5°
April	50.5°	48.3°
May	62.6°	62.2°
July	80.3°	70.5°
September	77.4°	50.1°
October	68.2°	33.3°
December	44.8°	2.4°

(c)



Answers will vary.

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100. $S = 23.1 + 0.442t + 4.3\cos\dfrac{\pi t}{6}$

(a) $t = 2;\ S = 23.1 + 0.442(2) + 4.3\cos\dfrac{\pi(2)}{6} = 26{,}134$ snowboards

(b) $t = 14;\ S = 23.1 + 0.442(14) + 4.3\cos\dfrac{\pi(14)}{6} \approx 31{,}438$ snowboards

(c) $t = 6;\ S = 23.1 + 0.442(6) + 4.3\cos\dfrac{\pi(6)}{6} \approx 21{,}452$ snowboards

(d) $t = 18;\ S = 23.1 + 0.442(18) + 4.3\cos\dfrac{\pi(18)}{6} \approx 26{,}756$ snowboards

101. $I = 5e^{-2t}\sin t$

$I(0.7) = 5e^{-1.4}\sin 0.7 \approx 0.79$ amp

102. As θ increases from $0°$ to $90°$, x decreases from 12 cm to 0 cm and y increases from 0 cm to 12 cm. Therefore, $\sin\theta = \dfrac{y}{12}$ increases from 0 to 1, $\cos\theta = \dfrac{y}{12}$ decreases from 1 to 0, and $\tan\theta = \dfrac{y}{x}$ increases without bound (and is undefined at $\theta = 90°$).

103. False. In each of the four quadrants, the sign of the secant function and the cosine function will be the same because they are reciprocals of each other.

104. False. The reference angle is always acute by definition. For θ in Quadrant II, $\theta' = 180° - \theta$. For θ in Quadrant III, $\theta' = \theta - 180°$. For θ in Quadrant IV, $\theta' = 360° - \theta$.

105. Answers will vary.

106. (a) $\sin t = y, \cos t = x$

(b) r is the hypotenuse of the triangle which is equal to the radius of the circle. So, $r = 1$.

(c) $\sin\theta = y, \cos\theta = x$

(d) $\sin t = \sin\theta$ and $\cos t = \cos\theta$

Section 4.5 Graphs of Sine and Cosine Functions

1. cycle

2. amplitude

3. phase shift

4. vertical translation

5. $y = 2\sin 5x$

Period: $\dfrac{2\pi}{5}$

Amplitude: $|2| = 2$

6. $y = 3\cos 2x$

Period: $\dfrac{2\pi}{2} = \pi$

Amplitude: $|3| = 3$

7. $y = \dfrac{3}{4}\cos\dfrac{\pi x}{2}$

Period: $\dfrac{2\pi}{b} = \dfrac{2\pi}{\pi/2} = 4$

Amplitude: $\left|\dfrac{3}{4}\right| = \dfrac{3}{4}$

8. $y = -5\sin\dfrac{\pi x}{3}$

Period: $\dfrac{2\pi}{b} = \dfrac{2\pi}{\pi/3} = 6$

Amplitude: $|-5| = 5$

9. $y = -\dfrac{1}{2}\sin\dfrac{5x}{4}$

Period: $\dfrac{2\pi}{b} = \dfrac{2\pi}{5/4} = \dfrac{8\pi}{5}$

Amplitude: $\left|-\dfrac{1}{2}\right| = \dfrac{1}{2}$

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10. $y = \frac{1}{4}\sin\frac{x}{6}$

Period: $\frac{2\pi}{b} = \frac{2\pi}{1/6} = 12\pi$

Amplitude: $\left|\frac{1}{4}\right| = \frac{1}{4}$

11. $y = -\frac{5}{3}\sin\frac{\pi x}{12}$

Period: $\frac{2\pi}{b} = \frac{2\pi}{\pi/12} = 24$

Amplitude: $\left|-\frac{5}{3}\right| = \frac{5}{3}$

12. $y = -\frac{2}{5}\cos 10\pi x$

Period: $\frac{2\pi}{b} = \frac{2\pi}{10\pi} = \frac{1}{5}$

Amplitude: $\left|-\frac{2}{5}\right| = \frac{2}{5}$

13. $f(x) = \cos x$

$g(x) = \cos 5\,x$

The period of *g* is one-fifth the period of *f*.

14. $f(x) = \sin x$

$g(x) = 2\sin x$

The amplitude of *g* is twice the amplitude of *f*.

15. $f(x) = \cos 2x$

$g(x) = -\cos 2x$

g is a reflection of the graph of *f* in the *x*-axis.

16. $f(x) = \sin 3x$

$g(x) = \sin(-3x)$

g is a reflection of the graph of *f* in the *y*-axis.

17. $f(x) = \sin x$

$g(x) = \sin(x - \pi)$

g is a horizontal shift to the right π units of the graph of *f*.

18. $f(x) = \cos x$

$g(x) = \cos(x + \pi)$

g is a horizontal shift to the left π units of the graph of *f*.

19. $f(x) = \sin 2x$

$f(x) = 3 + \sin 2x$

g is a vertical shift three units upward of the graph of *f*.

20. $f(x) = \cos 4x$

$g(x) = -2 + \cos 4x$

g is a vertical shift two units downward of the graph of *f*.

21. The graph of *g* has twice the amplitude as the graph of *f*. The period is the same.

22. The period of *g* is one-third the period of *f*.

23. The graph of *g* is a horizontal shift π units to the right of the graph of *f*.

24. The graph of *g* is a vertical shift two units upward of the graph of *f*.

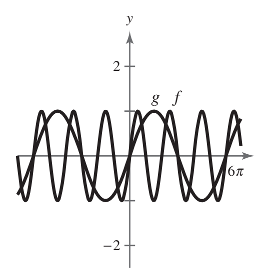
25. $f(x) = \sin x$

Period: $\frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$

Amplitude: 1

Symmetry: origin

Key points: Intercept	Maximum	Intercept	Minimum	Intercept
$(0, 0)$	$\left(\frac{\pi}{2}, 1\right)$	$(\pi, 0)$	$\left(\frac{3\pi}{2}, -1\right)$	$(2\pi, 0)$



Because $g(x) = \sin\left(\frac{x}{3}\right) = f\left(\frac{x}{3}\right)$, the graph of $g(x)$ is the graph of $f(x)$, but stretched horizontally by a factor of 3.

Generate key points for the graph of $g(x)$ by multiplying the *x*-coordinate of each key point of $f(x)$ by 3.

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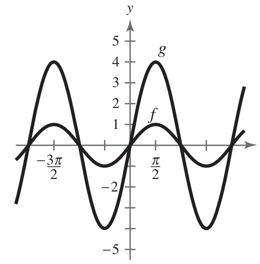
26. $f(x) = \sin x$



Period: $\frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$

Amplitude: 1

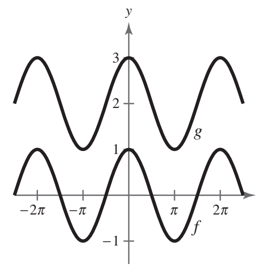
Symmetry: origin

Key points:	Intercept	Maximum	Intercept	Minimum	Intercept
	$(0, 0)$	$\left(\frac{\pi}{2}, 1\right)$	$(\pi, 0)$	$\left(\frac{3\pi}{2}, -1\right)$	$(2\pi, 0)$

Because $g(x) = 4 \sin x = 4f(x)$, the graph of $g(x)$ is the graph of $f(x)$, but stretched vertically by a factor of 4. The amplitude of $g(x)$ is four times the amplitude of $f(x)$.

Generate key points for the graph $g(x)$ by multiplying the x-coordinate of each key point of $f(x)$ by 4.

27. $f(x) = \cos x$



Period: $\frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$

Amplitude: $|1| = 1$

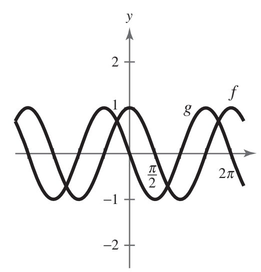
Symmetry: y-axis

Key points:	Maximum	Intercept	Minimum	Intercept	Maximum
	$(0, 1)$	$\left(\frac{\pi}{2}, 0\right)$	$(\pi, -1)$	$\left(\frac{3\pi}{2}, 0\right)$	$(2\pi, 1)$

Because $g(x) = 2 + \cos x = f(x) + 2$, the graph of $g(x)$ is the graph of $f(x)$, but translated upward by two units.

Generate key points of $g(x)$ by adding 2 to the y-coordinate of each key point of $f(x)$.

28. $f(x) = \cos x$



Period: $\frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$

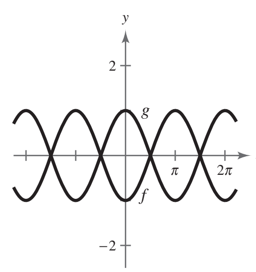
Amplitude: 1

Symmetry: y-axis

Key points:	Maximum	Intercept	Minimum	Intercept	Maximum
	$(0, 1)$	$\left(\frac{\pi}{2}, 0\right)$	$(\pi, -1)$	$\left(\frac{3\pi}{2}, 0\right)$	$(2\pi, 1)$

Because $g(x) = \cos\left(x + \frac{\pi}{2}\right) = f\left(x + \frac{\pi}{2}\right)$, the graph of $g(x)$ is the graph of $f(x)$, but with a phase shift (horizontal translation) of $-\frac{\pi}{2}$. Generate key points for the graph of $g(x)$ by shifting each key point of $f(x)$ $\frac{\pi}{2}$ units to the left.

29. $f(x) = -\cos x$



Period: $\frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$

Amplitude: 1

Symmetry: y-axis

Key points:	Minimum	Intercept	Maximum	Intercept	Minimum
	$(0, -1)$	$\left(\frac{\pi}{2}, 0\right)$	$(\pi, 1)$	$\left(\frac{3\pi}{2}, 0\right)$	$(2\pi, -1)$

Because $g(x) = -\cos(x - \pi) = f(x - \pi)$, the graph of $g(x)$ is the graph of $f(x)$, but with a phase shift (horizontal translation) of π. Generate key points for the graph of $g(x)$ by shifting each key point of $f(x)$ π units to the right.

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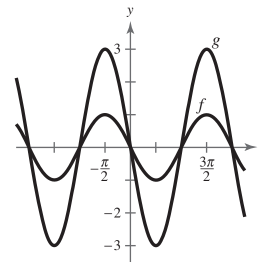
30. $f(x) = -\sin x$

Period: $\dfrac{2\pi}{b} = \dfrac{2\pi}{1} = 2\pi$

Amplitude: 1

Symmetry: origin

Key points: Intercept	Minimum	Intercept	Maximum	Intercept
$(0, 0)$	$\left(\dfrac{\pi}{2}, -1\right)$	$(\pi, 0)$	$\left(\dfrac{3\pi}{2}, 1\right)$	$(2\pi, 0)$



Because $g(x) = -3\sin x = 3f(x)$, generate key points for the graph of $g(x)$ by multiplying the y-coordinate of each key point of $f(x)$ by 3.

31. $y = 5\sin x$

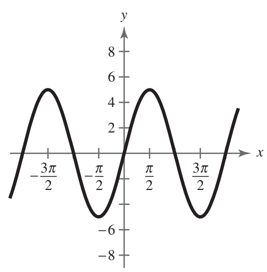
Period: 2π

Amplitude: 5

Key points:

$(0, 0), \left(\dfrac{\pi}{2}, 5\right), (\pi, 0),$

$\left(\dfrac{3\pi}{2}, -5\right), (2\pi, 0)$



32. $y = \dfrac{1}{4}\sin x$

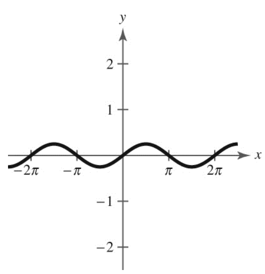
Period: 2π

Amplitude: $\dfrac{1}{4}$

Key points:

$(0, 0), \left(\dfrac{\pi}{2}, \dfrac{1}{4}\right), (\pi, 0),$

$\left(\dfrac{3\pi}{2}, -\dfrac{1}{4}\right), (2\pi, 0)$



33. $y = \dfrac{1}{3}\cos x$

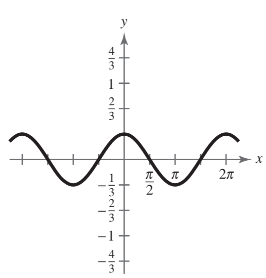
Period: 2π

Amplitude: $\dfrac{1}{3}$

Key points:

$\left(0, \dfrac{1}{3}\right), \left(\dfrac{\pi}{2}, 0\right), \left(\pi, -\dfrac{1}{3}\right),$

$\left(\dfrac{3\pi}{2}, 0\right), \left(2\pi, \dfrac{1}{3}\right)$



34. $y = 4\cos x$

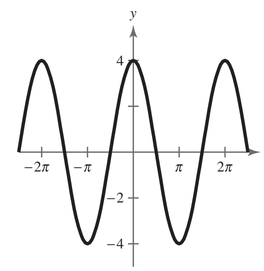
Period: 2π

Amplitude: 4

Key points:

$(0, 4), \left(\dfrac{\pi}{2}, 0\right), (\pi, -4),$

$\left(\dfrac{3\pi}{2}, 0\right), (2\pi, 4)$



35. $y = \cos\dfrac{x}{2}$

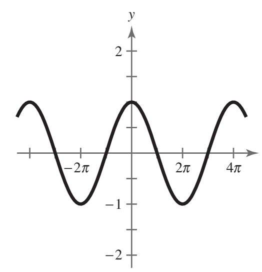
Period $\dfrac{2\pi}{1/2} = 4\pi$

Amplitude: 1

Key points:

$(0, 1), (\pi, 0), (2\pi, -1),$

$(3\pi, 0), (4\pi, 1)$



36. $y = \sin 4x$

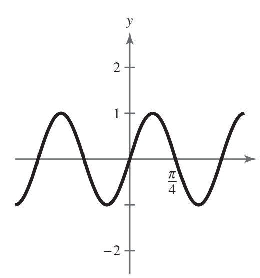
Period: $\dfrac{2\pi}{4} = \dfrac{\pi}{2}$

Amplitude: 1

Key points:

$(0, 0), \left(\dfrac{\pi}{8}, 1\right), \left(\dfrac{\pi}{4}, 0\right),$

$\left(\dfrac{3\pi}{8}, -1\right), \left(\dfrac{\pi}{2}, 0\right)$



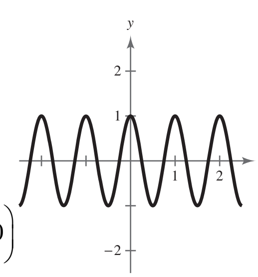
37. $y = \cos 2\pi x$

Period: $\dfrac{2\pi}{2\pi} = 1$

Amplitude: 1

Key points:

$(0, 1), \left(\dfrac{1}{4}, 0\right), \left(\dfrac{1}{2}, -1\right), \left(\dfrac{3}{4}, 0\right)$



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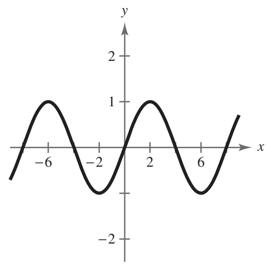
38. $y = \sin \dfrac{\pi x}{4}$

Period: $\dfrac{2\pi}{\pi/4} = 8$

Amplitude: 1

Key points:

$(0, 0), (2, 1), (4, 0),$
$(6, -1), (8, 0)$



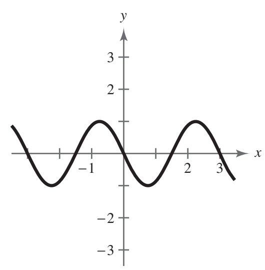
39. $y = -\sin \dfrac{2\pi x}{3}$

Period: $\dfrac{2\pi}{2\pi/3} = 3$

Amplitude: 1

Key points:

$(0, 0), \left(\dfrac{3}{4}, -1\right), \left(\dfrac{3}{2}, 0\right),$
$\left(\dfrac{9}{4}, 1\right), (3, 0)$



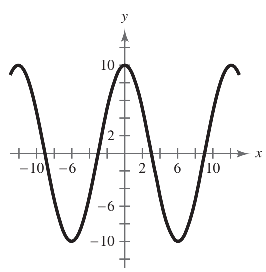
40. $y = 10 \cos \dfrac{\pi x}{6}$

Period: $\dfrac{2\pi}{\pi/6} = 12$

Amplitude: 10

Key points:

$(0, 10), (3, 0), (6, -10),$
$(9, 0), (12, 10)$



41. $y = \cos\left(x - \dfrac{\pi}{2}\right)$

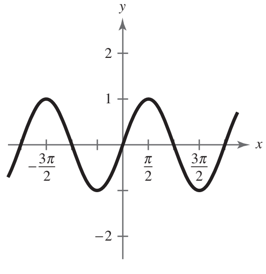
Period: 2π

Amplitude: 1

Shift: Set $x - \dfrac{\pi}{2} = 0$ and $x - \dfrac{\pi}{2} = 2\pi$

$x = \dfrac{\pi}{2} \qquad x = \dfrac{5\pi}{2}$

Key points: $\left(\dfrac{\pi}{2}, 1\right), (\pi, 0), \left(\dfrac{3\pi}{2}, -1\right), (2\pi, 0), \left(\dfrac{5\pi}{2}, 1\right)$



42. $y = \sin(x - 2\pi)$

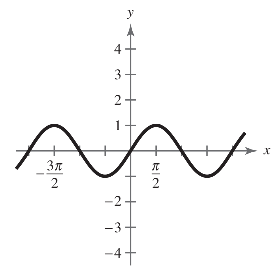
Period: 2π

Amplitude: 1

Shift: Set $x - 2\pi = 0$ and $x - 2\pi = 2\pi$

$x = 2\pi$ and $x = 4\pi$

Key points: $(2\pi, 0), \left(\dfrac{5\pi}{2}, 1\right), (3\pi, 0), \left(\dfrac{7\pi}{2}, -1\right), (4\pi, 0)$



43. $y = 3 \sin(x + \pi)$

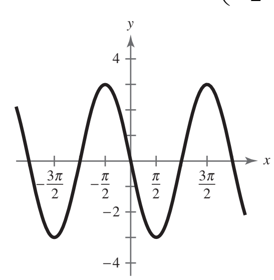
Period: 2π

Amplitude: 3

Shift: Set $x + \pi = 0$ and $x + \pi = 2\pi$

$x = -\pi \qquad x = \pi$

Key points: $(-\pi, 0), \left(-\dfrac{\pi}{2}, 3\right), (0, 0), \left(\dfrac{\pi}{2}, -3\right), (\pi, 0)$



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44. $y = -4\cos\left(x + \dfrac{\pi}{4}\right)$

Period: 2π

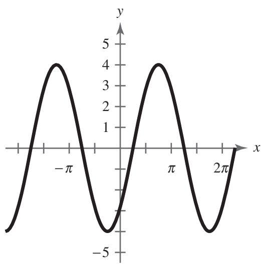
Amplitude: 4

Shift: Set $x + \dfrac{\pi}{4} = 0$ and $x + \dfrac{\pi}{4} = 2\pi$

$x = -\dfrac{\pi}{4}$ $\qquad x = \dfrac{7\pi}{4}$

Key points:

$\left(-\dfrac{\pi}{4}, -4\right), \left(\dfrac{\pi}{4}, 0\right), \left(\dfrac{3\pi}{4}, 4\right), \left(\dfrac{5\pi}{4}, 0\right), \left(\dfrac{7\pi}{4}, -4\right)$



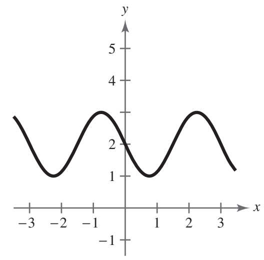
45. $y = 2 - \sin\dfrac{2\pi x}{3}$

Period: $\dfrac{2\pi}{2\pi/3} = 3$

Amplitude: 1

Key points:

$(0, 2), \left(\dfrac{3}{4}, 1\right), \left(\dfrac{3}{2}, 2\right), \left(\dfrac{9}{4}, 3\right), (3, 2)$



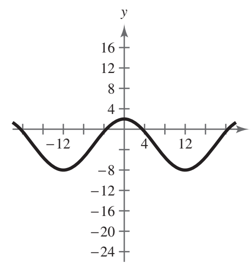
46. $y = -3 + 5\cos\dfrac{\pi t}{12}$

Period: $\dfrac{2\pi}{\pi/12} = 24$

Amplitude: 5

Key points:

$(0, 2), (6, -3), (12, -8), (18, -3), (24, 2)$



47. $y = 2 + 5\cos 6\pi x$

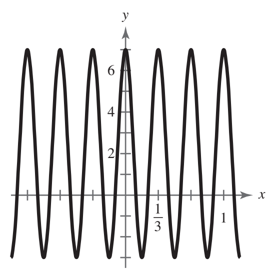
Period: $\dfrac{2\pi}{6\pi} = \dfrac{1}{3}$

Amplitude: 5

Shift: Set $6\pi x = 0$ and $6\pi x = 2\pi$

$x = 0$ $\qquad x = \dfrac{1}{3}$

Key points: $\left(\dfrac{1}{3}, 7\right), \left(\dfrac{5}{12}, 2\right), \left(\dfrac{1}{2}, -3\right), \left(\dfrac{7}{12}, 2\right), \left(\dfrac{1}{3}, 7\right)$



48. $y = 5 + 2\sin 3x$

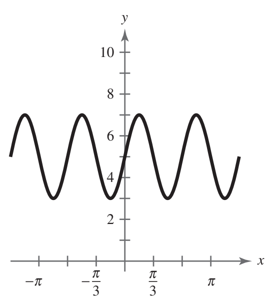
Period: $\dfrac{2\pi}{3}$

Amplitude: 2

Shift: Set $3x = 0$ and $3x = 2\pi$

$x = 0$ $\qquad x = \dfrac{2\pi}{3}$

Key points: $(0, 5), \left(\dfrac{\pi}{6}, 7\right), \left(\dfrac{\pi}{3}, 5\right), \left(\dfrac{\pi}{2}, 3\right), \left(\dfrac{2\pi}{3}, 5\right)$



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49. $y = 3\sin(x + \pi) - 3$

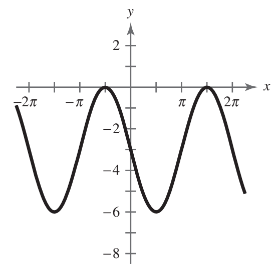
Period: 2π

Amplitude: 3

Shift: Set $x + \pi = 0$ and $x + \pi = 2\pi$

$x = -\pi \qquad x = \pi$

Key points: $(-\pi, -3), \left(-\dfrac{\pi}{2}, 0\right), (0, -3), \left(\dfrac{\pi}{2}, -6\right), (\pi, -3)$



50. $y = -3\sin(6x + \pi)$

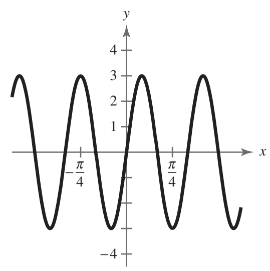
Period: $\dfrac{2\pi}{6} = \dfrac{\pi}{3}$

Amplitude: 3

Shift: Set $6x + \pi = 0$ and $6x + \pi = 2\pi$

$x = -\dfrac{\pi}{6} \qquad x = \dfrac{\pi}{6}$

Key points: $\left(-\dfrac{\pi}{6}, 0\right), \left(-\dfrac{\pi}{12}, -3\right), (0, 0), \left(\dfrac{\pi}{12}, 3\right), \left(\dfrac{\pi}{6}, 0\right)$



51. $y = \dfrac{2}{3}\cos\left(\dfrac{x}{2} - \dfrac{\pi}{4}\right)$

Period: $\dfrac{2\pi}{1/2} = 4\pi$

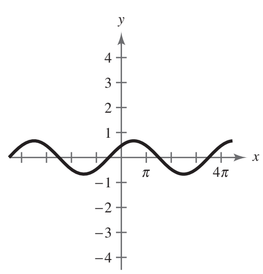
Amplitude: $\dfrac{2}{3}$

Shift: $\dfrac{x}{2} - \dfrac{\pi}{4} = 0$ and $\dfrac{\pi}{2} - \dfrac{\pi}{4} = 2\pi$

$x = \dfrac{\pi}{2} \qquad x = \dfrac{9\pi}{2}$

Key points:

$\left(\dfrac{\pi}{2}, \dfrac{2}{3}\right), \left(\dfrac{3\pi}{2}, 0\right), \left(\dfrac{5\pi}{2}, \dfrac{-2}{3}\right), \left(\dfrac{7\pi}{2}, 0\right), \left(\dfrac{9\pi}{2}, \dfrac{2}{3}\right)$



52. $y = 4\cos\left(\pi x + \dfrac{\pi}{2}\right) - 1$

Period: $\dfrac{2\pi}{\pi} = 2$

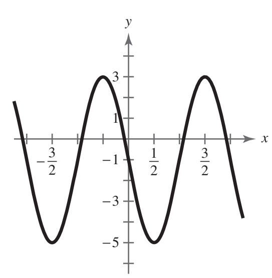
Amplitude: 4

Shift: Set $\pi x + \dfrac{\pi}{2} = 0$ and $\pi x + \dfrac{\pi}{2} = 2\pi$

$\pi x = -\dfrac{\pi}{2} \qquad \pi x = \dfrac{3\pi}{2}$

$x = -\dfrac{1}{2} \qquad x = \dfrac{3}{2}$

Key points: $\left(-\dfrac{1}{2}, 3\right), (0, -1), \left(\dfrac{1}{2}, -5\right), (1, -1), \left(\dfrac{3}{2}, 3\right)$



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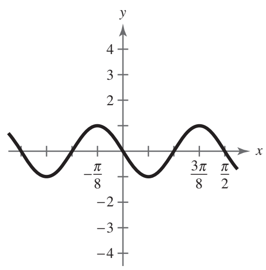
53. $g(x) = \sin(4x - \pi)$

(a) $g(x)$ is obtained by a horizontal shrink and a phase shift of $\frac{\pi}{4}$. One cycle of $g(x)$ corresponds to the interval $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$.

(b)

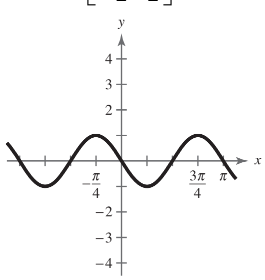


(c) $g(x) = f(4x - \pi)$ where $f(x) = \sin x$.

54. $g(x) = \sin(2x + \pi)$

(a) $g(x)$ is obtained by a horizontal shrink and a phase shift of $-\frac{\pi}{2}$. One cycle of $g(x)$ corresponds to the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

(b)

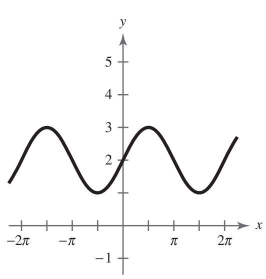


(c) $g(x) = f(2x + \pi)$ where $f(x) = \sin x$

55. $g(x) = \cos\left(x - \frac{\pi}{2}\right) + 2$

(a) $g(x)$ is obtained by shifting $f(x)$ two units upward and a phase shift of $\frac{\pi}{2}$. One cycle of $g(x)$ corresponds to the interval $[\pi, 3\pi]$.

(b)

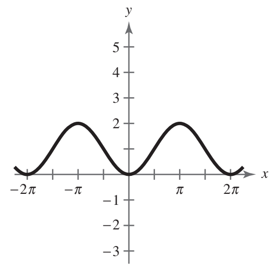


(c) $g(x) = f\left(x - \frac{\pi}{2}\right) + 2$ where $f(x) = \cos x$

56. $g(x) = 1 + \cos(x + \pi)$

(a) $g(x)$ is obtained by shifting $f(x)$ one unit upward and a phase shift of $-\pi$. One cycle of $g(x)$ corresponds to the interval $[-\pi, \pi]$.

(b)

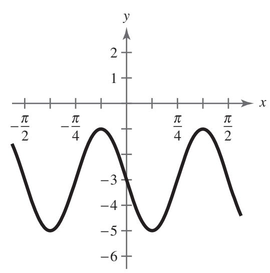


(c) $g(x) = f(x + \pi) + 1$ where $f(x) = \cos x$

57. $g(x) = 2\sin(4x - \pi) - 3$

(a) $g(x)$ is obtained by a vertical stretch, a horizontal shrink, a phase shift of $\frac{\pi}{4}$, and shifting $f(x)$ three units downward. One cycle of $g(x)$ corresponds to the interval $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$.

(b)

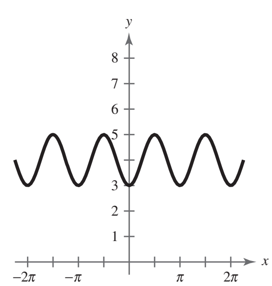


(c) $g(x) = 2f(4x - \pi) - 3$ where $f(x) = \sin x$

58. $g(x) = 4 - \sin(2x + \pi)$

(a) $g(x)$ is obtained by a horizontal shrink, a phase shift of $-\frac{\pi}{4}$, reflecting $f(x)$ over the x-axis and shifting $f(x)$ four units upward. One cycle of $g(x)$ corresponds to the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

(b)



(c) $g(x) = 4 - f(2x + \pi)$ where $f(x) = \sin x$

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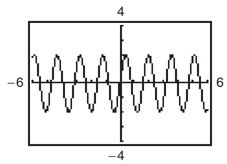
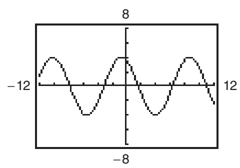
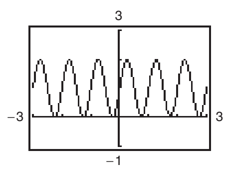
59. $y = -2\sin(4x + \pi)$



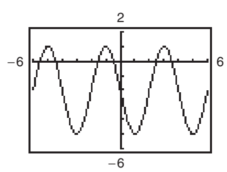
60. $y = -4\sin\left(\frac{2}{3}x - \frac{\pi}{3}\right)$



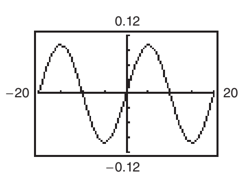
61. $y = \cos\left(2\pi x - \frac{\pi}{2}\right) + 1$



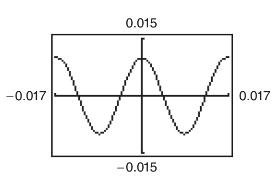
62. $y = 3\cos\left(\frac{\pi x}{2} + \frac{\pi}{2}\right) - 2$



63. $y = -0.1\sin\left(\frac{\pi x}{10} + \pi\right)$



64. $y = \frac{1}{100}\cos 120\pi t$



65. $f(x) = a\cos x + d$

Amplitude: $\frac{1}{2}[3 - (-1)] = 2 \Rightarrow a = 2$

$3 = 2\cos 0 + d$

$d = 3 - 2 = 1$

$a = 2, d = 1$

66. $f(x) = a\cos x + d$

Amplitude: $\frac{1 - (-3)}{2} = 2 \Rightarrow a = 2$

$1 = 2\cos 0 + d$

$d = 1 - 2 = -1$

$a = 2, d = -1$

67. $f(x) = a\cos x + d$

Amplitude: $\frac{1}{2}[8 - 0] = 4$

Reflected in the x-axis: $a = -4$

$0 = -4\cos 0 + d$

$d = 4$

$a = -4, d = 4$

68. $f(x) = a\cos x + d$

Amplitude: $\frac{-2 - (-4)}{2} = 1$

Reflected in the x-axis: $a = -1$

$-4 = -1\cos 0 + d$

$d = -3$

$a = -1, d = -3$

69. $y = a\sin(bx - c)$

Amplitude: $|a| = |3|$

Since the graph is reflected in the x-axis, we have $a = -3$.

Period: $\frac{2\pi}{b} = \pi \Rightarrow b = 2$

Phase shift: $c = 0$

$a = -3, b = 2, c = 0$

70. $y = a\sin(bx - c)$

Amplitude: $2 \Rightarrow a = 2$

Period: 4π

$\frac{2\pi}{b} = 4\pi \Rightarrow b = \frac{1}{2}$

Phase shift: $c = 0$

$a = 2, b = \frac{1}{2}, c = 0$

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71. $y = a\sin(bx - c)$

Amplitude: $a = 2$

Period: $2\pi \Rightarrow b = 1$

Phase shift: $bx - c = 0$ when $x = -\dfrac{\pi}{4}$

$$(1)\left(-\frac{\pi}{4}\right) - c = 0 \Rightarrow c = -\frac{\pi}{4}$$

$a = 2, b = 1, c = -\dfrac{\pi}{4}$

72. $y = a\sin(bx - c)$

Amplitude: $2 \Rightarrow a = 2$

Period: 4

$\dfrac{2\pi}{b} = 4 \Rightarrow b = \dfrac{\pi}{2}$

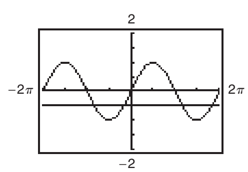
Phase shift: $\dfrac{c}{b} = 1 \Rightarrow \dfrac{c}{\pi/2} = 1 \Rightarrow c = \dfrac{\pi}{2}$

$a = 2, b = \dfrac{\pi}{2}, c = \dfrac{\pi}{2}$

73. $y_1 = \sin x$

$y_2 = -\dfrac{1}{2}$

In the interval $[-2\pi, 2\pi]$,



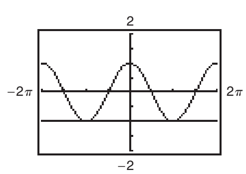
$y_1 = y_2$ when $x = -\dfrac{5\pi}{6}, -\dfrac{\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$.

74. $y_1 = \cos x$

$y_2 = -1$

In the interval $[-2\pi, 2\pi]$,

$y_1 = y_2$ when $x = \pi, -\pi$.



Answers for 75–78 are sample answers.

75. $f(x) = 2\sin(2x - \pi) + 1$

76. $f(x) = 3\sin\left(\dfrac{1}{2x} + \dfrac{\pi}{8}\right) - 1$

77. $f(x) = \cos(2x + 2\pi) - \frac{3}{2}$

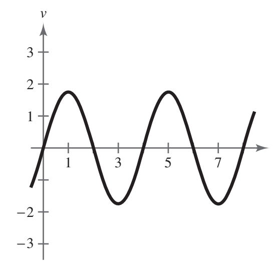
78. $f(x) = 3\cos\left(\dfrac{1}{2}x - \dfrac{\pi}{4}\right) + 2$

79. $v = 1.75\sin\dfrac{\pi t}{2}$

(a) Period $= \dfrac{2\pi}{\pi/2} = 4$ seconds

(b) $\dfrac{1 \text{ cycle}}{4 \text{ seconds}} \cdot \dfrac{60 \text{ seconds}}{1 \text{ minute}} = 15$ cycles per minute

(c)



80. $y = 0.85\sin\dfrac{\pi t}{3}$

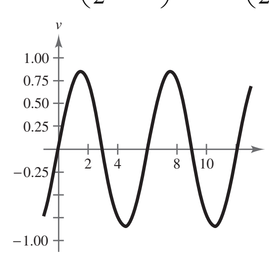
(a) Time for one cycle $= \dfrac{2\pi}{\pi/3} = 6$ seconds

(b) Cycles per min $= \dfrac{60}{6} = 10$ cycles per minute

(c) Amplitude: 0.85; Period: 6

Key points:

$(0, 0), \left(\dfrac{3}{2}, 0.85\right), (3, 0), \left(\dfrac{9}{2}, -0.85\right), (6, 0)$



81. $P = 100 - 20\cos\dfrac{5\pi t}{3}$

(a) Period: $\dfrac{2\pi}{(5\pi)/3} = \dfrac{6}{5}$ seconds

(b) $\dfrac{1 \text{ heartbeat}}{6/5 \text{ seconds}} \cdot \dfrac{60 \text{ seconds}}{1 \text{ minute}} = 50$ heartbeats per minute

82. $y = 0.001\sin 880\pi t$

(a) Period: $\dfrac{2\pi}{880\pi} = \dfrac{1}{440}$ seconds

(b) $f = \dfrac{1}{p} = 440$ cycles per second

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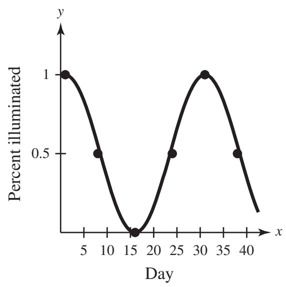
83. (a) and (c)



The model fits the data well.

(d) The period of the model is $p = 30$ days.

(e) Because March 12, 2018 is the 76th day of the year, $x = 76$. So, the percent of the moon's face illuminated is $y = 0.5\cos\left(\frac{\pi(71)}{15} - \frac{\pi}{15}\right) + 0.5 = 0.25 = 25\%$.

(b) Amplitude $a = \frac{1}{2}[\text{max. percent} - \text{min. percent}]$

$= \frac{1}{2}[1.0 - 0] = 0.5$

Period:

$p = (2^{\text{nd}} \text{ day of 1.0 percent} - 1^{\text{st}} \text{ day of 1.0 percent})$

$= 31 - 1 = 30$

$b = \frac{2\pi}{p} = \frac{2\pi}{30} = \frac{\pi}{15}$

Because the zero percent occurs at day 16, $x = 16$.

$bx - c = \pi$

$\left(\frac{\pi}{15}\right)(16) - c = \pi$

$\frac{16\pi}{15} - \pi = c$

$\frac{\pi}{15} = c$

The average percent of illumination is

$\frac{1}{2}(1.0 - 0) = 0.5$. So, $d = 0.5$.

So the model is $y = 0.5\cos\left(\frac{\pi x}{15} - \frac{\pi}{15}\right) + 0.5$.

84. (a) $y = a\cos(bt - c) + d$

Amplitude: $a = \frac{1}{2}[\text{max temp} - \text{min temp}] = \frac{1}{2}[78.6 - 13.8] = 32.4$

Period: $p = 2[\text{month of max temp} - \text{month of min temp}] = 2[7 - 1] = 12$

$b = \frac{2\pi}{p} = \frac{2\pi}{12} = \frac{\pi}{6}$

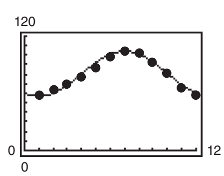
Because the maximum temperature occurs in the seventh month,

$\frac{c}{b} = 7$ so $c \approx 3.67$.

The average temperature is $\frac{1}{2}(78.6 + 13.8) = 46.2°$, so $d = 46.2$.

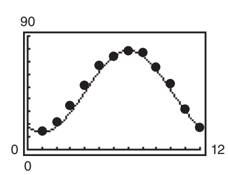
So, $I(t) = 32.4\cos\left(\frac{\pi}{6}t - 3.67\right) + 46.2$.

(b)



The model fits the data well.

(c)



The model fits the data well.

(d) The value of d in each model represents the average temperature.

Las Vegas: 80.6°; International Falls: 46.2°

(e) The period of each model is 12. This is what you would expect because the time period is one year (twelve months).

(f) The amplitude determines the variability. The greater the amplitude, the greater the temperature varies, so International Falls has the greater temperature variability, $a = 23.5$ for Las Vegas and $a = 32.4$ for International Falls.

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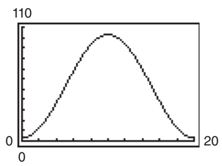
85. (a) Period $= \dfrac{2\pi}{\left(\dfrac{\pi}{10}\right)} = 20$ seconds

The wheel takes 20 seconds to revolve once.

(b) Amplitude: 50 feet

The radius of the wheel is 50 feet.

(c)



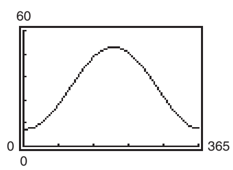
86. $C = 30.3 + 21.6\sin\left(\dfrac{2\pi t}{365} + 10.9\right)$

(a) Period $= \dfrac{2\pi}{\dfrac{2\pi}{365}} = 365$

Yes, this is what is expected because there are 365 days in a year.

(b) 30.3 gallons; the average daily fuel consumption is given by the amount of the vertical shift (from 0) which is given by the constant 30.3.

(c)



The consumption exceeds 40 gallons per day when $124 < t < 252$.

87. False. The graph of $g(x) = \sin(x + 2\pi)$ is the graph of $f(x) = \sin(x)$ translated to the *left* by one period, and the graphs are identical.

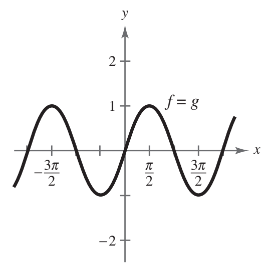
88. False. $y = \dfrac{1}{2}\cos 2x$ has an amplitude that is half that of $y = \cos x$. For $y = a\cos bx$, the amplitude is $|a|$.

89. True. $y = -\cos x$ is a reflection of $y = \sin\left(x + \dfrac{\pi}{2}\right)$ in the x-axis. So, $-\cos x = -\sin\left(x + \dfrac{\pi}{2}\right)$.

90. (a) For $c \neq 0$, the value of c shifts the graph of $f(x) = \sin x$ c units to the left when $c < 0$ and c units to the right when $c > 0$.

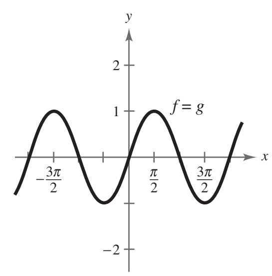
(b) An intercept of the graph $y = -\cos\left(x + \dfrac{\pi}{4}\right)$ occurs at $\left(\dfrac{\pi}{4}, 0\right)$. Because the graph of $y = \sin\left(x - \dfrac{\pi}{4}\right)$ also has an intercept at $\left(\dfrac{\pi}{4}, 0\right)$, the green graph is equivalent to $y = -\cos\left(x + \dfrac{\pi}{4}\right)$.

91.



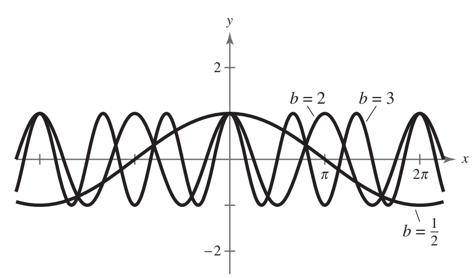
Because the graphs are the same, the conjecture is that $\sin(x) = \cos\left(x - \dfrac{\pi}{2}\right)$.

92.



Because the graphs are the same, the conjecture is that $\sin x = -\cos\left(x + \dfrac{\pi}{2}\right)$.

93.



As the value of b increases, the period decreases.

$b = \dfrac{1}{2} \rightarrow \dfrac{1}{2}$ cycle

$b = 2 \rightarrow 2$ cycles

$b = 3 \rightarrow 3$ cycles

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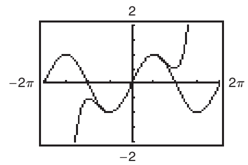
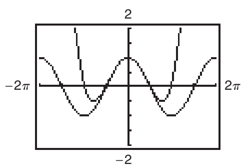
94. (a)



The graphs are nearly the same for $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

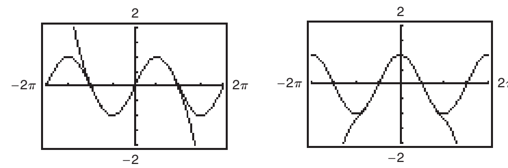
(b)



The graphs are nearly the same for $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

(c) $\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$

$\cos x \approx 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$



The graphs now agree over a wider range,

$-\frac{3\pi}{4} < x < \frac{3\pi}{4}$.

95. (a) $\sin\frac{1}{2} \approx \frac{1}{2} - \frac{(1/2)^3}{3!} + \frac{(1/2)^5}{5!} \approx 0.4794$

$\sin\frac{1}{2} \approx 0.4794$ (by calculator)

(b) $\sin 1 \approx 1 - \frac{1}{3!} + \frac{1}{5!} \approx 0.8417$

$\sin 1 \approx 0.8415$ (by calculator)

(c) $\sin\frac{\pi}{6} \approx 1 - \frac{(\pi/6)^3}{3!} + \frac{(\pi/6)^5}{5!} \approx 0.5000$

$\sin\frac{\pi}{6} = 0.5$ (by calculator)

(d) $\cos(-0.5) \approx 1 - \frac{(-0.5)^2}{2!} + \frac{(-0.5)^4}{4!} \approx 0.8776$

$\cos(-0.5) \approx 0.8776$ (by calculator)

(e) $\cos 1 \approx 1 - \frac{1}{2!} + \frac{1}{4!} \approx 0.5417$

$\cos 1 \approx 0.5403$ (by calculator)

(f) $\cos\frac{\pi}{4} \approx 1 - \frac{(\pi/4)^2}{2!} + \frac{(\pi/4)^2}{4!} = 0.7074$

$\cos\frac{\pi}{4} \approx 0.7071$ (by calculator)

The error in the approximation is not the same in each case. The error appears to increase as x moves farther away from 0.

Section 4.6 Graphs of Other Trigonometric Functions

1. odd; origin

2. vertical

3. reciprocal

4. damping

5. π

6. $x \neq n\pi$

7. $(-\infty, -1] \cup [1, \infty)$

8. 2π

9. $y = \sec 2x$

Period: $\frac{2\pi}{2} = \pi$

Matches graph (e).

10. $y = \tan\frac{x}{2}$

Period: $\frac{\pi}{b} = \frac{\pi}{1/2} = 2\pi$

Asymptotes: $x = -\pi$, $x = \pi$

Matches graph (c).

11. $y = \frac{1}{2}\cot \pi x$

Period: $\frac{\pi}{\pi} = 1$

Matches graph (a).

12. $y = -\csc x$

Period: 2π

Matches graph (d).

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13. $y = \frac{1}{2}\sec\frac{\pi x}{2}$

Period: $\frac{2\pi}{b} = \frac{2\pi}{\pi/2} = 4$

Asymptotes: $x = -1$, $x = 1$

Matches graph (f).

14. $y = -2\sec\frac{\pi x}{2}$

Period: $\frac{2\pi}{b} = \frac{2\pi}{\pi/2} = 4$

Asymptotes: $x = -1$, $x = 1$

Reflected in x-axis

Matches graph (b).

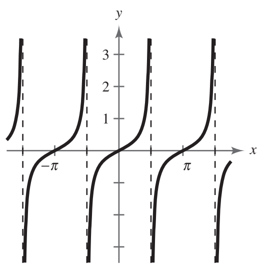
15. $y = \frac{1}{3}\tan x$

Period: π

Two consecutive asymptotes:

$x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$

x	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$
y	$-\frac{1}{3}$	0	$\frac{1}{3}$



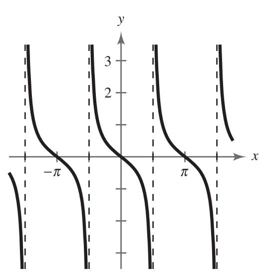
16. $y = -\frac{1}{2}\tan x$

Period: π

Two consecutive asymptotes:

$x = -\frac{\pi}{2}$, $x = \frac{\pi}{2}$

x	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$
y	$\frac{1}{2}$	0	$-\frac{1}{2}$



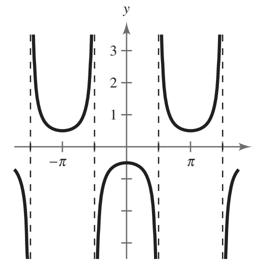
17. $y = -\frac{1}{2}\sec x$

Period: 2π

Two consecutive asymptotes:

$x = -\frac{\pi}{2}$, $x = \frac{\pi}{2}$

x	$-\frac{\pi}{3}$	0	$\frac{\pi}{3}$
y	-1	$-\frac{1}{2}$	-1



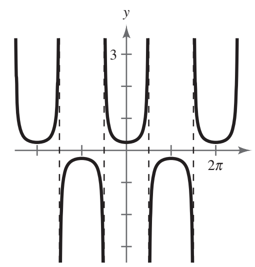
18. $y = \frac{1}{4}\sec x$

Period: 2π

Two consecutive asymptotes:

$x = -\frac{\pi}{2}$, $x = \frac{\pi}{2}$

x	$-\frac{\pi}{3}$	0	$\frac{\pi}{3}$
y	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$



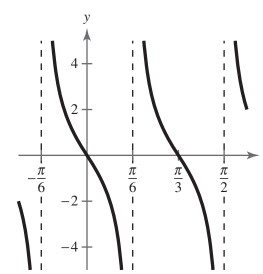
19. $y = -2\tan 3x$

Period: $\frac{\pi}{3}$

Two consecutive asymptotes:

$x = -\frac{\pi}{6}$, $x = \frac{\pi}{6}$

x	$-\frac{\pi}{3}$	0	$\frac{\pi}{3}$
y	0	0	0



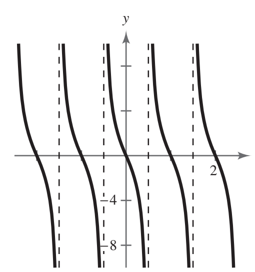
20. $y = -3\tan \pi x$

Period: $\frac{\pi}{\pi} = 1$

Two consecutive asymptotes:

$x = -\frac{1}{2}$, $x = \frac{1}{2}$

x	$-\frac{1}{4}$	0	$\frac{1}{4}$
y	3	0	−3



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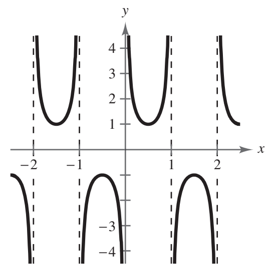
21. $y = \csc \pi x$

Period: $\dfrac{2\pi}{\pi} = 2$

Two consecutive asymptotes:

$x = 0,\ x = 1$

x	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{5}{6}$
y	2	1	2



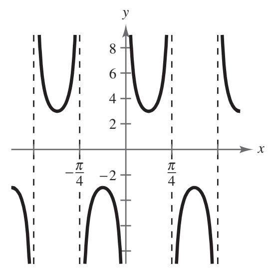
22. $y = 3 \csc 4x$

Period: $\dfrac{2\pi}{4} = \dfrac{\pi}{2}$

Two consecutive asymptotes:

$x = 0,\ x = \dfrac{\pi}{4}$

x	$\frac{\pi}{24}$	$\frac{\pi}{8}$	$\frac{5\pi}{24}$
y	6	3	6



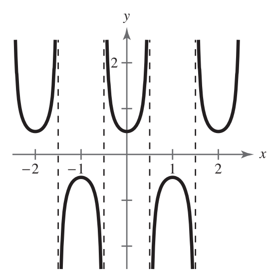
23. $y = \frac{1}{2} \sec \pi x$

Period: 2

Two consecutive asymptotes:

$x = -\frac{1}{2},\ x = \frac{1}{2}$

x	−1	0	1
y	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$



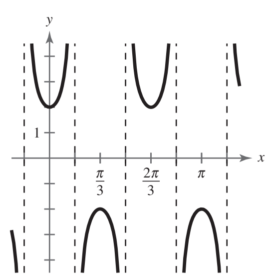
24. $y = 2 \sec 3x$

Period: $\dfrac{2\pi}{3}$

Two consecutive asymptotes:

$x = -\dfrac{\pi}{6},\ x = \dfrac{\pi}{6}$

x	$-\frac{\pi}{3}$	0	$\frac{\pi}{3}$
y	−2	2	−2



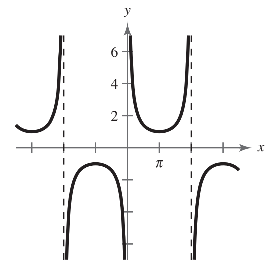
25. $y = \csc \dfrac{x}{2}$

Period: $\dfrac{2\pi}{1/2} = 4\pi$

Two consecutive asymptotes:

$x = 0,\ x = 2\pi$

x	$\frac{\pi}{3}$	π	$\frac{5\pi}{3}$
y	2	1	2



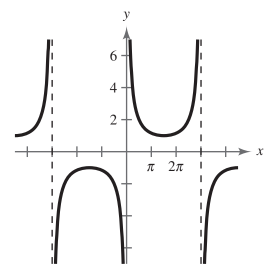
26. $y = \csc \dfrac{x}{3}$

Period: $\dfrac{2\pi}{1/3} = 6\pi$

Two consecutive asymptotes:

$x = 0,\ x = 3\pi$

x	$\frac{\pi}{2}$	$\frac{3\pi}{2}$	$\frac{5\pi}{2}$
y	2	1	2

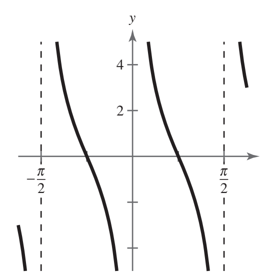


27. $y = 3 \cot 2x$

Period: $\dfrac{\pi}{2}$

Two consecutive asymptotes:

$x = -\dfrac{\pi}{2},\ x = \dfrac{\pi}{2}$



x	$-\frac{\pi}{6}$	$-\frac{\pi}{8}$	$\frac{\pi}{8}$	$\frac{\pi}{6}$
y	$-3\sqrt{3}$	−3	3	$3\sqrt{3}$

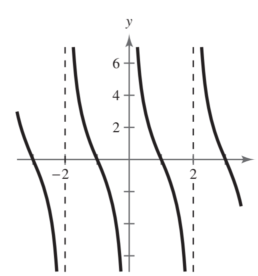
28. $y = 3 \cot \dfrac{\pi x}{2}$

Period: $\dfrac{\pi}{\pi/2} = 2$

Two consecutive asymptotes:

$x = 0,\ x = 2$

x	$\frac{1}{2}$	1	$\frac{3}{2}$
y	3	0	−3



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29. $y = \tan \frac{\pi x}{4}$

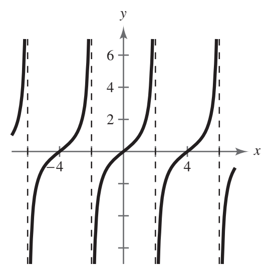
Period: $\frac{\pi}{\pi/4} = 4$

Two consecutive asymptotes:

$\frac{\pi x}{4} = -\frac{\pi}{2} \Rightarrow x = -2$

$\frac{\pi x}{4} = \frac{\pi}{2} \Rightarrow x = 2$

x	-1	0	1
y	-1	0	1



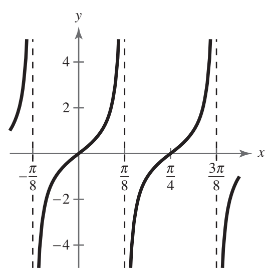
30. $y = \tan 4x$

Period: $\frac{\pi}{4}$

Two consecutive asymptotes

$x = -\frac{\pi}{8}, x = \frac{\pi}{8}$

x	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$
y	0	0	0



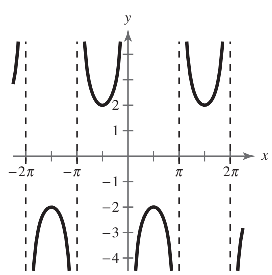
31. $y = 2\csc(x - \pi)$

Period: 2π

Two consecutive asymptotes:

$x = -\pi, x = \pi$

x	$-\frac{\pi}{2}$	$\frac{\pi}{2}$	$\frac{3\pi}{2}$
y	2	-2	-2



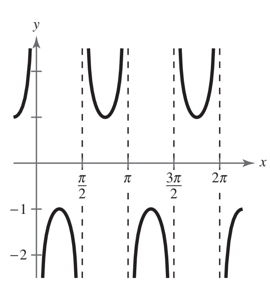
32. $y = \csc(2x - \pi)$

Period: $\frac{2\pi}{2} = \pi$

Two consecutive asymptotes:

$x = 0, x = \frac{\pi}{2}$

x	$\frac{\pi}{12}$	$\frac{\pi}{4}$	$\frac{5\pi}{12}$
y	-2	-1	-2



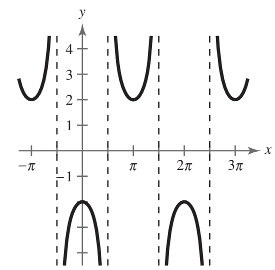
33. $y = 2\sec(x + \pi)$

Period: 2π

Two consecutive asymptotes:

$x = -\frac{\pi}{2}, x = \frac{\pi}{2}$

x	$-\frac{\pi}{3}$	0	$\frac{\pi}{3}$
y	-4	-2	-4



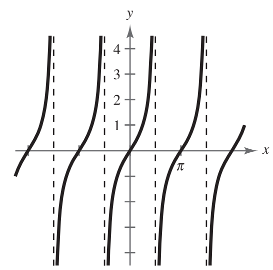
34. $y = \tan(x + \pi)$

Period: π

Two consecutive asymptotes:

$x = -\frac{\pi}{2}, x = \frac{\pi}{2}$

x	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$
y	-1	0	1



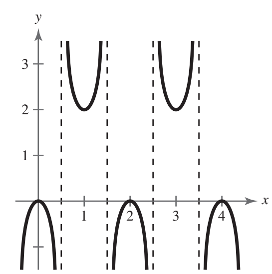
35. $y = -\sec \pi x + 1$

Period: $\frac{2\pi}{\pi} = 2$

Two consecutive asymptotes:

$x = -\frac{1}{2}, x = \frac{1}{2}$

x	$-\frac{1}{3}$	0	$\frac{1}{3}$
y	-1	0	1



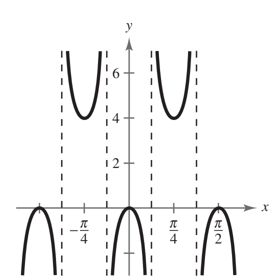
36. $y = -2\sec 4x + 2$

Period: $\frac{2\pi}{4} = \frac{\pi}{2}$

Two consecutive asymptotes:

$x = -\frac{\pi}{8}, x = \frac{\pi}{8}$

x	$-\frac{\pi}{12}$	0	$\frac{\pi}{12}$
y	-2	0	-2



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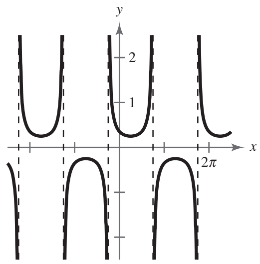
37. $y = \frac{1}{4}\csc\left(x + \frac{\pi}{4}\right)$

Period: 2π

Two consecutive asymptotes:

$x = -\frac{\pi}{4}, x = \frac{3\pi}{4}$

x	$-\frac{\pi}{12}$	$\frac{\pi}{4}$	$\frac{7\pi}{12}$
y	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$



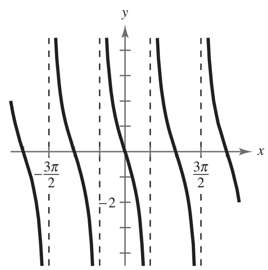
38. $y = 2\cot\left(x + \frac{\pi}{2}\right)$

Period: π

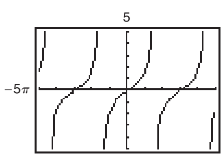
Two consecutive asymptotes:

$x = -\frac{\pi}{2}, x = \frac{\pi}{2}$

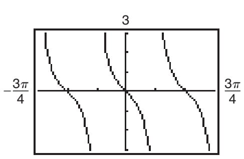
x	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$
y	2	0	-2



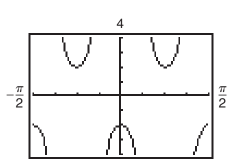
39. $y = \tan\frac{x}{3}$



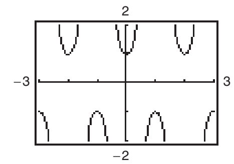
40. $y = -\tan 2x$



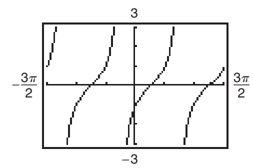
41. $y = -2\sec 4x = \frac{-2}{\cos 4x}$



42. $y = \sec \pi x \Rightarrow y = \frac{1}{\cos(\pi x)}$

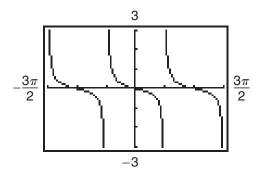


43. $y = \tan\left(x - \frac{\pi}{4}\right)$



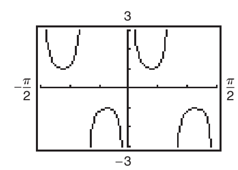
44. $y = \frac{1}{4}\cot\left(x - \frac{\pi}{2}\right)$

$= \frac{1}{4\tan\left(x - \frac{\pi}{2}\right)}$



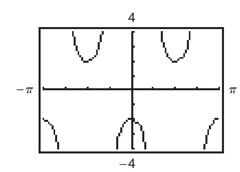
45. $y = -\csc(4x - \pi)$

$y = \frac{-1}{\sin(4x - \pi)}$

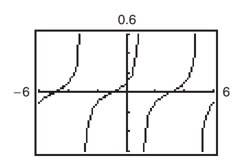


46. $y = 2\sec(2x - \pi) \Rightarrow$

$y = \frac{2}{\cos(2x - \pi)}$



47. $y = 0.1\tan\left(\frac{\pi x}{4} + \frac{\pi}{4}\right)$



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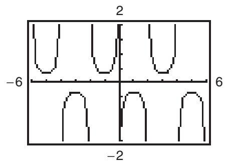
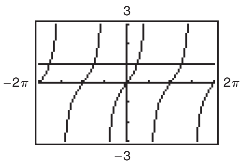
48. $y = \frac{1}{3}\sec\left(\frac{\pi x}{2} + \frac{\pi}{2}\right) \Rightarrow y = \frac{1}{3\cos\left(\frac{\pi x}{2} + \frac{\pi}{2}\right)}$



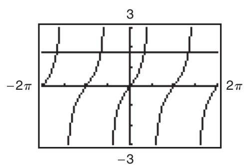
49. $\tan x = 1$

$x = -\frac{7\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}$



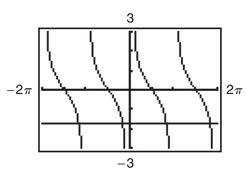
50. $\tan x = \sqrt{3}$

$x = -\frac{5\pi}{3}, -\frac{2\pi}{3}, \frac{\pi}{3}, \frac{4\pi}{3}$



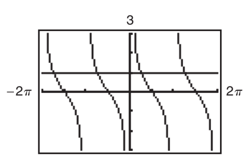
51. $\cot x = -\sqrt{3}$

$x = -\frac{7\pi}{6}, -\frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$



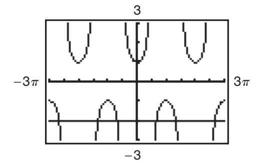
52. $\cot x = 1$

$x = -\frac{7\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}$



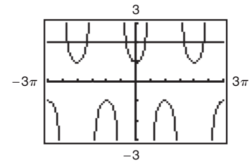
53. $\sec x = -2$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}, -\frac{2\pi}{3}, -\frac{4\pi}{3}$



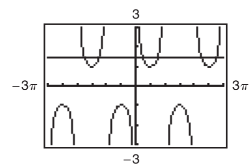
54. $\sec x = 2$

$x = -\frac{5\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{3}$



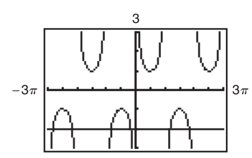
55. $\csc x = \sqrt{2}$

$x = -\frac{7\pi}{4}, -\frac{5\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$



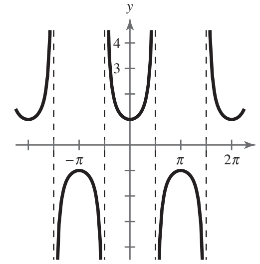
56. $\csc x = -2$

$x = -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$



57. $f(x) = \sec x = \frac{1}{\cos x}$

$$f(-x) = \sec(-x) = \frac{1}{\cos(-x)} = \frac{1}{\cos x} = f(x)$$



So, $f(x) = \sec x$ is an even function and the graph has y-axis symmetry.

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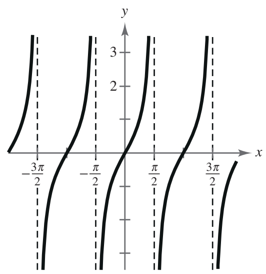
58. $f(x) = \tan x$

$\tan(-x) = -\tan x$

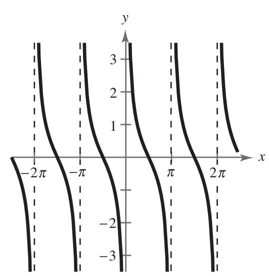
So, $f(x) = \tan x$ is an odd function and the graph is symmetric about the origin.



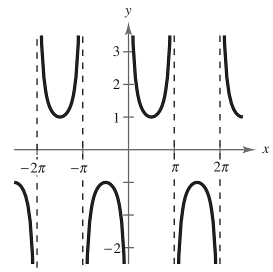
59. $g(x) = \cot x = \dfrac{1}{\tan x}$

$$\begin{aligned} g(-x) &= \cot(-x) \\ &= \frac{1}{\tan(-x)} \\ &= -\frac{1}{\tan x} \\ &= -g(x) \end{aligned}$$

So, $g(x) = \cot x$ is an odd function and the graph has origin symmetry.



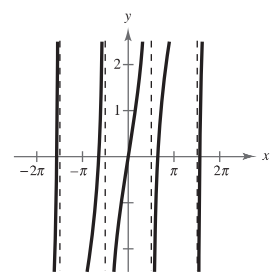
60. $g(x) = \csc x = \dfrac{1}{\sin x}$



$$\begin{aligned} g(-x) &= \csc(-x) \\ &= \frac{1}{\sin(-x)} \\ &= -\frac{1}{\sin x} \\ &= -g(x) \end{aligned}$$

So, $g(x) = \csc x$ is an odd function and the graph has origin symmetry.

61. $f(x) = x + \tan x$



$$\begin{aligned} f(-x) &= (-x) + \tan(-x) \\ &= -x - \tan x \\ &= -(x + \tan x) \\ &= -f(x) \end{aligned}$$

So, $f(x) = x + \tan x$ is an odd function and the graph has origin symmetry.

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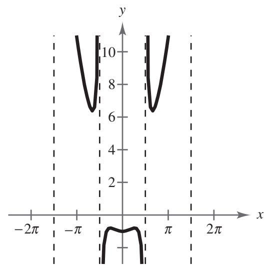
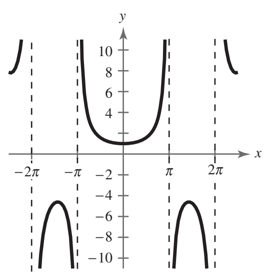
62. $f(x) = x^2 - \sec x = x^2 - \dfrac{1}{\cos x}$



$$\begin{aligned} f(-x) &= (-x)^2 - \sec(-x) \\ &= x^2 - \frac{1}{\cos(-x)} \\ &= x^2 - \frac{1}{\cos x} \\ &= x^2 - \sec x \\ &= f(x). \end{aligned}$$

So, $f(x) = x^2 - \sec x$ is an even function and the graph has y-axis symmetry.

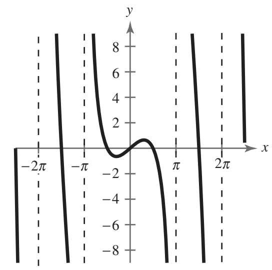
63. $g(x) = x \csc x = \dfrac{x}{\sin x}$



$$\begin{aligned} g(-x) &= (-x)\csc(-x) \\ &= \frac{-x}{\sin(-x)} \\ &= \frac{-x}{-\sin x} \\ &= \frac{x}{\sin x} \\ &= x \csc x \\ &= g(x) \end{aligned}$$

So, $g(x) = x \csc x$ is an even function and the graph has y-axis symmetry.

64. $g(x) = x^2 \cot x = \dfrac{x^2}{\tan x}$



$$\begin{aligned} g(-x) &= (-x)^2 \cot(-x) \\ &= \frac{x^2}{\tan(-x)} \\ &= -\frac{x^2}{\tan x} \\ &= -x^2 \cot x \\ &= -g(x) \end{aligned}$$

So, $g(x) = x^2 \cot x$ is an odd function and the graph has origin symmetry.

65. $f(x) = |x \cos x|$

Matches graph (d).

As $x \to 0, f(x) \to 0.$

66. $f(x) = x \sin x$

Matches graph (a)

As $x \to 0, f(x) \to 0.$

67. $g(x) = |x| \sin x$

Matches graph (b).

As $x \to 0, g(x) \to 0.$

68. $g(x) = |x| \cos x$

Matches graph (c).

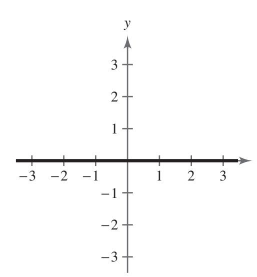
As $x \to 0, g(x) \to 0.$

69. $f(x) = \sin x + \cos\left(x + \dfrac{\pi}{2}\right)$

$g(x) = 0$

$f(x) = g(x)$

The functions are equal.



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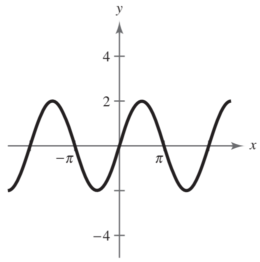
70. $f(x) = \sin x - \cos\left(x + \frac{\pi}{2}\right)$

$g(x) = 2 \sin x$

$f(x) = g(x)$

The functions are equal.

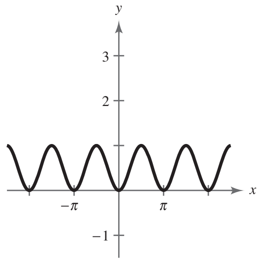


71. $f(x) = \sin^2 x$

$g(x) = \frac{1}{2}(1 - \cos 2x)$

$f(x) = g(x)$

The functions are equal.

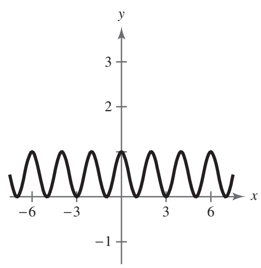


72. $f(x) = \cos^2 \frac{\pi x}{2}$

$g(x) = \frac{1}{2}(1 + \cos \pi x)$

$f(x) = g(x)$

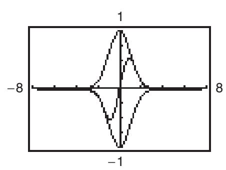
The functions are equal.



73. $g(x) = e^{-x^2/2} \sin x$

Damping factor: $e^{-x^2/2}$

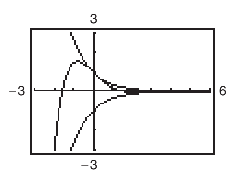
As $x \to \infty, g(x) \to 0.$



74. $f(x) = e^{-x} \cos x$

Damping factor: e^{-x}

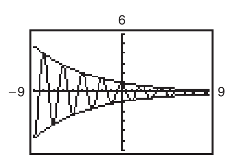
As $x \to \infty, f(x) \to 0.$



75. $f(x) = 2^{-x/4} \cos \pi x$

Damping factor: $y = 2^{-x/4}$

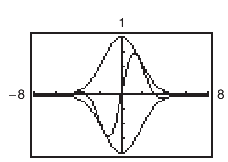
As $x \to \infty, f(x) \to 0.$



76. $h(x) = 2^{-x^2/4} \sin x$

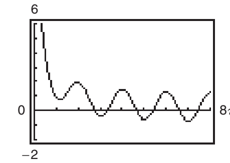
Damping factor: $2^{-x^2/4}$

As $x \to \infty, h(x) \to 0.$



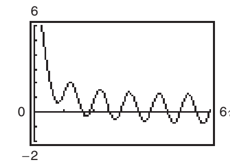
77. $y = \frac{6}{x} + \cos x, x > 0$

As $x \to 0, y \to \infty.$



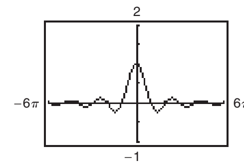
78. $y = \frac{4}{x} + \sin 2x, x > 0$

As $x \to 0, y \to \infty.$



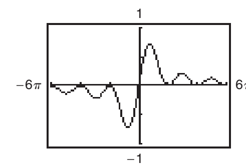
79. $g(x) = \frac{\sin x}{x}$

As $x \to 0, g(x) \to 1.$



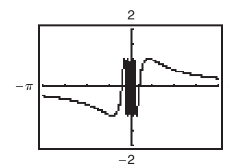
80. $f(x) = \frac{1 - \cos x}{x}$

As $x \to 0, f(x) \to 0.$



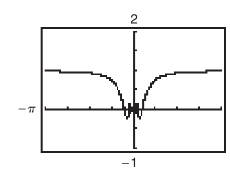
81. $f(x) = \sin \frac{1}{x}$

As $x \to 0, f(x)$ oscillates between -1 and 1.



82. $h(x) = x \sin \frac{1}{x}$

As $x \to 0, h(x) \to 0.$



83. (a) Period of $\cos \frac{\pi t}{6} = \frac{2\pi}{\pi/6} = 12$

Period of $\sin \frac{\pi t}{6} = \frac{2\pi}{\pi/6} = 12$

The period of $H(t)$ is 12 months.

The period of $L(t)$ is 12 months.

(b) From the graph, it appears that the greatest difference between high and low temperatures occurs in the summer. The smallest difference occurs in the winter.

(c) The highest high and low temperatures appear to occur about half of a month after the time when the sun is northernmost in the sky.

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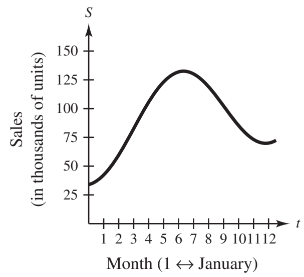
84. (a)

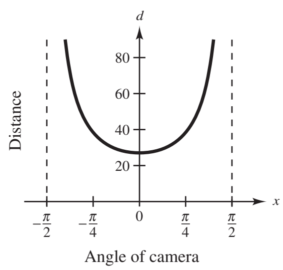


(b) $S(6) = 74 + 3(6) - 40\cos\left(\frac{6\pi}{6}\right)$

$= 74 + 18 - 40(-1) = 132 \Rightarrow 132{,}000$ units

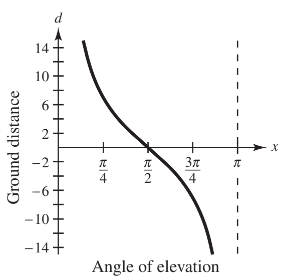
85. $\cos x = \frac{27}{d}$

$d = \frac{27}{\cos x} = 27\sec x, -\frac{\pi}{2} < x < \frac{\pi}{2}$



86. $\tan x = \frac{7}{d}$

$d = \frac{7}{\tan x} = 7\cot x$



87. True. Because

$y = \csc x = \frac{1}{\sin x},$

for a given value of x, the y-coordinate of $\csc x$ is the reciprocal of the y-coordinate of $\sin x$.

88. True.

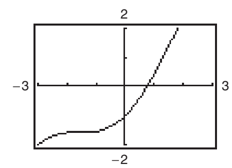
$y = \sec x = \frac{1}{\cos x}$

If the reciprocal of $y = \sin x$ is translated $\pi/2$ units to the left, then

$y = \frac{1}{\sin\left(x + \frac{\pi}{2}\right)} = \frac{1}{\cos x} = \sec x.$

89. $f(x) = x - \cos x$

(a)



The zero between 0 and 1 occurs at $x \approx 0.7391$.

(b) $x_n = \cos(x_{n-1})$

$x_0 = 1$

$x_1 = \cos 1 \approx 0.5403$

$x_2 = \cos 0.5403 \approx 0.8576$

$x_3 = \cos 0.8576 \approx 0.6543$

$x_4 = \cos 0.6543 \approx 0.7935$

$x_5 = \cos 0.7935 \approx 0.7014$

$x_6 = \cos 0.7014 \approx 0.7640$

$x_7 = \cos 0.7640 \approx 0.7221$

$x_8 = \cos 0.7221 \approx 0.7504$

$x_9 = \cos 0.7504 \approx 0.7314$

$\vdots$

This sequence appears to be approaching the zero of f: $x \approx 0.7391$.

90. (a) (i) $f(x) = \tan 2x$

The period of the graph is $\frac{\pi}{2}$ and the graph is increasing on $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$.

(b) (ii) $f(x) = \csc 4x$

The period of the graph $\frac{\pi}{2}$ and the graph has vertical asymptotes at multiples of $\frac{\pi}{4}$.

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91. $f(x) = \cot x$

(a) $x \to 0^+, f(x) \to \infty$

(b) $x \to 0^-, f(x) \to -\infty$

(c) $x \to \pi^+, f(x) \to \infty$

(d) $x \to \pi^-, f(x) \to -\infty$

92. $f(x) = \csc x$

(a) $x \to 0^+, f(x) \to \infty$

(b) $x \to 0^-, f(x) \to -\infty$

(c) $x \to \pi^+, f(x) \to -\infty$

(d) $x \to \pi^-, f(x) \to \infty$

93. $f(x) = \tan x$

(a) $x \to \frac{\pi}{2}^+, f(x) \to -\infty$

(b) $x \to \frac{\pi}{2}^-, f(x) \to \infty$

(c) $x \to -\frac{\pi}{2}^+, f(x) \to -\infty$

(d) $x \to -\frac{\pi}{2}^-, f(x) \to \infty$

94. $f(x) = \sec x$

(a) $x \to \frac{\pi}{2}^+, f(x) \to -\infty$

(b) $x \to \frac{\pi}{2}^-, f(x) \to \infty$

(c) $x \to -\frac{\pi}{2}^+, f(x) \to \infty$

(d) $x \to -\frac{\pi}{2}^-, f(x) \to -\infty$

Section 4.7 Inverse Trigonometric Functions

	Function	*Alternative Notation*	*Domain*	*Range*
1.	$y = \arcsin x$	$y = \sin^{-1} x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
2.	$y = \arccos x$	$y = \cos^{-1} x$	$-1 \le x \le 1$	$0 \le y \le \pi$
3.	$y = \arctan x$	$y = \tan^{-1} x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

4. inverse

5. $y = \arcsin \frac{1}{2} \Rightarrow \sin y = \frac{1}{2}$ for $-\frac{\pi}{2} \le y \le \frac{\pi}{2} \Rightarrow y = \frac{\pi}{6}$

6. $y = \arcsin 0 \Rightarrow \sin y = 0$ for $-\frac{\pi}{2} \le y \le \frac{\pi}{2} \Rightarrow y = 0$

7. $y = \arccos \frac{1}{2} \Rightarrow \cos y = \frac{1}{2}$ for $0 \le y \le \pi \Rightarrow y = \frac{\pi}{3}$

8. $y = \arccos 0 \Rightarrow \cos y = 0$ for $0 \le y \le \pi \Rightarrow y = \frac{\pi}{2}$

9. $y = \arctan \frac{\sqrt{3}}{3} \Rightarrow \tan y = \frac{\sqrt{3}}{3}$ for $-\frac{\pi}{2} < y < \frac{\pi}{2} \Rightarrow y = \frac{\pi}{6}$

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10. $y = \arctan(1) \Rightarrow \tan y = 1$ for $-\frac{\pi}{2} < y < \frac{\pi}{2} \Rightarrow y = \frac{\pi}{4}$

11. It is not possible to evaluate arcsin 3. The domain of the inverse sine function is $[-1, 1]$.

12. $y = \arctan(\sqrt{3}) \Rightarrow \tan y = \sqrt{3}$ for $-\frac{\pi}{2} < y < \frac{\pi}{2} \Rightarrow y = \frac{\pi}{3}$

13. $y = \arctan(-\sqrt{3}) \Rightarrow \tan y = -\sqrt{3}$ for $-\frac{\pi}{2} < y < \frac{\pi}{2} \Rightarrow y = -\frac{\pi}{3}$

14. It is not possible to evaluate $\cos^{-1}(-2)$. The domain of the inverse cosine function is $[-1, 1]$.

15. $y = \arccos\left(-\frac{1}{2}\right) \Rightarrow \cos y = -\frac{1}{2}$ for $0 \le y \le \pi \Rightarrow y = \frac{2\pi}{3}$

16. $y = \arcsin\left(\frac{\sqrt{2}}{2}\right) \Rightarrow \sin y = \frac{\sqrt{2}}{2}$ for $-\frac{\pi}{2} \le y \le \frac{\pi}{2} \Rightarrow y = \frac{\pi}{4}$

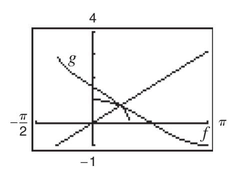
17. $y = \sin^{-1} -\frac{\sqrt{3}}{2} \Rightarrow \sin y = -\frac{\sqrt{3}}{2}$ for $-\frac{\pi}{2} \le y \le \frac{\pi}{2} \Rightarrow y = -\frac{\pi}{3}$

18. $y = \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) \Rightarrow \tan y = -\frac{\sqrt{3}}{3}$ for $-\frac{\pi}{2} < y < \frac{\pi}{2} \Rightarrow y = -\frac{\pi}{6}$

19. $f(x) = \cos x$

$g(x) = \arccos x$

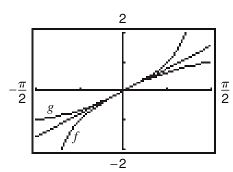
$y = x$



20. $f(x) = \tan x$

$g(x) = \arctan x$

$y = x$



21. $\arccos 0.37 = \cos^{-1}(0.37) \approx 1.19$

22. $\arcsin 0.65 = \sin^{-1}(0.65) \approx 0.71$

23. $\arcsin(-0.75) = \sin^{-1}(-0.75) \approx -0.85$

24. $\arccos(-0.7) = \cos^{-1}(-0.7) \approx 2.35$

25. $\arctan(-3) = \tan^{-1}(-3) \approx -1.25$

26. $\arctan 25 = \tan^{-1}(25) \approx 1.53$

27. It is not possible to evaluate $\sin^{-1} 1.36$. The domain of the inverse sine function is $[-1, 1]$.

28. $\cos^{-1} 0.26 = \cos^{-1} 0.26 \approx 1.31$

29. $\arccos(-0.41) = \cos^{-1}(-0.41) \approx 1.99$

30. $\arcsin(-0.125) = \sin^{-1}(-0.125) \approx -0.13$

31. $\arctan 0.92 = \tan^{-1} 0.92 \approx 0.74$

32. $\arctan 2.8 = \tan^{-1} 2.8 \approx 1.23$

33. $\arcsin \frac{7}{8} = \sin^{-1}\left(\frac{7}{8}\right) \approx 1.07$

34. It is not possible to evaluate $\arccos\left(-\frac{4}{3}\right)$. The domain of the inverse cosine function is $[-1, 1]$.

35. $\tan^{-1}\left(-\frac{95}{7}\right) \approx -1.50$

36. $\tan^{-1}\left(-\sqrt{372}\right) \approx -1.52$

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37. $\arctan\left(-\sqrt{3}\right) = -\dfrac{\pi}{3}$

$\tan\left(-\dfrac{\pi}{6}\right) = -\dfrac{\sqrt{3}}{3}$

$\tan\left(\dfrac{\pi}{4}\right) = 1$

38. $\arccos(-1) = \pi$

$\arccos\left(-\dfrac{1}{2}\right) = \dfrac{2\pi}{3}$

$\cos\left(\dfrac{\pi}{6}\right) = \dfrac{\sqrt{3}}{2}$

39. $\tan\theta = \dfrac{x}{4}$

$\theta = \arctan\dfrac{x}{4}$

40. $\cos\theta = \dfrac{4}{x}$

$\theta = \arccos\dfrac{4}{x}$

41. $\sin\theta = \dfrac{x+2}{5}$

$\theta = \arcsin\left(\dfrac{x+2}{5}\right)$

42. $\tan\theta = \dfrac{x+1}{10}$

$\theta = \arctan\dfrac{x+1}{10}$

43. $\cos\theta = \dfrac{x+3}{2x}$

$\theta = \arccos\dfrac{x+3}{2x}$

44. $\sin\theta = \dfrac{x-1}{x^2-1} = \dfrac{1}{x+1}$

$\theta = \arcsin\dfrac{1}{x+1},\ x \neq 1$

45. $\sin(\arcsin 0.3) = 0.3$

46. $\tan(\arctan 45) = 45$

47. It is not possible to evaluate $\cos\left[\arccos\left(-\sqrt{3}\right)\right]$. The domain of the inverse cosine function is $[-1, 1]$.

48. $\sin\left[\arcsin(-0.2)\right] = -0.2$

49. $\arcsin\left[\sin\left(\dfrac{9\pi}{4}\right)\right] = \arcsin\left(\dfrac{\sqrt{2}}{2}\right) = \dfrac{\pi}{4}.$

Note: $\dfrac{9\pi}{4}$ is not in the range of the arcsin function.

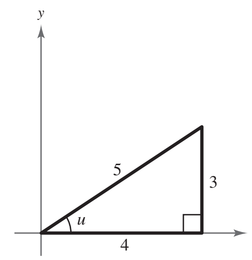
50. $\arccos\left(\cos\dfrac{7\pi}{2}\right) = \arccos 0 = \dfrac{\pi}{2}$

Note: $\dfrac{7\pi}{2}$ is not in the range of the arccosine function.

51. Let $u = \arctan\dfrac{3}{4}.$

$\tan u = \dfrac{3}{4},\ 0 < u < \dfrac{\pi}{2},$

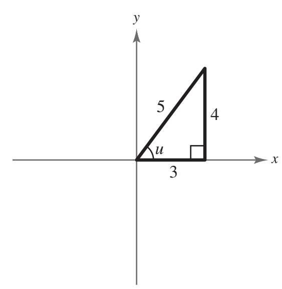
$\sin\left(\arctan\dfrac{3}{4}\right) = \sin u = \dfrac{3}{5}$



52. Let $u = \arcsin\dfrac{4}{5},$

$\sin u = \dfrac{4}{5},\ 0 < u < \dfrac{\pi}{2},$

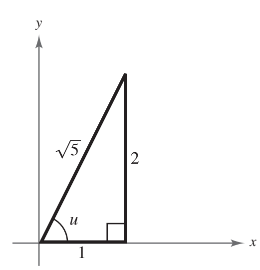
$\cos\left(\arcsin\dfrac{4}{5}\right) = \cos u = \dfrac{3}{5}.$



53. Let $u = \tan^{-1} 2,$

$\tan u = 2 = \dfrac{2}{1},\ 0 < u < \dfrac{\pi}{2},$

$\cos\left(\tan^{-1} 2\right) = \cos u = \dfrac{1}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}.$



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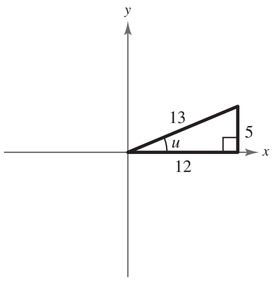
54. It is not possible to evaluate $\sin\left(\cos^{-1}\sqrt{5}\right)$. The domain of the inverse cosine function is $[-1, 1]$.

55. Let $u = \arcsin \frac{5}{13}$,

$\sin u = \frac{5}{13}, 0 < u < \frac{\pi}{2}$,

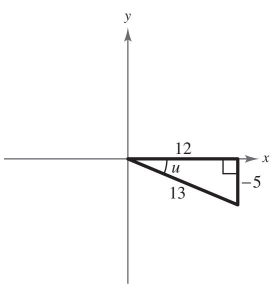
$\sec\left(\arcsin \frac{5}{13}\right) = \sec u = \frac{13}{12}$.



56. Let $u = \arctan\left(-\frac{5}{12}\right)$,

$\tan u = -\frac{5}{12}, -\frac{\pi}{2} < u < 0$,

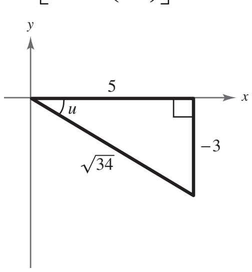
$\csc\left[\arctan\left(-\frac{5}{12}\right)\right] = \csc u = -\frac{13}{5}$.



57. Let $u = \arctan\left(-\frac{3}{5}\right)$,

$\tan u = -\frac{3}{5}, -\frac{\pi}{2} < y < 0$,

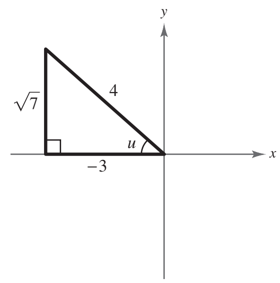
$\cot\left[\arctan\left(-\frac{3}{5}\right)\right] = \cot u = -\frac{5}{3}$.



58. Let $u = \arccos\left(-\frac{3}{4}\right)$.

$\cos u = -\frac{3}{4}, \frac{\pi}{2} < u < \pi$,

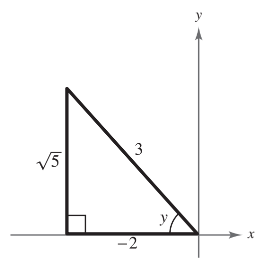
$\sec\left[\arccos\left(-\frac{3}{4}\right)\right] = \sec u = -\frac{4}{3}$



59. Let $u = \arccos\left(-\frac{2}{3}\right)$.

$\cos u = -\frac{2}{3}, \frac{\pi}{2} < u < \pi$,

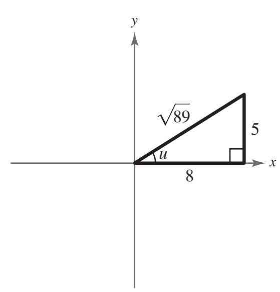
$\tan\left[\arccos\left(-\frac{2}{3}\right)\right] = \tan u = -\frac{\sqrt{5}}{2}$



60. Let $u = \arctan \frac{5}{8}$.

$\tan u = \frac{5}{8}, 0 < u < \frac{\pi}{2}$,

$\cot\left(\arctan \frac{5}{8}\right) = \cot u = \frac{8}{5}$



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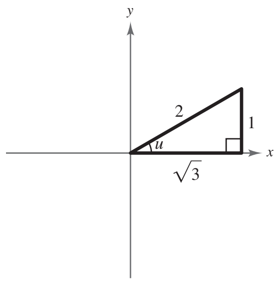
61. Let $u = \cos^{-1}\dfrac{\sqrt{3}}{2}$.

$\cos u = \dfrac{\sqrt{3}}{2}, 0 < u < \dfrac{\pi}{2},$

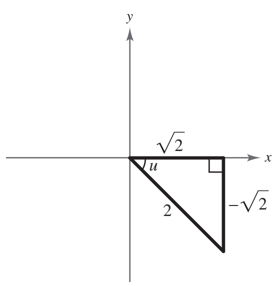
$\csc\left[\cos^{-1}\dfrac{\sqrt{3}}{2}\right] = \csc u = 2.$



62. Let $u = \sin^{-1}\left(-\dfrac{\sqrt{2}}{2}\right)$.

$\sin u = \left(-\dfrac{\sqrt{2}}{2}\right), -\dfrac{\pi}{2} < u < 0,$

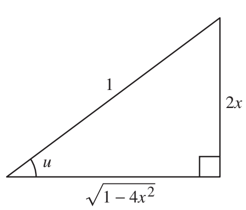
$\tan\left[\sin^{-1}\left(-\dfrac{\sqrt{2}}{2}\right)\right] = \tan u = \dfrac{-\sqrt{2}}{\sqrt{2}} = -1.$



63. Let $u = \arcsin(2x)$.

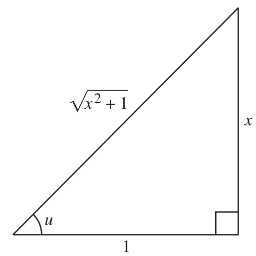
$\sin u = 2x = \dfrac{2x}{1},$

$\cos(\arcsin 2x) = \cos u = \sqrt{1 - 4x^2}$



64. Let $u = \arctan x$, $\tan u = x = \dfrac{x}{1},$

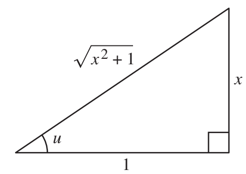
$\sin(\arctan x) = \sin u = \dfrac{x}{\sqrt{x^2 + 1}}$



65. Let $u = \arctan x$.

$\tan u = x = \dfrac{x}{1},$

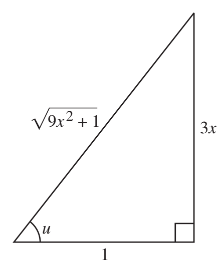
$\cot(\arctan x) = \cot u = \dfrac{1}{x}$



66. Let $u = \arctan 3x$.

$\tan u = 3x = \dfrac{3x}{1},$

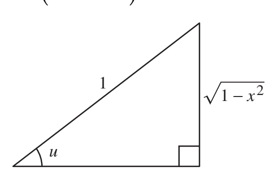
$\sec(\arctan 3x) = \sec u = \sqrt{9x^2 + 1}$



67. Let $u = \arccos x$.

$\cos u = x = \dfrac{x}{1},$

$\sin(\arccos x) = \sin u = \sqrt{1 - x^2}$



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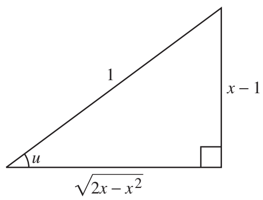
68. Let $u = \arcsin(x - 1)$.

$\sin u = x - 1 = \dfrac{x-1}{1}$,

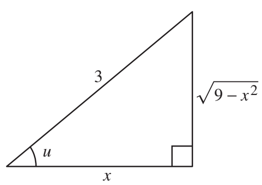
$\sec[\arcsin(x-1)] = \sec u = \dfrac{1}{\sqrt{2x - x^2}}$



69. Let $u = \arccos\left(\dfrac{x}{3}\right)$.

$\cos u = \dfrac{x}{3}$,

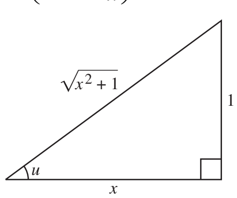
$\tan\left(\arccos\dfrac{x}{3}\right) = \tan u = \dfrac{\sqrt{9 - x^2}}{x}$



70. Let $u = \arctan\dfrac{1}{x}$.

$\tan u = \dfrac{1}{x}$,

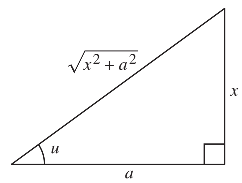
$\cot\left(\arctan\dfrac{1}{x}\right) = \cot u = x$



71. Let $u = \arctan\dfrac{x}{a}$.

$\tan u = \dfrac{x}{a}$,

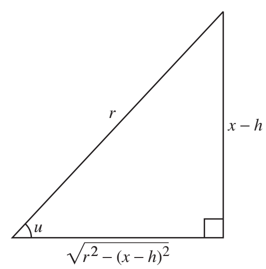
$\csc\left(\arctan\dfrac{x}{a}\right) = \csc u = \dfrac{\sqrt{x^2 + a^2}}{a}$



72. Let $u = \arcsin\dfrac{x-h}{r}$.

$\sin u = \dfrac{x-h}{r}$,

$\cos\left(\arcsin\dfrac{x-h}{r}\right) = \cos u = \dfrac{\sqrt{r^2 - (x-h)^2}}{r}$



73. $f(x) = \sin(\arctan 2x)$, $g(x) = \dfrac{2x}{\sqrt{1 + 4x^2}}$

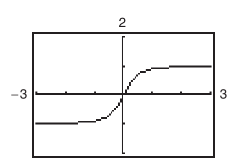
They are equal. Let $u = \arctan 2x$,

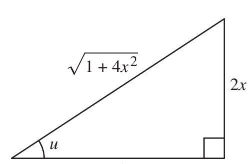
$\tan u = 2x = \dfrac{2x}{1}$,

and $\sin u = \dfrac{2x}{\sqrt{1 + 4x^2}}$.

$g(x) = \dfrac{2x}{\sqrt{1 + 4x^2}} = f(x)$

The graph has horizontal asymptotes at $y = \pm 1$.





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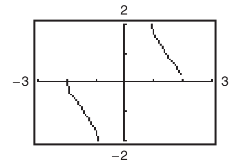
74. $f(x) = \tan\left(\arccos \dfrac{x}{2}\right)$

$g(x) = \dfrac{\sqrt{4 - x^2}}{x}$

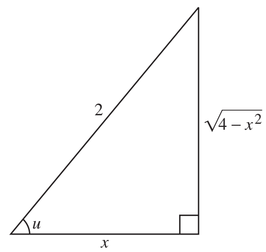


These are equal because:

Let $u = \arccos \dfrac{x}{2}$.

$f(x) = \tan\left(\arccos \dfrac{x}{2}\right) = \tan u = \dfrac{\sqrt{4 - x^2}}{x} = g(x)$

The graph has a horizontal asymptote at $x = 0$.

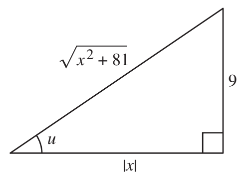


75. Let $u = \arctan \dfrac{9}{x}$.

$\tan u = \dfrac{9}{x}$ and $\sin u = \dfrac{9}{\sqrt{x^2 + 81}}, x > 0$

So,

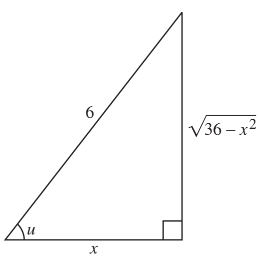
$\arctan \dfrac{9}{x} = \arcsin \dfrac{9}{\sqrt{x^2 + 81}}, x > 0.$



76. Let $u = \arcsin \dfrac{\sqrt{36 - x^2}}{6}$.

$\sin u = \dfrac{\sqrt{36 - x^2}}{6}$,

$\arcsin \dfrac{\sqrt{36 - x^2}}{6} = \arccos \dfrac{x}{6}$

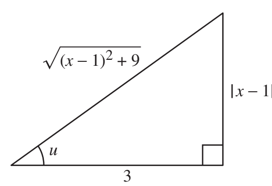


77. Let $u = \arccos \dfrac{3}{\sqrt{x^2 - 2x + 10}}$. Then,

$\cos u = \dfrac{3}{\sqrt{x^2 - 2x + 10}} = \dfrac{3}{\sqrt{(x - 1)^2 + 9}}$

and $\sin u = \dfrac{|x - 1|}{\sqrt{(x - 1)^2 + 9}}$.

So, $u = \arcsin \dfrac{|x - 1|}{\sqrt{x^2 - 2x + 10}}$.



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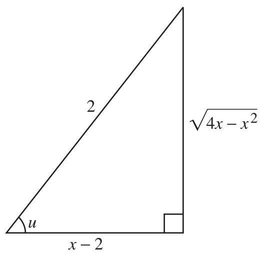
78. Let $u = \arccos \frac{x - 2}{2}, 2 < x < 4$

$\cos u = \frac{x - 2}{2},$

$\arccos \frac{x - 2}{2} = \arctan \frac{\sqrt{4x - x^2}}{x - 2}, 2 < x < 4$

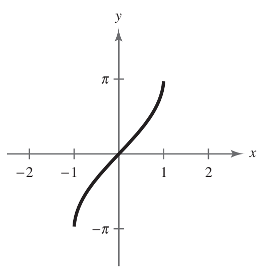


79. $g(x) = 2 \arcsin x$

Domain: $-1 \le x \le 1$

Range: $-\pi \le y \le \pi$

This is the graph of $f(x) = \arcsin(x)$ with a vertical stretch.

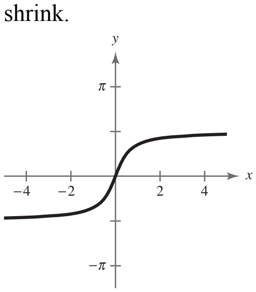


80. $f(x) = \arctan 2x$

Domain: all real numbers

Range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$

This is the graph of $g(x) = \arctan(x)$ with a horizontal shrink.

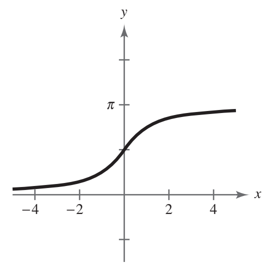


81. $f(x) = \frac{\pi}{2} + \arctan x$

Domain: all real numbers

Range: $0 < y \le \pi$

This is the graph of $y = \arctan x$ shifted upward $\pi/2$ units.

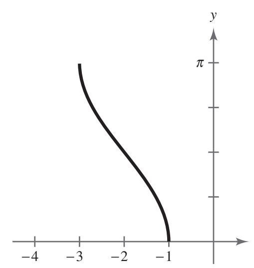


82. $g(t) = \arccos(t + 2)$

Domain: $-3 \le t \le -1$

Range: $0 \le y \le \pi$

This is the graph of $y = \arccos t$ shifted two units to the left.

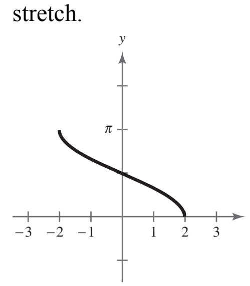


83. $h(v) = \arccos \frac{v}{2}$

Domain: $-2 \le v \le 2$

Range: $0 \le y \le \pi$

This is the graph of $h(v) = \arccos v$ with a horizontal stretch.



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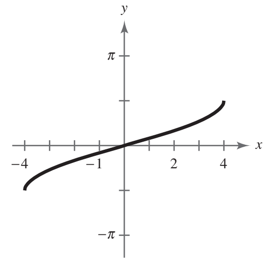
84. $f(x) = \arcsin \dfrac{x}{4}$

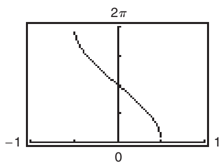
Domain: $-4 \le x \le 4$

Range: $-\dfrac{\pi}{2} \le y \le \dfrac{\pi}{2}$

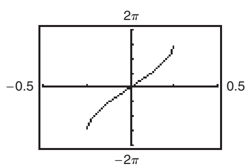
This is the $f(x) = \arcsin x$ with a horizontal stretch.



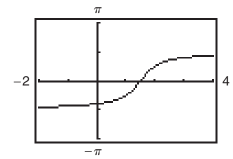
85. $f(x) = 2 \arccos(2x)$



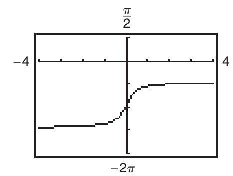
86. $f(x) = \pi \arcsin(4x)$



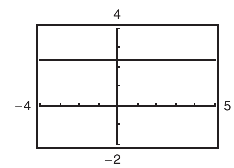
87. $f(x) = \arctan(2x - 3)$



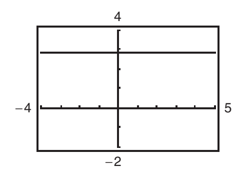
88. $f(x) = -3 + \arctan(\pi x)$



89. $f(x) = \pi - \sin^{-1}\left(\dfrac{2}{3}\right) \approx 2.412$



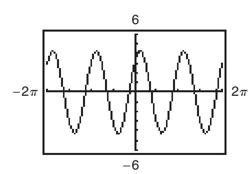
90. $f(x) = \dfrac{\pi}{2} + \cos^{-1}\left(\dfrac{1}{\pi}\right) \approx 2.82$



91. $f(t) = 3 \cos 2t + 3 \sin 2t = \sqrt{3^2 + 3^2} \sin\left(2t + \arctan \dfrac{3}{3}\right)$

$= 3\sqrt{2} \sin(2t + \arctan 1)$

$= 3\sqrt{2} \sin\left(2t + \dfrac{\pi}{4}\right)$

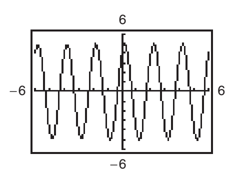


The graph implies that the identity is true.

92. $f(t) = 4 \cos \pi t + 3 \sin \pi t$

$= \sqrt{4^2 + 3^2} \sin\left(\pi t + \arctan \dfrac{4}{3}\right)$

$= 5 \sin\left(\pi t + \arctan \dfrac{4}{3}\right)$



The graph implies that $A \cos \omega t + B \sin \omega t = \sqrt{A^2 + B^2} \sin\left(\omega t + \arctan \dfrac{A}{B}\right)$ is true.

93. $\dfrac{\pi}{2}$

94. 0

95. $\dfrac{\pi}{2}$

96. $-\dfrac{\pi}{2}$

97. π

98. $-\dfrac{\pi}{2}$

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99. (a) $\sin\theta = \dfrac{5}{s}$

$\theta = \arcsin\dfrac{5}{s}$

(b) $s = 40$: $\theta = \arcsin\dfrac{5}{40} \approx 0.13$

$s = 20$: $\theta = \arcsin\dfrac{5}{20} \approx 0.25$

100. (a) $\tan\theta = \dfrac{s}{750}$

$\theta = \arctan\dfrac{s}{750}$

(b) When $s = 300$,

$\theta = \arctan\dfrac{300}{750} \approx 0.38 \approx 21.8°$.

When $s = 1200$,

$\theta = \arctan\dfrac{1200}{750} \approx 1.01 \approx 58.0°$.

101. (a) $\tan\theta = \dfrac{5.5}{8.5}$

$\theta \approx 32.9°$

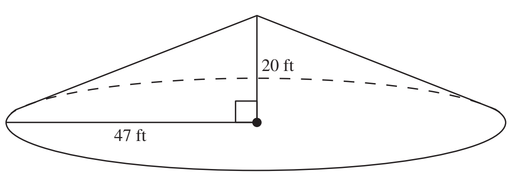
(b) $\tan 32.9° = \dfrac{h}{10}$

$h = 10\tan 32.9°$

$h \approx 6.49$ meters

The height is about 6.5 meters.

102. (a)



(b) $\tan\theta = \dfrac{20}{47}$

$\theta \approx 23.05°$

The angle is about 23.1 degrees.

(c) $\tan 23.05° = \dfrac{h}{30}$

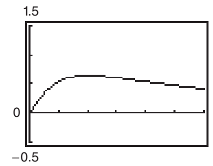
$h = 30\tan 23.05°$

$h \approx 12.77$ feet

The height is about 12.8 feet.

103. $\beta = \arctan\dfrac{3x}{x^2 + 4}$

(a)



(b) β is maximum when $x = 2$ feet.

(c) The graph has a horizontal asymptote at $\beta = 0$. As x increases, β decreases.

104. (a) $\sin\theta = \dfrac{6}{x}$

$\theta = \arcsin\dfrac{6}{x}$

(b) $x = 12$ miles

$\theta = \arcsin\dfrac{6}{12} \approx 30.0°$

$x = 7$ mile

$\theta = \arcsin\dfrac{6}{7} \approx 59.0°$

105. (a) $\tan\theta = \dfrac{x}{20}$

$\theta = \arctan\dfrac{x}{20}$

(b) $x = 5$: $\theta = \arctan\dfrac{5}{20} \approx 14.0°$

$x = 12$: $\theta = \arctan\dfrac{12}{20} \approx 31.0°$

106. False.

$\dfrac{5\pi}{6}$ is not in the range of $\arcsin(x)$.

$\arcsin\dfrac{1}{2} = \dfrac{\pi}{6}$

107. True. $-\dfrac{\pi}{4}$ is in the range of the arctangent function.

$\tan\left(-\dfrac{\pi}{4}\right) = -1 \Leftrightarrow \arctan(-1) = -\dfrac{\pi}{4}$.

108. False. The domain of $\arctan x$ is all real x, but the domain of both $\arcsin x$ and $\arccos x$ are $-1 \le x \le 1$.

$$\arctan(-1) = -\frac{\pi}{4} \ne \frac{\arcsin(-1)}{\arcsin(-1)} = \frac{\left(-\frac{\pi}{2}\right)}{\pi} = -\frac{1}{2}$$

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109. False. $\sin^{-1} x \neq \dfrac{1}{\sin x}$

The function $\sin^{-1} x$ is equivalent to $\arcsin x$, which is the inverse sine function. The expression, $\dfrac{1}{\sin x}$ is the reciprocal of the sine function and is equivalent to $\csc x$.

110. (a) Because the graph of $y = \arcsin x$ lies below the graph of $y = \arccos x$, when x is greater than or equal to -1 and less than $\sqrt{2}/2$,

$\arcsin x < \arccos x$ for $-1 \leq x < \sqrt{2}/2$.

(b) The graphs intersect at the point $\left(\sqrt{2}/2, \pi/4\right)$, so

$\arcsin x = \arccos x$ when $x = \dfrac{\sqrt{2}}{2}$.

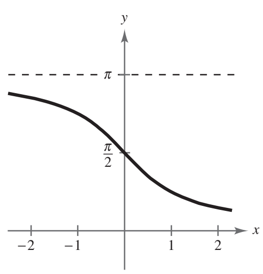
(c) Because the graph of $y = \arcsin x$ lies above the graph of $y = \arccos x$ when x is greater than $\dfrac{\sqrt{2}}{2}$ and less than or equal to 1,

$\arcsin x > \arccos x$ for $\sqrt{2}/2 < x \leq 1$.

111. $y = \operatorname{arccot} x$ if and only if $\cot y = x$.

Domain: $(-\infty, \infty)$

Range: $(0, \pi)$

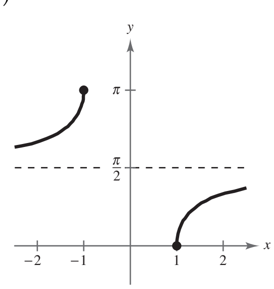


112. $y = \operatorname{arcsec} x$ if and only if $\sec y = x$ where

$x \leq -1 \cup x \geq 1$ and $0 \leq y < \dfrac{\pi}{2}$ and $\dfrac{\pi}{2} < y \leq \pi$.

Domain: $(-\infty, -1] \cup [1, \infty)$

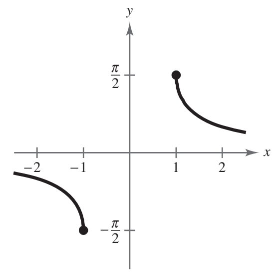
Range: $\left[0, \dfrac{\pi}{2}\right) \cup \left(\dfrac{\pi}{2}, \pi\right]$



113. $y = \operatorname{arccsc} x$ if and only if $\csc y = x$.

Domain: $(-\infty, -1] \cup [1, \infty)$

Range: $\left[-\dfrac{\pi}{2}, 0\right) \cup \left(0, \dfrac{\pi}{2}\right]$



114. Answers will vary.

115. $y = \operatorname{arcsec} \sqrt{2} \Rightarrow \sec y = \sqrt{2}$ and

$0 \leq y < \dfrac{\pi}{2} \cup \dfrac{\pi}{2} < y \leq \pi \Rightarrow y = \dfrac{\pi}{4}$

116. $y = \operatorname{arcsec} 1 \Rightarrow \sec y = 1$ and

$0 \leq y < \dfrac{\pi}{2} \cup \dfrac{\pi}{2} < y \leq \pi \Rightarrow y = 0$

117. $y = \operatorname{arccot}(-1) \Rightarrow \cot y = -1$ and

$0 < y < \pi \Rightarrow y = \dfrac{3\pi}{4}$

118. $y = \operatorname{arccot}\left(-\sqrt{3}\right) \Rightarrow \cot y = -\sqrt{3}$ and

$0 < y < \pi \Rightarrow y = \dfrac{5\pi}{6}$

119. $y = \operatorname{arccsc}(-1) \Rightarrow \csc y = -1$ and

$-\dfrac{\pi}{2} \leq y < 0 \cup 0 < y \leq \dfrac{\pi}{2} \Rightarrow y = -\dfrac{\pi}{2}$

120. $y = \operatorname{arccsc}\left(\dfrac{2\sqrt{3}}{3}\right) \Rightarrow \csc y = \dfrac{2\sqrt{3}}{3}$ and

$-\dfrac{\pi}{2} \leq y < 0 \cup 0 < y \leq \dfrac{\pi}{2} \Rightarrow y = \dfrac{\pi}{3}$

121. $\operatorname{arcsec} 2.54 = \arccos\left(\dfrac{1}{2.54}\right) \approx 1.17$

122. $\operatorname{arcsec}(-1.52) = \arccos\left(-\dfrac{1}{1.52}\right) \approx 2.29$

123. $\operatorname{arccsc}\left(-\frac{25}{3}\right) = \arcsin\left(-\frac{3}{25}\right) \approx -0.12$

124. $\operatorname{arccsc}(-12) = \arcsin\left(-\frac{1}{12}\right) \approx -0.08$

125. $\operatorname{arccot} 5.25 = \arctan\left(\dfrac{1}{5.25}\right) \approx 0.19$

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126. $\text{arccot}\left(-\frac{16}{7}\right) = \arctan\left(-\frac{7}{16}\right) \approx -0.41$

Because -0.41 is not in the domain $(0, \pi)$,

$\text{arccot}\left(-\frac{16}{7}\right) \approx -0.41 + \pi \approx 2.73.$

127. Area $= \arctan b - \arctan a$

(a) $a = 0, b = 1$

$$\text{Area} = \arctan 1 - \arctan 0 = \frac{\pi}{4} - 0 = \frac{\pi}{4}$$

(b) $a = -1, b = 1$

$$\text{Area} = \arctan 1 - \arctan(-1) = \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$$

(c) $a = 0, b = 3$

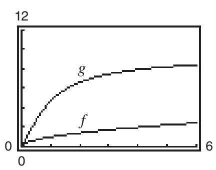
$$\text{Area} = \arctan 3 - \arctan 0 \approx 1.25 - 0 = 1.25$$

(d) $a = -1, b = 3$

$$\text{Area} = \arctan 3 - \arctan(-1) \approx 1.25 - \left(-\frac{\pi}{4}\right) \approx 2.03$$

128. $f(x) = \sqrt{x}$

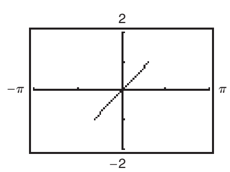
$g(x) = 6 \arctan x$



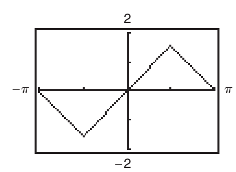
As x increases to infinity, g approaches 3π, but f has no maximum. Using the solve feature of the graphing utility, you find $a \approx 87.54$.

129. $f(x) = \sin(x), f^{-1}(x) = \arcsin(x)$

(a) $f \circ f^{-1} = \sin(\arcsin x)$



$f^{-1} \circ f = \arcsin(\sin x)$



(b) The graphs coincide with the graph of $y = x$ only for certain values of x.

$f \circ f^{-1} = x$ over its entire domain, $-1 \le x \le 1$.

$f^{-1} \circ f = x$ over the region $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, corresponding to the region where $\sin x$ is one-to-one and has an inverse.

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130. (a) Let $y = \arcsin(-x)$. Then,

$$\sin y = -x$$
$$-\sin y = x$$
$$\sin(-y) = x$$
$$-y = \arcsin x$$
$$y = -\arcsin x.$$

So, $\arcsin(-x) = -\arcsin x$.

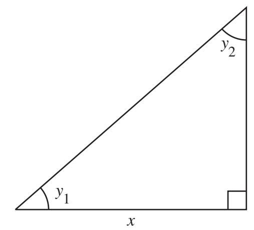
(b) Let $y = \arctan(-x)$. Then,

$$\tan y = -x, -\frac{\pi}{2} < y < \frac{\pi}{2}$$
$$-\tan y = x$$
$$\tan(-y) = x, -\frac{\pi}{2} < -y < \frac{\pi}{2}$$
$$\arctan(\tan(-y)) = \arctan x$$
$$-y = \arctan x$$
$$y = -\arctan x$$

So, $\arctan(-x) = -\arctan(x)$.

(c) Let $y_2 = \frac{\pi}{2} - y_1$.

$$\arctan x + \arctan\frac{1}{x} = y_1 + y_2 = y_1 + \left(\frac{\pi}{2} - y_1\right) = \frac{\pi}{2}$$

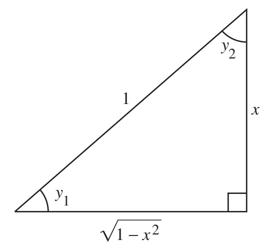


(d) Let $\alpha = \arcsin x$ and $\beta = \arccos x$, then $\sin\alpha = x$ and $\cos\beta = x$. So, $\sin\alpha = \cos\beta$ which implies that α and β are complementary angles and you have

$$\alpha + \beta = \frac{\pi}{2}$$
$$\arcsin x + \arccos x = \frac{\pi}{2}.$$

(e) $\arcsin x = \arcsin\frac{x}{1}$

$$= \arctan\frac{x}{\sqrt{1 - x^2}}$$



Section 4.8 Applications and Models

1. bearing

2. harmonic motion

3. period

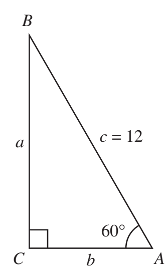
4. frequency

5. Given: $A = 60°, c = 12$

$$\sin A = \frac{a}{c} \Rightarrow a = c\sin A = 12\sin 60° = \frac{12\sqrt{3}}{2} = 6\sqrt{3} \approx 10.39$$

$$\cos A = \frac{b}{c} \Rightarrow b = c\cos A = 12\cos 60° = 12\left(\frac{1}{2}\right) = 6$$

$$B = 90° - 60° = 30°$$

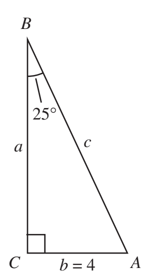


6. Given: $B = 25°, b = 4$

$$A = 90° - B = 90° - 25° = 65°$$

$$\sin B = \frac{b}{c} \Rightarrow c = \frac{b}{\sin B} = \frac{4}{\sin 25°} \approx 9.46$$

$$\tan B = \frac{b}{a} \Rightarrow a = \frac{b}{\tan B} = \frac{4}{\tan 25°} \approx 8.58$$

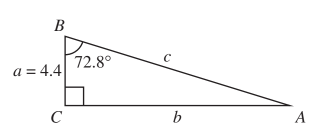


7. Given: $B = 72.8°, a = 4.4$

$$\cos B = \frac{a}{c} \Rightarrow c = \frac{a}{\cos B} = \frac{4.4}{\cos 72.8°} \approx 14.88$$

$$\tan B = \frac{b}{a} \Rightarrow b = a\tan B = 4.4\tan 72.8° \approx 14.21$$

$$A = 90° - 72.8° = 17.2°$$



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8. Given: $A = 8.4°, a = 40.5$

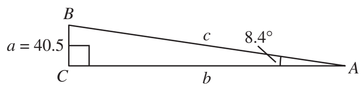
$B = 90° - A$

$= 90° - 8.4° = 81.6°$

$\tan A = \dfrac{a}{b} \Rightarrow b = \dfrac{a}{\tan A}$

$= \dfrac{40.5}{\tan 8.4°} \approx 274.27$

$\sin A = \dfrac{a}{c} \Rightarrow c = \dfrac{a}{\sin A} = \dfrac{40.5}{\sin 8.4°} \approx 277.24$

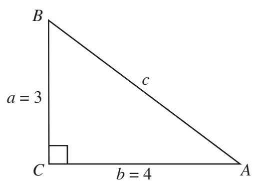


9. Given: $a = 3, b = 4$

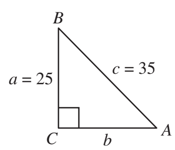
$a^2 + b^2 = c^2 \Rightarrow c^2 = (3)^2 + (4)^2 \Rightarrow c = 5$

$\tan A = \dfrac{a}{b} \Rightarrow A = \tan^{-1}\left(\dfrac{a}{b}\right) = \tan^{-1}\left(\dfrac{3}{4}\right) \approx 36.87°$

$B = 90° - 36.87° = 53.13°$



10. Given: $a = 25, c = 35$



$b = \sqrt{c^2 - a^2}$

$= \sqrt{35^2 - 25^2}$

$= \sqrt{600} \approx 24.49$

$\sin A = \dfrac{a}{c} \Rightarrow A = \arcsin\dfrac{a}{c} = \arcsin\dfrac{25}{35} \approx 45.58°$

$\cos B = \dfrac{a}{c} \Rightarrow B = \arccos\dfrac{a}{c} = \arccos\dfrac{25}{35} \approx 44.42°$

11. Given: $b = 15.70, c = 55.16$

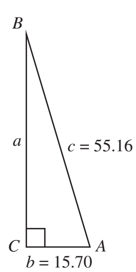
$a = \sqrt{55.16^2 - 15.7^2} \approx 52.88$

$\cos A = \dfrac{b}{c}$

$\cos A = \dfrac{15.7}{55.15}$

$A = \arccos\dfrac{15.7}{55.15} \approx 73.46°$

$B = 90° - 73.46° \approx 16.54°$



12. Given: $b = 1.32, c = 9.45$

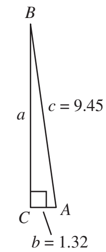
$a = \sqrt{c^2 - b^2} = \sqrt{87.5601} \approx 9.36$

$\cos A = \dfrac{b}{c} \Rightarrow A = \arccos\dfrac{b}{c} = \arccos\dfrac{1.32}{9.45} \approx 81.97°$

$\sin B = \dfrac{b}{c} \Rightarrow B = \arcsin\dfrac{b}{c}$

$= \arcsin\dfrac{1.32}{9.45}$

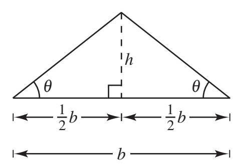
$\approx 8.03°$



13. $\theta = 45°, b = 6$

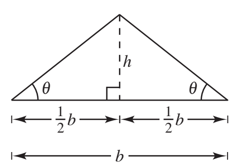
$\tan\theta = \dfrac{h}{(1/2)b} \Rightarrow h = \dfrac{1}{2}b\tan\theta$

$h = \dfrac{1}{2}(6)\tan 45° = 3.00$ units



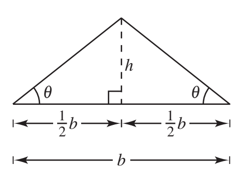
14. $\tan\theta = \dfrac{h}{(1/2)b} \Rightarrow h = \dfrac{1}{2}b\tan\theta$

$h = \dfrac{1}{2}(14)\tan 22° \approx 2.83$ units



15. $\tan\theta = \dfrac{h}{(1/2)b} \Rightarrow h = \dfrac{1}{2}b\tan\theta$

$h = \dfrac{1}{2}(8)\tan 32° \approx 2.50$ units



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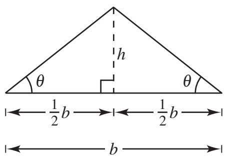
16. $\tan\theta = \dfrac{h}{(1/2)b} \Rightarrow h = \dfrac{1}{2}b\tan\theta$

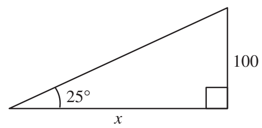
$h = \dfrac{1}{2}(11)\tan 27° \approx 2.80$ units



17. $\tan 25° = \dfrac{100}{x}$

$x = \dfrac{100}{\tan 25°}$

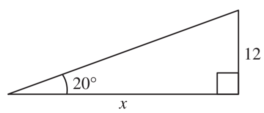
≈ 214.45 feet



18. $\tan 20° = \dfrac{12}{x}$

$x = \dfrac{12}{\tan 20°}$

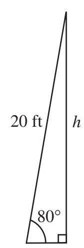
≈ 32.97 feet



19. $\sin 80° = \dfrac{h}{20}$

$20\sin 80° = h$

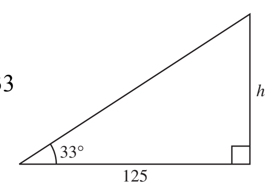
$h \approx 19.7$ feet



20. $\tan 33° = \dfrac{h}{125}$

$h = 125\tan 33$

≈ 81.2 feet



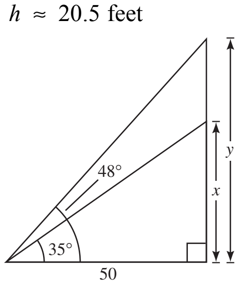
21. Let the height of the church $= x$ and the height of the church and steeple $= y$.

$\tan 35° = \dfrac{x}{50}$ and $\tan 48° = \dfrac{y}{50}$

$x = 50\tan 35° \approx 35.01$ and $y = 50\tan 48° \approx 55.53$

$h = y - x = 55.53 - 35.01 = 20.52.$

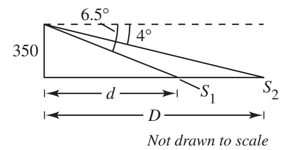
$h \approx 20.5$ feet



22. $\tan 6.5° = \dfrac{350}{d} \Rightarrow d \approx 3071.91$ ft

$\tan 4° = \dfrac{350}{D} \Rightarrow D \approx 5005.23$ ft

Distance between ships: $D - d \approx 1933.3$ ft

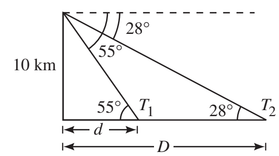


23. $\cot 55 = \dfrac{d}{10} \Rightarrow d \approx 7$ kilometers

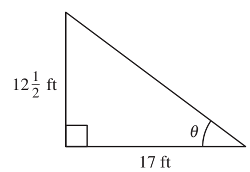
$\cot 28° = \dfrac{D}{10} \Rightarrow D \approx 18.8$ kilometers

Distance between towns:

$D - d = 18.8 - 7 = 11.8$ kilometers



24. (a)

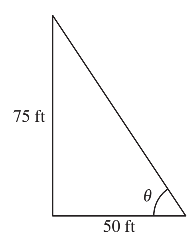


(b) $\tan\theta = \dfrac{12.5}{17}$

(c) $\theta = \arctan\left(\dfrac{12.5}{17}\right) \approx 36.3°$

25. $\tan\theta = \frac{75}{50}$

$\theta = \arctan\frac{3}{2} \approx 56.3°$



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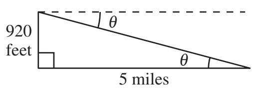
26. 1200 feet + 120 feet − 400 feet = 920 feet

5 miles = 5 miles$\left(\dfrac{5280 \text{ feet}}{1 \text{ mile}}\right)$ = 26,400 feet

$$\tan\theta = \frac{920}{26{,}400}$$

$$\theta = \arctan\left(\frac{920}{26{,}400}\right) \approx 2.0°$$

Not drawn to scale



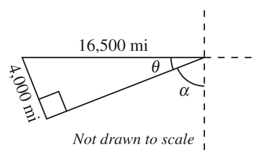
27. 12,500 + 4000 = 16,500

$$\sin\theta = \frac{4000}{16{,}500}$$

$$\theta = \arcsin\left(\frac{4000}{16{,}500}\right)$$

$$\theta \approx 14.03°$$

Angle of depression = $\alpha \approx 90° - 14.03° = 75.97°$



28. (a) $l^2 = (h + 17)^2 + 100^2$

$$l = \sqrt{(h + 17)^2 + 10{,}000}$$

$$= \sqrt{h^2 + 34h + 10{,}289}$$

(b) $\cos\theta = \dfrac{100}{l}$

$$\theta = \arccos\left(\frac{100}{l}\right)$$

(c) $\cos\theta = \dfrac{100}{l}$

$$\cos 35° = \frac{100}{l}$$

$$l \approx 122.077$$

$$l^2 = 100^2 + (h + 17)^2$$

$$l^2 = h^2 + 34h + 10.289$$

$$0 = h^2 + 34h - 4613.794$$

$$h \approx 53.02 \text{ feet}$$

29. $\tan 57° = \dfrac{a}{x} \Rightarrow x = a \cot 57°$

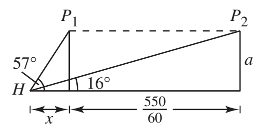
$$\tan 16° = \frac{a}{x + (55/6)}$$

$$\tan 16° = \frac{a}{a\cot 57° + (55/6)}$$

$$\cot 16° = \frac{a\cot 57° + (55/6)}{a}$$

$$a\cot 16° - a\cot 57° = \frac{55}{6} \Rightarrow a \approx 3.23 \text{ miles}$$

$$\approx 17{,}054 \text{ feet}$$



30. (a) $\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$

$$\sin 60° = \frac{h}{30}$$

$$h = 15\sqrt{3} \approx 25.98 \text{ feet}$$

(b) $\tan\theta = \dfrac{25.981}{d}$

$$\theta = \arctan\left(\frac{25.981}{d}\right)$$

(c) When $\theta = 25°$,

$$\tan 25° = \frac{25.981}{d}$$

$$d = \frac{25.981}{\tan 25°}$$

$$d \approx 55.72 \text{ feet.}$$

When $\theta = 30°$,

$$\tan 30° = \frac{25.981}{d}$$

$$d = \frac{25.981}{\tan 30°}$$

$$d \approx 45.00$$

$$45 \text{ ft} \le d \le 55.7 \text{ feet.}$$

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31. (a) $l^2 = (200)^2 + (150)^2$

$l = 250$ feet

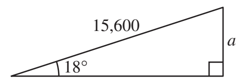
$\tan A = \dfrac{150}{200} \Rightarrow A = \arctan\left(\dfrac{150}{200}\right) \approx 36.87°$

$\tan B = \dfrac{200}{150} \Rightarrow B = \arctan\left(\dfrac{200}{150}\right) \approx 53.13°$

(b) $250 \text{ ft} \times \dfrac{\text{mile}}{5280 \text{ ft}} \times \dfrac{\text{hour}}{35 \text{ miles}} \times \dfrac{3600 \text{ sec}}{\text{hour}} \approx 4.87$ seconds

32. (a) Because the airplane speed is

$\left(260 \dfrac{\text{ft}}{\text{sec}}\right)\left(60 \dfrac{\text{sec}}{\text{min}}\right) = 15{,}600 \dfrac{\text{ft}}{\text{min}}$,

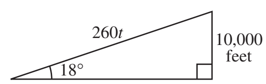


after one minute its distance travelled is 15,600 feet.

$\sin 18° = \dfrac{a}{15{,}600}$

$a = 15{,}600 \sin 18° \approx 4820.7$ ft

(b) Let t be the time for the plane to reach 10,000 ft.



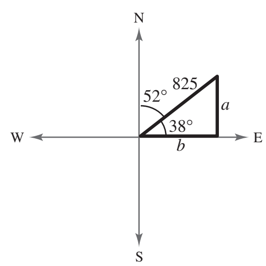
$\sin 18° = \dfrac{10{,}000}{260t}$

$s = \dfrac{10{,}000}{260(\sin 18°)} \approx 124.5$ seconds

33. The plane has traveled $1.5(550) = 825$ miles.

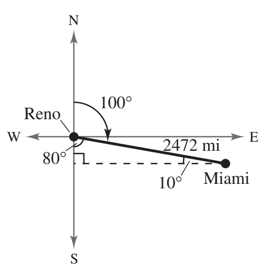
$\sin 38° = \dfrac{a}{825} \Rightarrow a \approx 508$ miles north

$\cos 38° = \dfrac{b}{825} \Rightarrow b \approx 650$ miles east

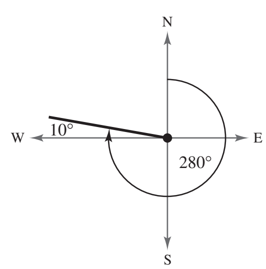


34. (a) Reno is $2472 \sin 10° = 429.26$ miles north of Miami.

Reno is $2472 \cos 10° = 2434.44$ miles west of Miami.



(b) The return heading is $280°$.



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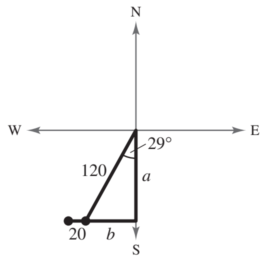
35.



(a) $\cos 29° = \dfrac{a}{120} \Rightarrow a \approx 104.95$ nautical miles south

$\sin 29° = \dfrac{b}{120} \Rightarrow b \approx 58.18$ nautical miles west

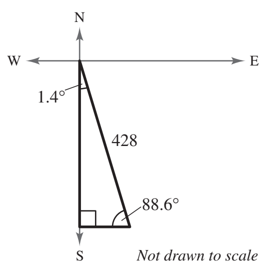
(b) $\tan\theta = \dfrac{20 + b}{a} \approx \dfrac{78.18}{104.95} \Rightarrow \theta \approx 36.7°$

Bearing: S 36.7° W

Distance: $d \approx \sqrt{104.95^2 + 78.18^2}$

≈ 130.9 nautical miles from port

36.



(a) $t = \frac{428}{20} = 21.4$ hours

(b) After 12 hours, the yacht will have traveled 240 nautical miles.

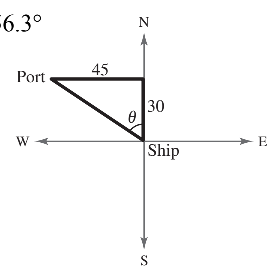
$240 \sin 1.4° \approx 5.86$ miles east

$240 \cos 1.4° \approx 239.93$ miles south

(c) Bearing from N is 178.6°.

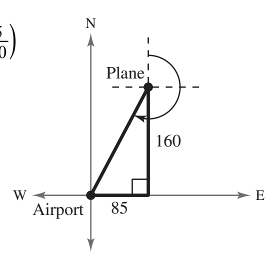
37. $\tan\theta = \frac{45}{30} \Rightarrow \theta \approx 56.3°$

Bearing: N 56.31°



38. Bearing $= 180° + \arctan\left(\frac{85}{160}\right)$

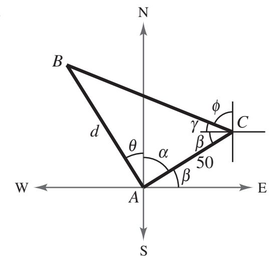
$= 208.0°$



39. $\theta = 32°, \phi = 68°$

(a) $\alpha = 90° - 32° = 58°$

Bearing from A to C: N 58° E



(b) $\beta = \theta = 32°$

$\gamma = 90° - \phi = 22°$

$C = \beta + \gamma = 54°$

$\tan C = \dfrac{d}{50} \Rightarrow \tan 54°$

$= \dfrac{d}{50} \Rightarrow d \approx 68.82$ meters

40. $\tan 14° = \dfrac{d}{x} \Rightarrow x = d \cot 14°$

$\tan 34° = \dfrac{d}{y} = \dfrac{d}{30 - x}$

$= \dfrac{d}{30 - d\cot 14°}$

$\cot 34° = \dfrac{30 - d\cot 14°}{d}$

$d\cot 34° = 30 - d\cot 14°$

$d = \dfrac{30}{\cot 34° + \cot 14°}$

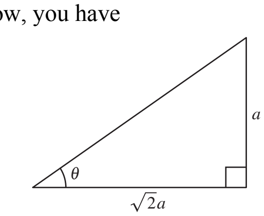
≈ 5.46 kilometers

41. The diagonal of the base has a length of $\sqrt{a^2 + a^2} = \sqrt{2}a$. Now, you have

$\tan\theta = \dfrac{a}{\sqrt{2}a} = \dfrac{1}{\sqrt{2}}$

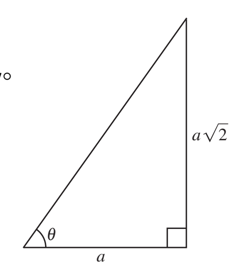
$\theta = \arctan\dfrac{1}{\sqrt{2}}$

$\theta \approx 35.3°$.



42. $\tan\theta = \dfrac{a\sqrt{2}}{a} = \sqrt{2}$

$\theta = \arctan\sqrt{2} \approx 54.7°$



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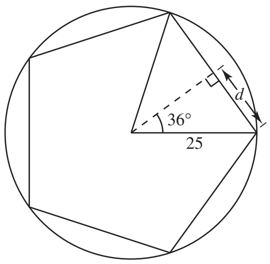
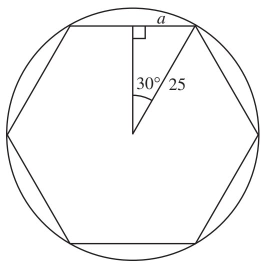
43. $\sin 36° = \dfrac{d}{25} \Rightarrow d \approx 14.69$

Length of side: $2d \approx 29.4$ inches



44.



$\sin 30° = \dfrac{a}{25}$

$a = 25 \sin 30° = 12.5$

Length of side $= 2a = 2(12.5)$

$= 25$ inches

45. Use $d = a \sin \omega t$ because $d = 0$ when $t = 0$.

Period: $\dfrac{2\pi}{\omega} = 2 \Rightarrow \omega = \pi$

So, $d = 4 \sin(\pi t)$.

46. Use $d = a \sin \omega t$ because $d = 0$ when $t = 0$.

Amplitude: $|a| = 3$

Period: $\dfrac{2\pi}{\omega} = 6 \Rightarrow \omega = \dfrac{\pi}{3}$

So, $d = 3 \sin\left(\dfrac{\pi t}{3}\right)$.

47. Use $d = a \cos \omega t$ because $d = 3$ when $t = 0$.

Period: $\dfrac{2\pi}{\omega} = 1.5 \Rightarrow \omega = \dfrac{4\pi}{3}$

So, $d = 3 \cos\left(\dfrac{4\pi}{3}t\right) = 3 \cos\left(\dfrac{4\pi t}{3}\right)$.

48. Use $d = a \cos \omega t$ because $d = 2$ when $t = 0$.

Amplitude: $|a| = 2$

Period: $\dfrac{2\pi}{\omega} = 10 \Rightarrow \omega = \dfrac{\pi}{5}$

So, $d = 2\cos\left(\dfrac{\pi t}{5}\right)$.

49. $d = a \sin \omega t$

Frequency $= \dfrac{\omega}{2\pi}$

$262 = \dfrac{\omega}{2\pi}$

$\omega = 2\pi(262) = 524\pi$

50. At $t = 0$, buoy is at its high point $\Rightarrow d = a \cos \omega t$.

Distance from high to low $= 2|a| = 3.5$

$|a| = \dfrac{7}{4}$

Returns to high point every 10 seconds:

Period: $\dfrac{2\pi}{\omega} = 10 \Rightarrow \omega = \dfrac{\pi}{5}$

$d = \dfrac{7}{4} \cos \dfrac{\pi t}{5}$

51. $d = 9 \cos \dfrac{6\pi}{5}t$

(a) Maximum displacement $=$ amplitude $= 9$

(b) Frequency $= \dfrac{\omega}{2\pi} = \dfrac{\frac{6\pi}{5}}{2\pi}$

$= \dfrac{3}{5}$ cycle per unit of time

(c) $d = 9 \cos \dfrac{6\pi}{5}(5) = 9$

(d) $9 \cos \dfrac{6\pi}{5}t = 0$

$\cos \dfrac{6\pi}{5}t = 0$

$\dfrac{6\pi}{5}t = \arccos 0$

$\dfrac{6\pi}{5}t = \dfrac{\pi}{2}$

$t = \dfrac{5}{12}$

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52. $d = \frac{1}{2}\cos 20\pi t$

(a) Maximum displacement: $|a| = \left|\frac{1}{2}\right| = \frac{1}{2}$

(b) Frequency:

$\frac{\omega}{2\pi} = \frac{20\pi}{2\pi} = 10$ cycles per unit of time

(c) $d = \frac{1}{2}\cos 100\pi \approx \frac{1}{2}$

(d) $\frac{1}{2}\cos 20\pi t = 0$

$$\cos 20\pi t = 0$$
$$20\pi t = \arccos 0$$
$$20\pi t = \frac{\pi}{2}$$
$$t = \frac{\pi}{2}\cdot\frac{1}{20\pi} = \frac{1}{40}$$

53. $d = \frac{1}{4}\sin 6\pi t$

(a) Maximum displacement = amplitude = $\frac{1}{4}$

(b) Frequency $= \frac{\omega}{2\pi} = \frac{6\pi}{2\pi}$

= 3 cycles per unit of time

(c) $d = \frac{1}{4}\sin 30\pi \approx 0$

(d) $\frac{1}{4}\sin 6\pi t = 0$

$$\sin 6\pi t = 0$$
$$6\pi t = \arcsin 0$$
$$6\pi t = \pi$$
$$t = \frac{1}{6}$$

54. $d = \frac{1}{64}\sin 792\pi t$

(a) Maximum displacement: $|a| = \left|\frac{1}{64}\right| = \frac{1}{64}$

(b) Frequency:

$\frac{\omega}{2\pi} = \frac{792\pi}{2\pi} = 396$ cycles per unit of time

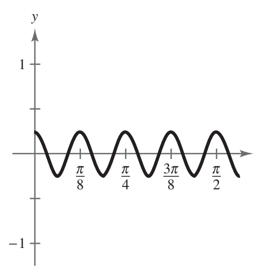
(c) $d = \frac{1}{64}\sin(3960\pi) = 0$

(d) $\frac{1}{64}\sin 792\pi t = 0$

$$\sin 792\pi t = 0$$
$$792\pi t = \arcsin 0$$
$$792\pi t = \pi$$
$$t = \frac{\pi}{792\pi} = \frac{1}{792}$$

55. $y = \frac{1}{4}\cos 16t,\ t > 0$

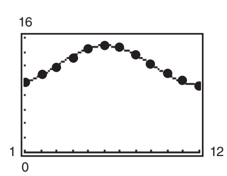
(a)



(b) Period: $\frac{2\pi}{16} = \frac{\pi}{8}$

(c) $\frac{1}{4}\cos 16t = 0$ when $16t = \frac{\pi}{2} \Rightarrow t = \frac{\pi}{32}$

56. (a)



(b) $H(t) = 12.13 + 2.77\sin\left(\frac{\pi t}{6} - 1.60\right)$

period $= \frac{2\pi}{b} = \frac{2\pi}{\frac{\pi}{6}} = 12$. Yes, the period is the number of months in a year.

(c) The amplitude is $a = |2.77| = 2.77$. The amplitude would represent the change in the maximum number of hours of sunlight. Therefore a maximum of $12.13 + 2.77 = 14.9$ hours to a minimum of $12.13 - 2.77 = 9.36$ hours.

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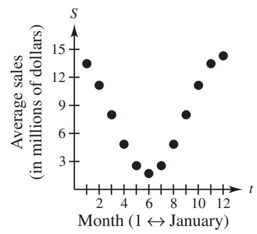
57. (a)



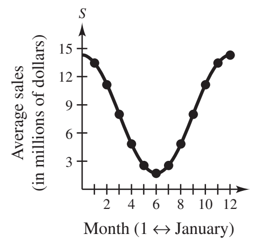
(b) $a = \frac{1}{2}(14.3 - 1.7) = 6.3$

$\frac{2\pi}{b} = 12 \Rightarrow b = \frac{\pi}{6}$

Shift: $d = 14.3 - 6.3 = 8$

$S = d + a \cos bt$

$S = 8 + 6.3 \cos\left(\frac{\pi t}{6}\right)$



Note: Another model is $S = 8 + 6.3 \sin\left(\frac{\pi t}{6} + \frac{\pi}{2}\right)$.

The model is a good fit.

(c) The period is $\frac{2\pi}{(\pi/6)} = 12$. Yes, sales of outwear are seasonal.

(d) The amplitude is the maximum displacement from average sales of $8 million.

58. (a) The maximum displacement from $d = 0$ is 4, so the amplitude is 4 centimeters**.**

(b) The period is the time for the graph to complete one cycle is 3 seconds.

(c) Because the spring is at maximum displacement when $t = 0$, the equation is of the form $d = a \cos wt$.

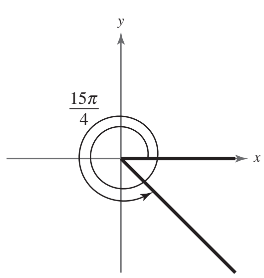
59. False. The tower isn't vertical and so the triangle formed is not a right triangle.

60. False. N 24° E means 24 degrees east of north.

Review Exercises for Chapter 4

1. $\theta = \frac{15\pi}{4}$

(a)



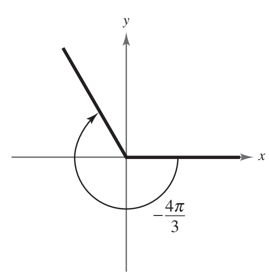
(b) Quadrant IV

(c) $\frac{15\pi}{4} - 2\pi = \frac{7\pi}{4}$

$\frac{7\pi}{4} - 2\pi = -\frac{\pi}{4}$

2. $\theta = -\frac{4\pi}{3}$

(a)



(b) Quadrant II

(c) Coterminal angles:

$-\frac{4\pi}{3} + 2\pi = \frac{2\pi}{3}$

$-\frac{4\pi}{3} - 2\pi = -\frac{10\pi}{3}$

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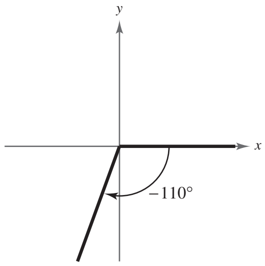
3. $\theta = -110°$

(a)



(b) Quadrant III

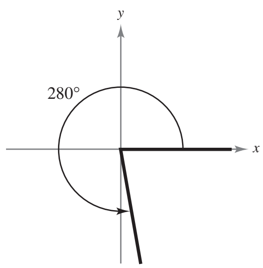
(c) Coterminal angles:

$-110° + 360° = 250°$

$-110° - 360° = -470°$

4. $\theta = 280°$

(a)



(b) Quadrant IV

(c) $280° + 360° = 640°$

$280° - 360° = -80°$

5. $450° = 450° \cdot \dfrac{\pi \text{ rad}}{180°} = \dfrac{5\pi}{2} \approx 7.854$ radians

6. $190° = 190° \cdot \dfrac{\pi \text{ rad}}{180°} \approx 3.316$ radians

7. $-16° = -16° \cdot \dfrac{\pi \text{ rad}}{180°} \approx -0.279$ radians

8. $-112° = -112° \cdot \dfrac{\pi \text{ rad}}{180°} = -\dfrac{28\pi}{45} \approx -1.955$ radians

9. $\dfrac{3\pi}{10} = \dfrac{3\pi}{10} \cdot \dfrac{180°}{\pi \text{ rad}} = 54°$

10. $-\dfrac{11\pi \text{ rad}}{6} \cdot \dfrac{180°}{\pi \text{ rad}} = -330°$

11. $-3.5 \text{ rad} = -3.5 \text{ rad} \cdot \dfrac{180°}{\pi \text{ rad}} \approx -200.535°$

12. $5.7 \text{ rad} \cdot \dfrac{180°}{\pi \text{ rad}} \approx 326.586°$

13. $198.4° = 198° + 0.4(60)' = 198° 24'$

14. $-5.96° = -\left(5° + 0.96(60)'\right)$

$= -\left(5° 57' + 0.6(60)'\right)$

$= -5° 57' 36''$

15. $138° = \dfrac{138\pi}{180} = \dfrac{23\pi}{30}$ radians

$s = r\theta = 20\left(\dfrac{23\pi}{30}\right) \approx 48.17$ inches

16. (a) Angular speed $= \dfrac{\left(33\frac{1}{3}\right)(2\pi) \text{ radians}}{1 \text{ minute}}$

$= 66\frac{2}{3}\pi$ radians per minute

(b) Linear speed $= \dfrac{6\left(66\frac{2}{3}\pi\right) \text{ inches}}{1 \text{ minute}}$

$= 400\pi$ inches per minute

17. $150° = \dfrac{150\pi}{180} = \dfrac{5\pi}{6}$ radians

$A = \dfrac{1}{2}r^2\theta = \dfrac{1}{2}(20)^2\left(\dfrac{5\pi}{6}\right) = \dfrac{500\pi}{3} \approx 523.6$ square inches

18. $A = \dfrac{1}{2}\theta r^2 = \dfrac{1}{2}\left(\dfrac{2\pi}{3}\right)(7.5)^2 = \dfrac{75\pi}{4} \approx 58.9$ square millimeters

19. $t = \dfrac{2\pi}{3}$ corresponds to the point $\left(-\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right)$.

20. $t = \dfrac{7\pi}{4}$ corresponds to the point

$(x, y) = \left(\dfrac{\sqrt{2}}{2}, -\dfrac{\sqrt{2}}{2}\right)$.

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21. $t = \frac{7\pi}{6}$ corresponds to the point $(x, y) = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$.

22. $t = -\frac{4\pi}{3}$ corresponds to the point $(x, y) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

23. $t = \frac{3\pi}{4}$ corresponds to the point $(x, y) = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

$\sin\frac{3\pi}{4} = y = \frac{\sqrt{2}}{2}$ $\qquad$ $\csc\frac{3\pi}{4} = \frac{1}{y} = \sqrt{2}$

$\cos\frac{3\pi}{4} = x = -\frac{\sqrt{2}}{2}$ $\qquad$ $\sec\frac{3\pi}{4} = \frac{1}{x} = -\sqrt{2}$

$\tan\frac{3\pi}{4} = \frac{y}{x} = -1$ $\qquad$ $\cot\frac{3\pi}{4} = \frac{x}{y} = -1$

24. $t = -\frac{2\pi}{3}$ corresponds to the point $(x, y) = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

$\sin\left(-\frac{2\pi}{3}\right) = y = -\frac{\sqrt{3}}{2}$ $\qquad$ $\csc\left(-\frac{2\pi}{3}\right) = \frac{1}{y} = -\frac{2\sqrt{3}}{3}$

$\cos\left(-\frac{2\pi}{3}\right) = x = -\frac{1}{2}$ $\qquad$ $\sec\left(-\frac{2\pi}{3}\right) = \frac{1}{x} = -2$

$\tan\left(-\frac{2\pi}{3}\right) = \frac{y}{x} = \sqrt{3}$ $\qquad$ $\cot\left(-\frac{2\pi}{3}\right) = \frac{x}{y} = \frac{\sqrt{3}}{3}$

25. $\sin\frac{11\pi}{4} = \sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$

26. $\cos 4\pi = \cos 0 = 1$

27. $\cos\left(-\frac{17\pi}{6}\right) = \cos\frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$

28. $\sin\left(-\frac{13\pi}{3}\right) = \sin\frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$

29. $\sec\left(\frac{12\pi}{5}\right) = \frac{1}{\cos\left(\frac{12\pi}{5}\right)} \approx 3.2361$

30. $\sin\left(-\frac{\pi}{9}\right) \approx -0.3420$

31. $\tan 33 \approx -75.3130$

32. $\csc 10.5 = \frac{1}{\sin 10.5} \approx -1.1368$

33. opp $= 4$, adj $= 5$, hyp $= \sqrt{4^2 + 5^2} = \sqrt{41}$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4}{\sqrt{41}} = \frac{4\sqrt{41}}{41}$ $\qquad$ $\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{41}}{4}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{5}{\sqrt{41}} = \frac{5\sqrt{41}}{41}$ $\qquad$ $\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{41}}{5}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{5}$ $\qquad$ $\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{5}{4}$

34. adj $= 4$, hyp $= 8$, opp $= \sqrt{8^2 - 4^2} = \sqrt{48} = 4\sqrt{3}$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$ $\qquad$ $\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{8}{4\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{4}{8} = \frac{1}{2}$ $\qquad$ $\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{8}{4} = 2$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4\sqrt{3}}{4} = \sqrt{3}$ $\qquad$ $\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{4}{4\sqrt{3}} = \frac{\sqrt{3}}{3}$

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35. $\tan 33° \approx 0.6494$

36. $\sec 79.3° = \dfrac{1}{\cos 79.3°} \approx 5.3860$

37. $\cot 15° 14' = \dfrac{1}{\tan\left(15 + \frac{14}{60}\right)}$

≈ 3.6722

38. $\cos 78° 11' 58'' = \cos\left(78 + \frac{11}{60} + \frac{58}{3600}\right)^{\circ}$

≈ 0.2045

39. $\sin\theta = \dfrac{1}{3}$

(a) $\csc\theta = \dfrac{1}{\sin\theta} = 3$

(b) $\sin^2\theta + \cos^2\theta = 1$

$\left(\dfrac{1}{3}\right)^2 + \cos^2\theta = 1$

$\cos^2\theta = 1 - \dfrac{1}{9}$

$\cos^2\theta = \dfrac{8}{9}$

$\cos\theta = \sqrt{\dfrac{8}{9}}$

$\cos\theta = \dfrac{2\sqrt{2}}{3}$

(c) $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{3}{2\sqrt{2}} = \dfrac{3\sqrt{2}}{4}$

(d) $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{1/3}{\left(2\sqrt{2}\right)/3} = \dfrac{1}{2\sqrt{2}} = \dfrac{\sqrt{2}}{4}$

40. $\csc\theta = 5$

(a) $\sin\theta = \dfrac{1}{\csc\theta} = \dfrac{1}{5}$

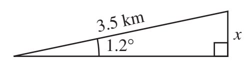
(b) $\cot\theta = \sqrt{\csc^2\theta - 1} = \sqrt{25 - 1} = 2\sqrt{6}$

(c) $\tan\theta = \dfrac{1}{\cot\theta} = \dfrac{1}{2\sqrt{6}} = \dfrac{\sqrt{6}}{12}$

(d) $\sec\left(90° - \theta\right) = \csc\theta = 5$

41. $\sin 1.2° = \dfrac{x}{3.5}$

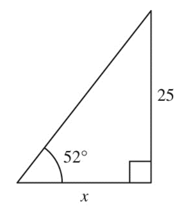
$x = 3.5 \sin 1.2° \approx 0.0733$ kilometer or 73.3 meters



Not drawn to scale

42. $\tan 52° = \dfrac{25}{x}$

$x = \dfrac{25}{\tan 52°} \approx 19.5$ feet



43. $x = 12, y = 16, r = \sqrt{144 + 256} = \sqrt{400} = 20$

$\sin\theta = \dfrac{y}{r} = \dfrac{4}{5}$ $\quad\csc\theta = \dfrac{r}{y} = \dfrac{5}{4}$

$\cos\theta = \dfrac{x}{r} = \dfrac{3}{5}$ $\quad\sec\theta = \dfrac{r}{x} = \dfrac{5}{3}$

$\tan\theta = \dfrac{y}{x} = \dfrac{4}{3}$ $\quad\cot\theta = \dfrac{x}{y} = \dfrac{3}{4}$

44. $x = 3, y = -4$

$r = \sqrt{3^2 + (-4)^2} = 5$

$\sin\theta = \dfrac{y}{r} = -\dfrac{4}{5}$ $\quad\csc\theta = \dfrac{r}{y} = -\dfrac{5}{4}$

$\cos\theta = \dfrac{x}{r} = \dfrac{3}{5}$ $\quad\sec\theta = \dfrac{r}{x} = \dfrac{5}{3}$

$\tan\theta = \dfrac{y}{x} = -\dfrac{4}{3}$ $\quad\cot\theta = \dfrac{x}{y} = -\dfrac{3}{4}$

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45. $x = 0.3, y = 0.4$

$$r = \sqrt{(0.3)^2 + (0.4)^2} = 0.5$$

$$\sin\theta = \frac{y}{r} = \frac{0.4}{0.5} = \frac{4}{5} = 0.8 \qquad \csc\theta = \frac{r}{y} = \frac{0.5}{0.4} = \frac{5}{4} = 1.25$$

$$\cos\theta = \frac{x}{r} = \frac{0.3}{0.5} = \frac{3}{5} = 0.6 \qquad \sec\theta = \frac{r}{x} = \frac{0.5}{0.3} = \frac{5}{3} \approx 1.67$$

$$\tan\theta = \frac{y}{x} = \frac{0.4}{0.3} = \frac{4}{3} \approx 1.33 \qquad \cot\theta = \frac{x}{y} = \frac{0.3}{0.4} = \frac{3}{4} = 0.75$$

46. $x = -\frac{10}{3}, y = -\frac{2}{3}$

$$r = \sqrt{\left(-\frac{10}{3}\right)^2 + \left(-\frac{2}{3}\right)^2} = \frac{2\sqrt{26}}{3}$$

$$\sin\theta = \frac{y}{r} = \frac{-2/3}{\left(2\sqrt{26}\right)/3} = -\frac{\sqrt{26}}{26} \qquad \csc\theta = \frac{r}{y} = \frac{\left(2\sqrt{26}\right)/3}{-2/3} = -\sqrt{26}$$

$$\cos\theta = \frac{x}{r} = \frac{-10/3}{\left(2\sqrt{26}\right)/3} = -\frac{5\sqrt{26}}{26} \qquad \sec\theta = \frac{r}{x} = \frac{\left(2\sqrt{26}\right)/3}{-10/3} = -\frac{\sqrt{26}}{5}$$

$$\tan\theta = \frac{y}{x} = \frac{-2/3}{-10/3} = \frac{1}{5} \qquad \cot\theta = \frac{x}{y} = \frac{-10/3}{-2/3} = 5$$

47. $\sec\theta = \frac{6}{5}, \tan\theta < 0 \Rightarrow \theta$ is in Quadrant IV.

$$r = 6, x = 5, y = -\sqrt{36 - 25} = -\sqrt{11}$$

$$\sin\theta = \frac{y}{r} = -\frac{\sqrt{11}}{6}$$

$$\cos\theta = \frac{x}{r} = \frac{5}{6}$$

$$\tan\theta = \frac{y}{x} = -\frac{\sqrt{11}}{5}$$

$$\csc\theta = \frac{r}{y} = -\frac{6\sqrt{11}}{11}$$

$$\cot\theta = -\frac{5\sqrt{11}}{11}$$

48. $\csc\theta = \frac{3}{2}, \cos\theta < 0 \Rightarrow \theta$ is in Quadrant II.

$$\sin\theta = \frac{1}{\csc\theta} = \frac{2}{3}$$

$$\cos\theta = -\sqrt{1 - \sin^2\theta} = -\frac{\sqrt{5}}{3}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = -\frac{2\sqrt{5}}{5}$$

$$\sec\theta = \frac{1}{\cos\theta} = -\frac{3\sqrt{5}}{5}$$

$$\cot\theta = \frac{1}{\tan\theta} = -\frac{\sqrt{5}}{2}$$

49. $\cos\theta = \frac{x}{r} = \frac{-2}{5} \Rightarrow y^2 = 21$

$\sin\theta > 0 \Rightarrow \theta$ is in Quadrant II $\Rightarrow y = \sqrt{21}$

$$\sin\theta = \frac{y}{r} = \frac{\sqrt{21}}{5}$$

$$\tan\theta = \frac{y}{x} = -\frac{\sqrt{21}}{2}$$

$$\csc\theta = \frac{r}{y} = \frac{5}{\sqrt{21}} = \frac{5\sqrt{21}}{21}$$

$$\sec\theta = \frac{r}{x} = \frac{5}{-2} = -\frac{5}{2}$$

$$\cot\theta = \frac{x}{y} = \frac{-2}{\sqrt{21}} = -\frac{2\sqrt{21}}{21}$$

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50. $\sin\theta = -\frac{2}{4} = -\frac{1}{2}, \cos\theta > 0$

θ is in Quadrant IV.

$$\csc\theta = \frac{1}{\sin\theta} = -2$$

$$\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \left(\frac{1}{4}\right)} = \frac{\sqrt{3}}{2}$$

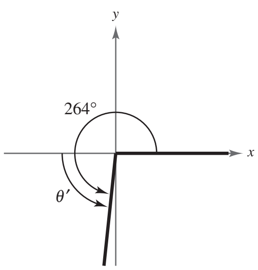
$$\sec\theta = \frac{1}{\cos\theta} = \frac{2\sqrt{3}}{3}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = -\frac{\sqrt{3}}{3}$$

$$\cot\theta = \frac{1}{\tan\theta} = -\sqrt{3}$$

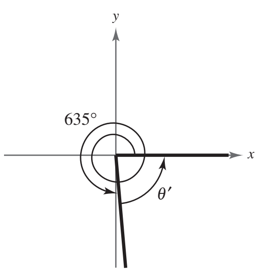
51. $\theta = 264°$

$= 264° - 180° = 84°$



52. $\theta = 635° = 720° - 85°$

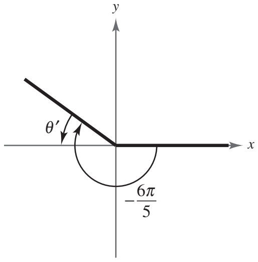
$\theta' = 85°$



53. $\theta = -\frac{6\pi}{5}$

$$-\frac{6\pi}{5} + 2\pi = \frac{4\pi}{5}$$

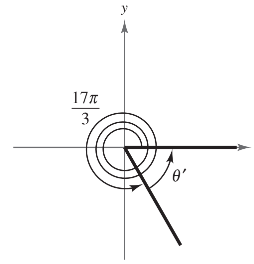
$$\theta' = \pi - \frac{4\pi}{5} = \frac{\pi}{5}$$



54. $\theta = \frac{17\pi}{3} = \frac{18\pi}{3} - \frac{\pi}{3}$

$$= 6\pi - \frac{\pi}{3}$$

$$\theta' = \frac{\pi}{3}$$



55. $\sin(-150°) = -\frac{1}{2}$

$$\cos(-150°) = -\frac{\sqrt{3}}{2}$$

$$\tan(-150°) = \frac{-1/2}{-\sqrt{3}/2} = \frac{\sqrt{3}}{3}$$

56. $\sin 495° = \sin 45° = \frac{\sqrt{2}}{2}$

$$\cos 495° = -\cos 45° = -\frac{\sqrt{2}}{2}$$

$$\tan 495° = -\tan 45° = -1$$

57. $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$$\cos\frac{\pi}{3} = \frac{1}{2}$$

$$\tan\frac{\pi}{3} = \sqrt{3}$$

58. $\sin\left(-\frac{5\pi}{4}\right) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$$\cos\left(-\frac{5\pi}{4}\right) = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$

$$\tan\left(-\frac{5\pi}{4}\right) = -\tan\frac{\pi}{4} = \frac{2}{-\sqrt{2}/2} = -1$$

59. $\sin 106° \approx 0.9613$

60. $\tan 37° \approx 0.7536$

61. $\tan\left(-\frac{17\pi}{15}\right) \approx -0.4452$

62. $\cos\left(-\frac{25\pi}{7}\right) \approx 0.2225$

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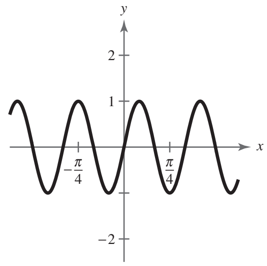
63. $y = \sin 6x$

Amplitude: 1

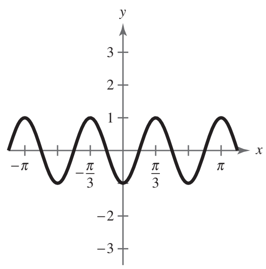
Period: $\dfrac{2\pi}{6} = \dfrac{\pi}{3}$



64. $y = -\cos 3x$

Amplitude: 1

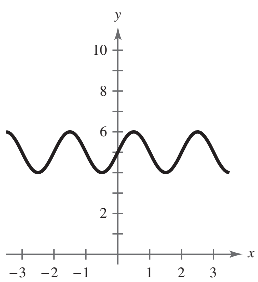
Period: $\dfrac{2\pi}{3}$



65. $y = 5 + \sin \pi x$

Amplitude: 1

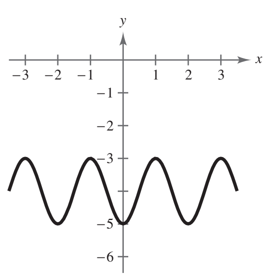
Period: $\dfrac{2\pi}{\pi} = 2$



66. $y = -4 - \cos \pi x$

Amplitude: 1

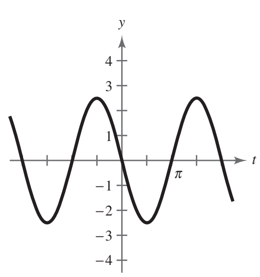
Period: $\dfrac{2\pi}{\pi} = 2$



67. $g(t) = \frac{5}{2}\sin(t - \pi)$

Amplitude: $\frac{5}{2}$

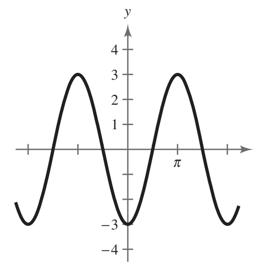
Period: 2π



68. $g(t) = 3\cos(t + \pi)$

Amplitude: 3

Period: 2π



69. $y = a \sin bx$

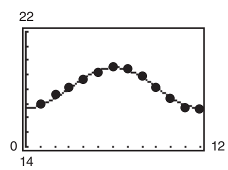
(a) $a = 2,$

$\dfrac{2\pi}{b} = \dfrac{1}{264} \Rightarrow b = 528\pi$

$y = 2\sin 528\pi x$

(b) $f = \dfrac{1}{1/264} = 264$ cycles per second

70. (a) $S(t) = 18.10 - 1.41\sin\left(\dfrac{\pi t}{6} + 1.55\right)$



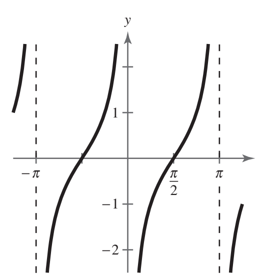
(b) Period $= \dfrac{2\pi}{\pi/6} = (2)(6) = 12$

12 months = 1 year, so this is expected.

(c) Amplitude: 1.41

The amplitude represents the maximum change in time of sunset from the average time $(d = 18.10)$.

71. $f(t) = \tan\left(t + \dfrac{\pi}{2}\right)$



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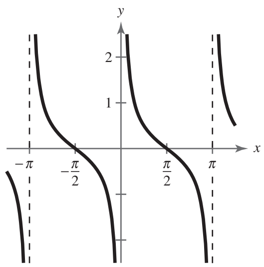
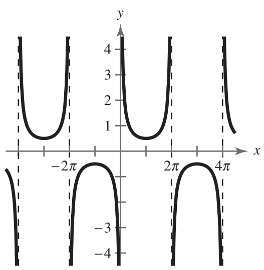
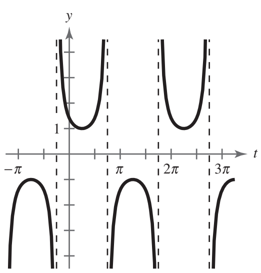
72. $f(x) = \frac{1}{2}\cot x$



73. $f(x) = \dfrac{1}{2}\csc\dfrac{x}{2}$



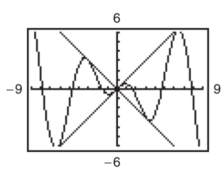
74. $h(t) = \sec\left(t - \dfrac{\pi}{4}\right)$



75. $f(x) = x\cos x$

Damping factor: x

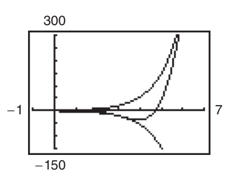
As $x \to \infty$, $f(x)$ oscillates.



76. $g(x) = e^x \cos x$

Damping factor: e^x

As $x \to \infty$, $f(x)$ oscillates.



77. $\arcsin(-1) = -\dfrac{\pi}{2}$

78. $\cos^{-1}(1) = 0$

79. $\operatorname{arccot}\sqrt{3} = \dfrac{\pi}{6}$

80. $\operatorname{arcsec}\left(-\sqrt{2}\right) = \dfrac{3\pi}{4}$

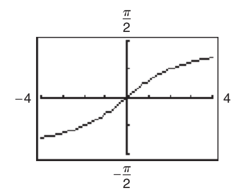
81. $\tan^{-1}(-1.3) \approx -0.92$ radian

82. $\arccos 0.372 \approx 1.19$ radians

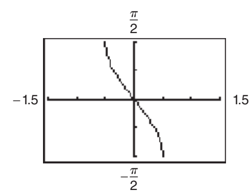
83. $\operatorname{arccot} 15.5 = \arctan\dfrac{1}{15.5} \approx 0.06$

84. $\operatorname{arccsc}(-4.03) = \arcsin\left(-\dfrac{1}{4.03}\right) \approx -0.25$

85. $f(x) = \arctan\left(\dfrac{x}{2}\right) = \tan^{-1}\left(\dfrac{x}{2}\right)$

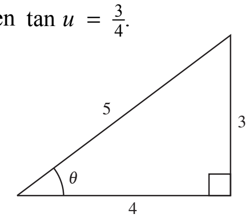


86. $f(x) = -\arcsin 2x$



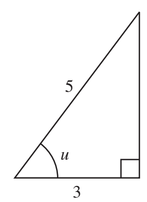
87. Let $u = \arctan\frac{3}{4}$ then $\tan u = \frac{3}{4}$.

$\cos\left(\arctan\frac{3}{4}\right) = \frac{4}{5}$



88. Let $u = \arccos\frac{3}{5}$.

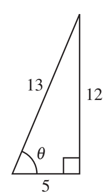
$\tan\left(\arccos\frac{3}{5}\right) = \tan u = \frac{4}{3}$



89. Let $u = \arctan\frac{12}{5}$

then $\tan u = \frac{12}{5}$.

$\sec\left(\arctan\frac{12}{5}\right) = \frac{13}{5}$



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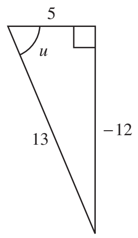
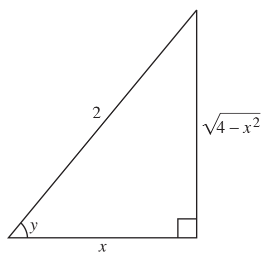
90. Let $u = \arcsin\left(-\frac{12}{13}\right)$.

$\cot\left[\arcsin\left(-\frac{12}{13}\right)\right] = \cot u = -\frac{5}{12}$



91. Let $y = \arccos\left(\dfrac{x}{2}\right)$. Then

$\cos y = \dfrac{x}{2}$ and $\tan y = \tan\left(\arccos\left(\dfrac{x}{2}\right)\right) = \dfrac{\sqrt{4 - x^2}}{x}$.



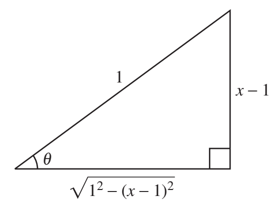
92. $\sec(\arcsin(x - 1))$

$\theta = \arcsin(x - 1) \Rightarrow -\dfrac{\pi}{2} \le \theta \le \dfrac{\pi}{2}$

$\sin\theta = x - 1$

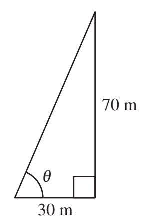
$\cos\theta = \sqrt{1^2 - (x - 1)^2}$

$\sec\theta = \dfrac{1}{\sqrt{1 - (x - 1)^2}}$



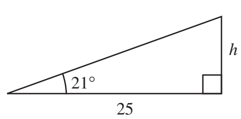
93. $\tan\theta = \frac{70}{30}$

$\theta = \arctan\left(\frac{70}{30}\right) \approx 66.8°$



94. $\tan 21° = \dfrac{h}{25}$

$h = 25 \tan 21° \approx 9.6$ feet



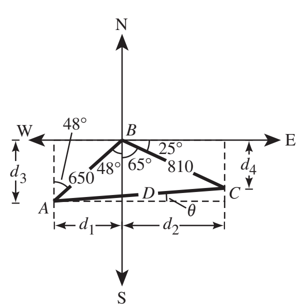
95. $\left.\begin{array}{l} \sin 48° = \dfrac{d_1}{650} \Rightarrow d_1 \approx 483 \\ \cos 25° = \dfrac{d_2}{810} \Rightarrow d_2 \approx 734 \end{array}\right\} d_1 + d_2 \approx 1217$

$\left.\begin{array}{l} \cos 48° = \dfrac{d_3}{650} \Rightarrow d_3 \approx 435 \\ \sin 25° = \dfrac{d_4}{810} \Rightarrow d_4 \approx 342 \end{array}\right\} d_3 - d_4 \approx 93$

$\tan\theta \approx \dfrac{93}{1217} \Rightarrow \theta \approx 4.4°$

$\sec 4.4° \approx \dfrac{D}{1217} \Rightarrow D \approx 1217 \sec 4.4° \approx 1221$

The distance is 1221 miles and the bearing is 85.6°.



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96. Amplitude: $\frac{1.5}{2} = 0.75$ inches

Period: 3 seconds

$d = a \cos bt$

$a = 0.75$

$b = \frac{2\pi}{3}$

$d = 0.75 \cos\left(\frac{2\pi t}{3}\right)$

97. False. For each θ there corresponds exactly one value of y.

98. False. The range of arctan is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, so

$\arctan(-1) = -\frac{\pi}{4}$.

99. $f(\theta) = \sec\theta$ is undefined at the zeros of $g(\theta) = \cos\theta$ because $\sec\theta = \frac{1}{\cos\theta}$.

100. (a)

θ	0.1	0.4	0.7	1.0	1.3
$\tan\left(\theta - \frac{\pi}{2}\right)$	−9.9666	−2.3652	−1.1872	−0.6421	−0.2776
$-\cot\theta$	−9.9666	−2.3652	−1.1872	−0.6421	−0.2776

(b) $\tan\left(\theta - \frac{\pi}{2}\right) = -\cot\theta$

101. The ranges of the other four trigonometric functions are not bounded.

For $y = \tan x$ and $y = \cot x$, the range is $(-\infty, \infty)$.

For $y = \sec x$ and $y = \csc x$,

the range is $(-\infty, -1] \cup [1, \infty)$.

102. $y = Ae^{-kt} \cos bt = \frac{1}{5}e^{-t/10} \cos 6t$

(a) A is changed from $\frac{1}{5}$ to $\frac{1}{3}$: The displacement is increased.

(b) k is changed from $\frac{1}{10}$ to $\frac{1}{3}$: The friction damps the oscillations more rapidly.

(c) b is changed from 6 to 9: The frequency of oscillation is increased.

Problem Solving for Chapter 4

1. (a) $8{:}57 - 6{:}45 = 2$ hours 12 minutes $= 132$ minutes

$\frac{132}{48} = \frac{11}{4}$ revolutions

$\theta = \left(\frac{11}{4}\right)(2\pi) = \frac{11\pi}{2}$ radians or 990°

(b) $s = r\theta = 47.25(5.5\pi) \approx 816.42$ feet

2. Gear 1: $\frac{24}{32}(360°) = 270° = \frac{3\pi}{2}$ radians

Gear 2: $\frac{24}{26}(360°) \approx 332.3° \approx \frac{24\pi}{13}$ radians

Gear 3: $\frac{24}{22}(360°) \approx 392.7° \approx \frac{24\pi}{11}$ radians

Gear 4: $\frac{40}{32}(360°) = 450° = \frac{5\pi}{2}$ radians

Gear 5: $\frac{24}{19}(360°) \approx 454.7° \approx \frac{48\pi}{19}$ radians

3. If you alter the model so that $h = 1$ when $t = 0$, you can use either a sine or a cosine model.

$a = \frac{1}{2}[\max - \min] = \frac{1}{2}[101 - 1] = 50$

$d = \frac{1}{2}[\max + \min] = \frac{1}{2}[101 + 1] = 51$

$b = 8\pi$

Cosine model: $h = 51 - 50\cos(8\pi t)$

Sine model: $h = 51 - 50\sin\left(8\pi t + \frac{\pi}{2}\right)$

Notice that you needed the horizontal shift so that the sine value was one when $t = 0$.

Another model would be: $h = 51 + 50\sin\left(8\pi t + \frac{3\pi}{2}\right)$

Here you wanted the sine value to be 1 when $t = 0$.

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4. Given that f is periodic with period $c \rightarrow f(t + c) = f(t)$

(a) $f(t - 2c + 2c) = f(t)$

True. $f(t - 2c) = f(t)$

(b) $f\left(t + \frac{1}{2}c\right)$ has a period of c, while $f\left(\frac{1}{2}t\right)$ has a period of $\frac{c}{\frac{1}{2}} = 2c$.

False. $f\left(t + \frac{1}{2}c\right) \neq f\left(\frac{1}{2}t\right)$.

(c) $f\left[\frac{1}{2}(t + c)\right] = f\left(\frac{1}{2}t + \frac{1}{2}c\right)$ and $f\left(\frac{1}{2}t\right)$ both have a period of $\frac{c}{\frac{1}{2}} = 2c$, but

$f\left[\frac{1}{2}(t + c)\right] = f\left(\frac{1}{2}t + \frac{1}{2}c - 2c\right) \neq f\left(\frac{1}{2}t\right)$

False. $f\left[\frac{1}{2}(t + c)\right] \neq f\left(\frac{1}{2}t\right)$.

(d) $f\left[\frac{1}{2}(t + 4c)\right] = f\left(\frac{1}{2}t + 2c\right)$ and $f\left(\frac{1}{2}t\right)$ both have a period of $\frac{c}{\frac{1}{2}} = 2c$ and

$f\left[\frac{1}{2}(t + 4c)\right] = f\left(\frac{1}{2}t + 2c - 2c\right) = f\left(\frac{1}{2}t\right)$

True. $f\left[\frac{1}{2}(t + 4c)\right] = f\left(\frac{1}{2}t\right)$.

5. (a) $\sin 39° = \frac{3000}{d}$

$d = \frac{3000}{\sin 39°} \approx 4767$ feet

(b) $\tan 39° = \frac{3000}{x}$

$x = \frac{3000}{\tan 39°} \approx 3705$ feet

(c) $\tan 63° = \frac{w + 3705}{3000}$

$3000 \tan 63° = w + 3705$

$w = 3000 \tan 63° - 3705 \approx 2183$ feet

6. (a) ΔABC, ΔADE, and ΔAFG are all similar triangles because they all have the same angles. $\angle A$ is part of all three triangles and $\angle C = \angle E = \angle G = 90°$. So, $\angle B = \angle D = \angle F$.

(b) Because the triangles are similar, the ratios of corresponding sides are equal.

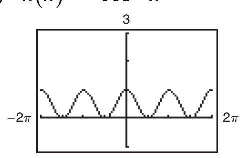
$\frac{BC}{AB} = \frac{DE}{AD} = \frac{FG}{AF}$

(c) Because the ratios: $\frac{\text{opp}}{\text{hyp}} = \frac{BC}{AB} = \frac{DE}{AD} = \frac{FG}{AF} = \sin A$ it does not matter which triangle is used to calculate $\sin A$.

Any triangle similar to these three triangles could be used to find $\sin A$. The value of $\sin A$ would not change.

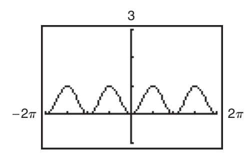
(d) Because the values of all six trigonometric functions can be found by taking the ratios of the sides of a right triangle, similar triangles would yield the same values.

7. (a) $h(x) = \cos^2 x$



h is even.

(b) $h(x) = \sin^2 x$



h is even.

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8. Given: f is an even function and g is an odd function.

(a) $h(x) = [f(x)]^2$

$h(-x) = [f(-x)]^2$

$= [f(x)]^2$ since f is even

$= h(x)$

So, h is an even function.

(b) $h(x) = [g(x)]^2$

$h(-x) = [g(-x)]^2$

$= [-g(x)]^2$ since g is odd

$= [g(x)]^2$

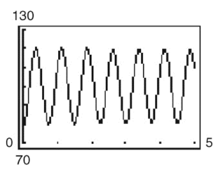
$= h(x)$

So, h is an even function.

Conjecture: The square of either an even function or an odd function is an even function.

9. $P = 100 - 20\cos\left(\frac{8\pi}{3}t\right)$

(a)



(b) Period $= \frac{2\pi}{8\pi/3} = \frac{6}{8} = \frac{3}{4}$ sec

This is the time between heartbeats.

(c) Amplitude: 20

The blood pressure ranges between $100 - 20 = 80$ and $100 + 20 = 120$.

(d) Pulse rate $= \frac{60 \text{ sec/min}}{\frac{3}{4} \text{ sec/beat}} = 80$ beats/min

(e) Period $= \frac{60}{64} = \frac{15}{16}$ sec

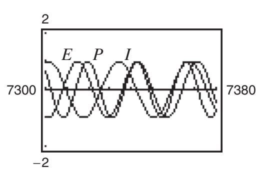
$64 = \frac{60}{2\pi/b} \Rightarrow b = \frac{64}{60} \cdot 2\pi = \frac{32}{15}\pi$

10. Physical (23 days): $P = \sin\frac{2\pi t}{23}, t \geq 0$

Emotional (28 days): $E = \sin\frac{2\pi t}{28}, t \geq 0$

Intellectual (33 days): $I = \sin\frac{2\pi t}{33}, t \geq 0$

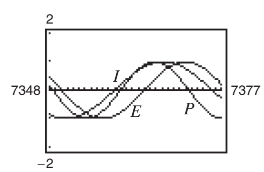
(a)



(b) Number of days since birth until September 1, 2008:

$t = \underbrace{365 \times 20}_{\text{20 years}} + \underbrace{5}_{\text{leap year}} + \underbrace{11}_{\text{remaining July days}} + \underbrace{31}_{\text{August days}} + \underbrace{1}_{\text{days in September}}$

$t = 7348$



All three drop early in the month, then peak toward the middle of the month, and drop again toward the latter part of the month.

(c) For September 22, 2008, use $t = 7369$.

$P \approx 0.631$

$E \approx 0.901$

$I \approx 0.945$

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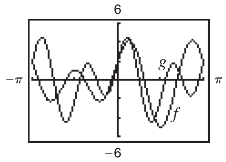
11. $f(x) = 2\cos 2x + 3\sin 3x$

$g(x) = 2\cos 2x + 3\sin 4x$

(a)



(b) The period of $f(x)$ is 2π.

The period of $g(x)$ is π.

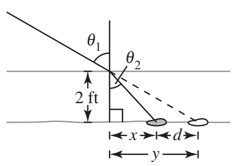
(c) $h(x) = A\cos\alpha x + B\sin\beta x$ is periodic because the sine and cosine functions are periodic.

12. (a) Both graphs have a period of 2 and intersect when $x = 5.35$. They should also intersect when $x = 5.35 - 2 = 3.35$ and $x = 5.35 + 2 = 7.35$.

(b) The graphs intersect when $x = 5.35 - 3(2) = -0.65$.

(c) Because $13.35 = 5.35 + 4(2)$ and $-4.65 = 5.35 - 5(2)$ the graphs will intersect again at these values. So, $f(13.35) = g(-4.65)$.

13.



(a) $\dfrac{\sin\theta_1}{\sin\theta_2} = 1.333$

$\sin\theta_2 = \dfrac{\sin\theta_1}{1.333} = \dfrac{\sin 60^\circ}{1.333} \approx 0.6497$

$\theta_2 = 40.5^\circ$

(b) $\tan\theta_2 = \dfrac{x}{2} \Rightarrow x = 2\tan 40.52^\circ \approx 1.71$ feet

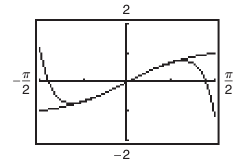
$\tan\theta_1 = \dfrac{y}{2} \Rightarrow y = 2\tan 60^\circ \approx 3.46$ feet

(c) $d = y - x = 3.46 - 1.71 = 1.75$ feet

(d) As you move closer to the rock, θ_1 decreases, which causes y to decrease, which in turn causes d to decrease.

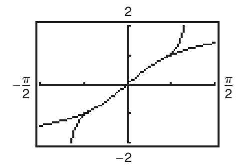
14. $\arctan x \approx x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7}$

(a)



The graphs are nearly the same for $-1 \le x \le 1$.

(b)



The accuracy of the approximation improved slightly by adding the next term $(x^9/9)$.

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Practice Test for Chapter 4

1. Express 350° in radian measure.

2. Express $(5\pi)/9$ in degree measure.

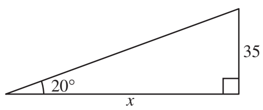
3. Convert $135° 14' 12''$ to decimal form.

4. Convert $-22.569°$ to $D° M' S''$ form.

5. If $\cos\theta = \frac{2}{3}$, use the trigonometric identities to find $\tan\theta$.

6. Find θ given $\sin\theta = 0.9063$.

7. Solve for x in the figure below.



8. Find the magnitude of the reference angle for $\theta = (6\pi)/5$.

9. Evaluate $\csc 3.92$.

10. Find $\sec\theta$ given that θ lies in Quadrant III and $\tan\theta = 6$.

11. Graph $y = 3\sin\frac{x}{2}$.

12. Graph $y = -2\cos(x - \pi)$.

13. Graph $y = \tan 2x$.

14. Graph $y = -\csc\left(x + \frac{\pi}{4}\right)$.

15. Graph $y = 2x + \sin x$, using a graphing calculator.

16. Graph $y = 3x\cos x$, using a graphing calculator.

17. Evaluate $\arcsin 1$.

18. Evaluate $\arctan(-3)$.

19. Evaluate $\sin\left(\arccos\frac{4}{\sqrt{35}}\right)$.

20. Write an algebraic expression for $\cos\left(\arcsin\frac{x}{4}\right)$.

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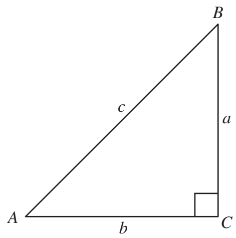
For Exercises 21–23, solve the right triangle.



21. $A = 40°, c = 12$

22. $B = 6.84°, a = 21.3$

23. $a = 5, b = 9$

24. A 20-foot ladder leans against the side of a barn. Find the height of the top of the ladder if the angle of elevation of the ladder is 67°.

25. An observer in a lighthouse 250 feet above sea level spots a ship off the shore. If the angle of depression to the ship is 5°, how far out is the ship?

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