CHAPTER 3
Exponential and Logarithmic Functions

C H A P T E R 3
Exponential and Logarithmic Functions

Section 3.1 Exponential Functions and Their Graphs

1. algebraic

2. transcendental

3. One-to-One

4. natural exponential; natural

5. $A = P\left(1 + \frac{r}{n}\right)^{nt}$

6. $A = Pe^{rt}$

7. $f(1.4) = (0.9)^{1.4} \approx 0.863$

8. $f(-\pi) = 4.7^{-\pi} \approx 0.008$

9. $f\left(\frac{2}{5}\right) = 3^{2/5} \approx 1.552$

10. $f\left(\frac{3}{10}\right) = \left(\frac{2}{3}\right)^{5\left(\frac{3}{10}\right)} \approx 0.544$

11. $f(-1.5) = 5000\left(2^{-1.5}\right)$
≈ 1767.767

12. $f(24) = 200(1.2)^{12\cdot 24}$
$\approx 1.274 \times 10^{25}$

13. $f(x) = 2^x$

Increasing
Asymptote: $y = 0$

Intercept: $(0, 1)$

Matches graph (d).

14. $f(x) = 2^x + 1$

Increasing
Asymptote: $y = 1$

Intercept: $(0, 2)$

Matches graph (c).

15. $f(x) = 2^{-x}$

Decreasing
Asymptote: $y = 0$

Intercept: $(0, 1)$

Matches graph (a).

16. $f(x) = 2^{x-2}$

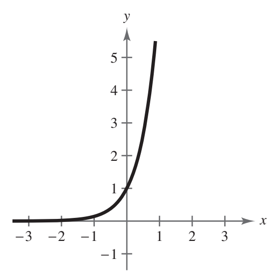
Increasing
Asymptote: $y = 0$

Intercept: $\left(0, \frac{1}{4}\right)$

Matches graph (b).

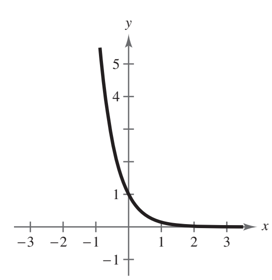
17. $f(x) = 7^x$

x	-2	-1	0	1	2
$f(x)$	0.020	0.143	1	7	49



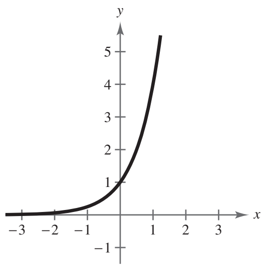
18. $f(x) = 7^{-x}$

x	-2	-1	0	1	2
$f(x)$	49	7	1	0.143	0.020



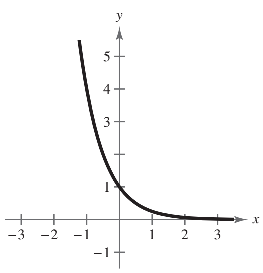
19. $f(x) = \left(\frac{1}{4}\right)^{-x}$

x	-2	-1	0	1	2
$f(x)$	0.063	0.25	1	4	16



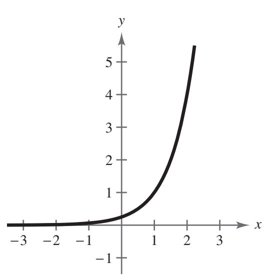
20. $f(x) = \left(\frac{1}{4}\right)^{x}$

x	-2	-1	0	1	2
$f(x)$	16	4	1	0.25	0.063



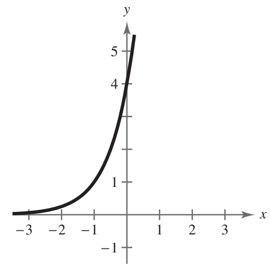
21. $f(x) = 4^{x-1}$

x	-2	-1	0	1	2
$f(x)$	0.016	0.063	0.25	1	4



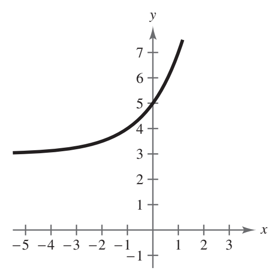
22. $f(x) = 4^{x+1}$

x	-2	-1	0	1	2
$f(x)$	0.25	1	4	16	64



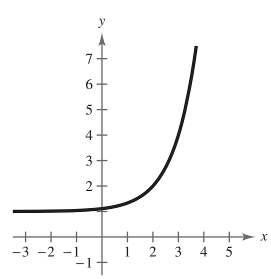
23. $f(x) = 2^{x+1} + 3$

x	-3	-2	-1	0	1
$f(x)$	3.25	3.5	4	5	7



24. $f(x) = 3^{x-2} + 1$

x	-1	0	1	2	3
$f(x)$	1.037	1.111	1.333	2	4



25.
$$\begin{aligned} 3^{x+1} &= 27 \\ 3^{x+1} &= 3^3 \\ x + 1 &= 3 \\ x &= 2 \end{aligned}$$

26.
$$\begin{aligned} 2^{x-2} &= 64 \\ 2^{x-2} &= 2^6 \\ x - 2 &= 6 \\ x &= 8 \end{aligned}$$

27. $\left(\frac{1}{2}\right)^x = 32$

$\left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^{-5}$

$x = -5$

28. $5^{x-2} = \frac{1}{125}$

$5^{x-2} = 5^{-3}$

$x - 2 = -3$

$x = -1$

29. $f(x) = 3^x, g(x) = 3^x + 1$

Because $g(x) = f(x) + 1,$ the graph of g can be obtained by shifting the graph of f one unit upward.

30. $f(x) = \left(\frac{7}{2}\right)^x, g(x) = -\left(\frac{7}{2}\right)^{-x}$

Because $g(x) = -f(-x),$ the graph of g can be obtained by reflecting the graph of f in the x-axis and y-axis (reflect the graph of f in the origin).

31. $f(x) = 10^x, g(x) = 10^{-x+3}$

Because $g(x) = f(-x + 3),$ the graph of g can be obtained by reflecting the graph of f in the y-axis and shifting f three units to the right. (**Note:** This is equivalent to shifting f three units to the left and then reflecting the graph in the y-axis.)

32. $f(x) = 0.3^x, g(x) = -0.3^x + 5$

$g(x) = -f(x) + 5,$ so the graph of g can be obtained by reflecting the graph of f in the x-axis and shifting the resulting graph five units upward.

33. $f(x) = e^x$

$f(1.9) = e^{1.9} \approx 6.686$

34. $f(x) = 1.5e^{(1/2)x}$

$= 1.5e^{120} \approx 1.956 \times 10^{52}$

35. $f(6) = 5000e^{0.06(6)} \approx 7166.647$

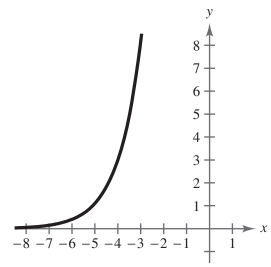
36. $f(x) = 250e^{0.05x}$

$= 250e^{0.05(20)} \approx 679.570$

37. $f(x) = 3e^{x+4}$

x	-8	-7	-6	-5	-4
$f(x)$	0.055	0.149	0.406	1.104	3

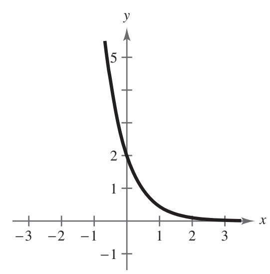
Asymptote: $y = 0$



38. $f(x) = 2e^{-1.5x}$

x	-2	-1	0	1	2
$f(x)$	40.171	8.963	2	0.446	0.100

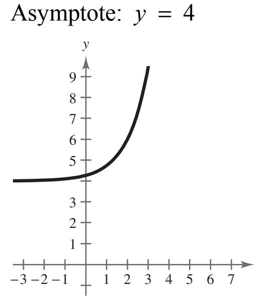
Asymptote: $y = 0$



39. $f(x) = 2e^{x-2} + 4$

x	-2	-1	0	1	2
$f(x)$	4.037	4.100	4.271	4.736	6

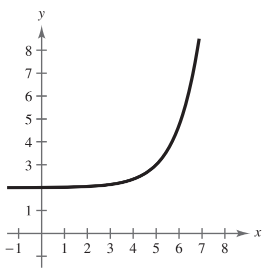
Asymptote: $y = 4$



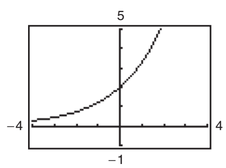
40. $f(x) = 2 + e^{x-5}$

x	0	2	4	5	6
$f(x)$	2.007	2.050	2.368	3	4.718

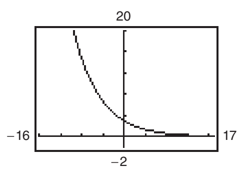
Asymptote: $y = 2$



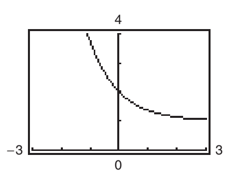
41. $s(t) = 2e^{0.5t}$



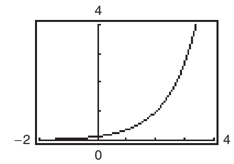
42. $s(t) = 3e^{-0.2t}$



43. $g(x) = 1 + e^{-x}$



44. $h(x) = e^{x-2}$



45.
$$\begin{aligned} e^{3x+2} &= e^3 \\ 3x + 2 &= 3 \\ 3x &= 1 \\ x &= \tfrac{1}{3} \end{aligned}$$

46.
$$\begin{aligned} e^{2x-1} &= e^4 \\ 2x - 1 &= 4 \\ 2x &= 5 \\ x &= \tfrac{5}{2} \end{aligned}$$

47.
$$\begin{aligned} e^{x^2-3} &= e^{2x} \\ x^2 - 3 &= 2x \\ x^2 - 2x - 3 &= 0 \\ (x - 3)(x + 1) &= 0 \\ x = 3 \quad \text{or} \quad x &= -1 \end{aligned}$$

48.
$$\begin{aligned} e^{x^2+6} &= e^{5x} \\ x^2 + 6 &= 5x \\ x^2 - 5x + 6 &= 0 \\ (x - 3)(x - 2) &= 0 \\ x = 3 \text{ or } x &= 2 \end{aligned}$$

49. $P = \$1500, r = 2\%, t = 10$ years

Compounded n times per year: $A = P\left(1 + \frac{r}{n}\right)^{nt} = 1500\left(1 + \frac{0.02}{n}\right)^{10n}$

Compounded continuously: $A = Pe^{rt} = 1500e^{0.02(10)}$

n	1	2	4	12	365	Continuous
A	\$1828.49	\$1830.29	\$1831.19	\$1831.80	\$1832.09	\$1832.10

50. $P = \$2500, r = 3.5\%, t = 10$ years

Compounded n times per year: $A = P\left(1 + \frac{r}{n}\right)^{nt} = 2500\left(1 + \frac{0.035}{n}\right)^{10n}$

Compounded continuously: $A = Pe^{rt} = 2500e^{0.035(10)}$

n	1	2	4	12	365	Continuous
A	\$3526.50	\$3536.95	\$3542.27	\$3545.86	\$3547.61	\$3547.67

51. $P = \$2500, r = 4\%, t = 20$ years

Compounded n times per year: $A = P\left(1 + \frac{r}{n}\right)^{nt} = 2500\left(1 + \frac{0.04}{n}\right)^{20n}$

Compounded continuously: $A = Pe^{rt} = 2500e^{0.04(20)}$

n	1	2	4	12	365	Continuous
A	\$5477.81	\$5520.10	\$5541.79	\$5556.46	\$5563.61	\$5563.85

52. $P = \$1000, r = 6\%, t = 40$ years

Compounded n times per year: $A = 1000\left(1 + \frac{0.06}{n}\right)^{40n}$

Compounded continuously: $A = 1000e^{0.06(40)}$

n	1	2	4	12	365	Continuous
A	\$10,285.72	\$10,640.89	\$10,828.46	\$10,957.45	\$11,021.00	\$11,023.18

53. $A = Pe^{rt} = 12{,}000e^{0.04t}$

t	10	20	30	40	50
A	\$17,901.90	\$26,706.49	\$39,841.40	\$59,436.39	\$88,668.67

54. $A = Pe^{rt} = 12{,}000e^{0.06t}$

t	10	20	30	40	50
A	\$21,865.43	\$39,841.40	\$72,595.77	\$132,278.12	\$241,026.44

55. $A = Pe^{rt} = 12{,}000e^{0.065t}$

t	10	20	30	40	50
A	\$22,986.49	\$44,031.56	\$84,344.25	\$161,564.86	\$309,484.08

56. $A = Pe^{rt} = 12{,}000e^{0.035t}$

t	10	20	30	40	50
A	\$17,028.81	\$24,165.03	\$34,291.81	\$48,662.40	\$69,055.23

57. $A = 30{,}000e^{(0.05)(25)} \approx \$104{,}710.29$

58. $A = 5000e^{(0.075)(50)} \approx \$212{,}605.41$

59. $C(t) = 29.88(1.04)^t$

Ten years from today, $t = 10$: $C(10) = 29.88(1.04)^{10} \approx \$44.23$

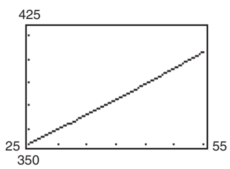
60. $V(t) = 100e^{4.6052t}$

(a) $V(1) \approx 10{,}000$ computers

(b) $V(1.5) \approx 100{,}004$ computers

(c) $V(2) \approx 1{,}000{,}060$ computers

61. (a)



(b)

t	25	26	27	28
P (in millions)	350.281	352.107	353.943	355.788

t	29	30	31	32
P (in millions)	357.643	359.508	361.382	363.266

t	33	34	35	36
P (in millions)	365.160	367.064	368.977	370.901

t	37	38	39	40
P (in millions)	372.835	374.779	376.732	378.697

t	41	42	43	44
P (in millions)	380.671	382.656	384.651	386.656

t	45	46	47	48
P (in millions)	388.672	390.698	392.735	394.783

t	49	50	51	52
P (in millions)	396.841	398.910	400.989	403.080

t	53	54	55
P (in millions)	405.182	407.294	409.417

(c) Using the model and extending the table beyond the year 2055, the population will exceed 430 million in 2064.

t	55	56	57	58	59	60	61	62	63	64	65
P (in millions)	409.417	411.552	413.698	415.854	418.022	420.202	422.393	424.595	426.808	429.034	431.270

62. (a) Because the coefficient of the model $P = 57.59e^{0.0051t}$ is positive, the population is increasing.

(b) 2003: Let $t = 3$: $P = 57.59e^{0.0051(3)} \approx 58.478$

The population of Italy in 2000 was 58,478,000 people.

2015: Let $t = 15$: $P = 57.59e^{0.0051(15)} \approx 62.169$

The population of Italy in 2012 was about 62,169,000 people.

(c) 2020: Let $t = 20$: $P = 57.59e^{0.0051(20)} \approx 63.774$

The population of Italy in 2020 will be about 63,774,000 people.

2025: Let $t = 25$: $P = 57.59e^{0.0051(25)} \approx 65.421$

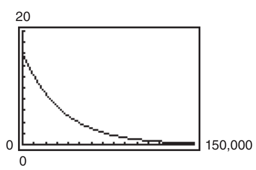
The population of Italy in 2025 will be about 65,421,000 people.

63. $Q = 16\left(\frac{1}{2}\right)^{t/24,100}$

(a) $Q(0) = 16$ grams

(b) $Q(75,000) \approx 1.85$ grams

(c)



64. $Q = 10\left(\frac{1}{2}\right)^{t/5715}$

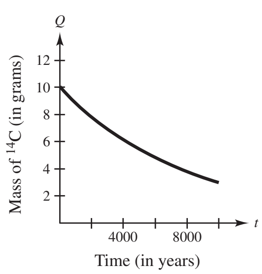
(a) When $t = 0$: $Q = 10\left(\frac{1}{2}\right)^{0/5715}$

$= 10(1) = 10$ grams

(b) When $t = 2000$: $Q = 10\left(\frac{1}{2}\right)^{2000/5715}$

≈ 7.85 grams

(c)



65. (a) $V(t) = 49,810\left(\frac{7}{8}\right)^t$ where t is the number of years since it was purchased.

(b) $V(4) = 49,810\left(\frac{7}{8}\right)^4 \approx 29,197.71$

After 4 years, the value of the van is about \$29,198.

66. (a) $C(t) = 300(0.75)^t$

(b) $C(8) \approx 30$ milligrams per milliliter

67. True. The line $y = -2$ is a horizontal asymptote for the graph of $f(x) = 10^x - 2$. As $x \to -\infty$, $f(x) \to -2$ but never reaches -2.

68. False, $e \neq \dfrac{271,801}{99,990}$. e is an irrational number.

69.
$$\begin{aligned} f(x) &= 3^{x-2} \\ &= 3^x 3^{-2} \\ &= 3^x\left(\frac{1}{3^2}\right) \\ &= \frac{1}{9}(3^x) \\ &= h(x) \end{aligned}$$

So, $f(x) \neq g(x)$, but $f(x) = h(x)$.

70.
$$\begin{aligned} g(x) &= 2^{2x+6} \\ &= 2^{2x} \cdot 2^6 \\ &= 64(2^{2x}) \\ &= 64(2^2)^x \\ &= 64(4^x) \\ &= h(x) \end{aligned}$$

So, $g(x) = h(x)$ but $g(x) \neq f(x)$.

71.
$$\begin{aligned} f(x) &= 16(4^{-x}) \\ &= 4^2(4^{-x}) \\ &= 4^{2-x} \\ &= \left(\tfrac{1}{4}\right)^{-(2-x)} \\ &= \left(\tfrac{1}{4}\right)^{x-2} \\ &= g(x) \end{aligned} \qquad \text{and} \qquad \begin{aligned} f(x) &= 16(4^{-x}) \\ &= 16(2^2)^{-x} \\ &= 16(2^{-2x}) \\ &= h(x) \end{aligned}$$

So, $f(x) = g(x) = h(x)$.

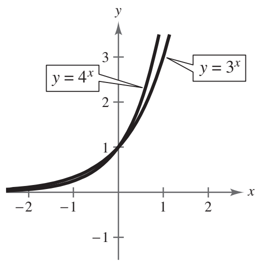
72. $f(x) = e^{-x} + 3$

$g(x) = e^{3-x} = e^{3} \cdot e^{-x}$

$h(x) = -e^{x-3} = -(e^{x} \cdot e^{-3})$

So, none are equal.

73. $y = 3^x$ and $y = 4^x$

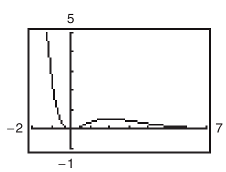


x	-2	-1	0	1	2
3^x	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9
4^x	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16

(a) $4^x < 3^x$ when $x < 0$.

(b) $4^x > 3^x$ when $x > 0$.

74. (a) $f(x) = x^2e^{-x}$



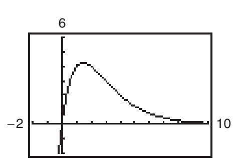
Decreasing: $(-\infty, 0), (2, \infty)$

Increasing: $(0, 2)$

Relative maximum: $(2, 4e^{-2})$

Relative minimum: $(0, 0)$

(b) $g(x) = x2^{3-x}$

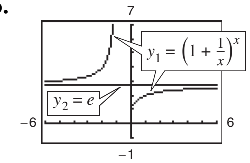


Decreasing: $(1.44, \infty)$

Increasing: $(-\infty, 1.44)$

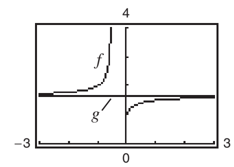
Relative maximum: $(1.44, 4.25)$

75.



As x increases, the graph of y_1 approaches e, which is y_2.

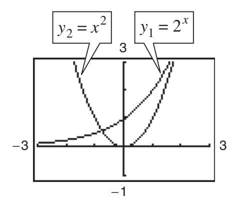
76. $f(x) = \left(1 + \dfrac{0.5}{x}\right)^x$ and $g(x) = e^{0.5}$ (Horizontal line)



As $x \to \infty$, $f(x) \to g(x)$.

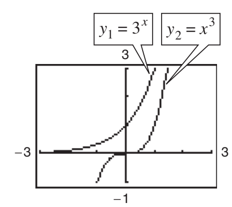
As $x \to -\infty$, $f(x) \to g(x)$.

77. (a)



At $x = 2$, both functions have a value of 4. The function y_1 increases for all values of x. The function y_2 is symmetric with respect to the y-axis.

(b)



Both functions are increasing for all values of x. For $x > 0$, both functions have a similar shape. The function y_2 is symmetric with respect to the origin.

In both viewing windows, the constant raised to a variable power increases more rapidly than the variable raised to a constant power.

78. The graph of $y = a^x$ is increasing, so graphs (d), (e), and (f) represent $y = 2^x$, $y = e^x$, and $y = 10^x$.

The greater the value of *a*, the quicker the graph increases. Because $10 > e > 2$, $y = 10^x$ matches graph (d), $y = e^x$ matches graph (e), and $y = 2^x$ matches graph (f).

Graph (c) is a reflection in the *y*-axis of graph (d), so $y = 10^{-x}$ matches graph (c).

Graph (b) is a reflection in the *y*-axis of graph (e), so $y = e^{-x}$ matches graph (b).

Graph (a) is a reflection in the *y*-axis of graph (f), so $y = 2^{-x}$ matches graph (a)

79. The functions (c) $h(x) = 3^x$ and (d) $k(x) = 2^{-x}$ are exponential.

80. $P = \$3000, r = 6\%, t = 10$ years

$$A = P\left(1 + \frac{r}{n}\right)^{nt} = 3000\left(1 + \frac{0.06}{n}\right)^{10n}$$

(a) $n = 365$
$A \approx \$5466.09$

(b) $n = 365(24) = 8760$
$A \approx \$5466.35$

(c) $n = 365(24)(60) = 525{,}600$
$A \approx \$5466.36$

(d) $n = 365(24)(60)(60) = 31{,}536{,}000$
$A \approx \$5466.38$

Increasing the number of compoundings does not result in unlimited growth because the balance increases slowly from compounding by the day to compounding by the second.

Section 3.2 Logarithmic Functions and Their Graphs

1. logarithmic

2. 10

3. natural; *e*

4. $a^{\log_a x} = x$

5. $x = y$

6. positive real numbers

7. $\log_4 16 = 2 \Rightarrow 4^2 = 16$

8. $\log_9 \frac{1}{81} = -2 \Rightarrow 9^{-2} = \frac{1}{81}$

9. $\log_{12} 12 = 1 \Rightarrow 12^1 = 12$

10. $\log_{32} 4 = \frac{2}{5} \Rightarrow 32^{2/5} = 4$

11. $5^3 = 125 \Rightarrow \log_5 125 = 3$

12. $9^{3/2} = 27 \Rightarrow \log_9 27 = \frac{3}{2}$

13. $4^{-3} = \frac{1}{64} \Rightarrow \log_4 \frac{1}{64} = -3$

14. $24^0 = 1 \Rightarrow \log_{24} 1 = 0$

15. $f(x) = \log_2 x$

$f(64) = \log_2 64 = 6$ because $2^6 = 64$

16. $f(x) = \log_{25} x$

$f(5) = \log_{25} 5 = \frac{1}{2}$ because $25^{1/2} = 5$

17. $f(x) = \log_8 x$

$f(1) = \log_8 1 = 0$ because $8^0 = 1$

18. $f(x) = \log x$

$f(10) = \log 10 = 1$ because $10^1 = 10$

19. $g(x) = \log_a x$

$g(a^2) = \log_a a^{-2}$

$= -2$

20. $g(x) = \log_b x$

$g(b^{-3}) = \log_b \sqrt{b} = \log_b b^{1/2} = \frac{1}{2}$

21. $f(x) = \log x$

$f\left(\frac{7}{8}\right) = \log\left(\frac{7}{8}\right) \approx -0.058$

22. $f(x) = \log x$

$f\left(\frac{1}{500}\right) = \log \frac{1}{500} \approx -2.699$

23. $f(x) = \log x$

$f(12.5) = \log 12.5 \approx 1.097$

24. $f(x) = \log x$

$f(96.75) = \log 96.75 \approx 1.986$

25. $\log_8 8 = 1$ because $8^1 = 8$

26. $\log_\pi \pi^2 = 2$ because $\pi^2 = \pi^2$

27. $\log_{7.5} 1 = 0$ because $7.5^0 = 1$

28. $5^{\log_5 3} = 3$ because $a^{\log_a x} = x$, $5^{\log_5 3} = 3$

29. $\log_5(x + 1) = \log_5 6$

$x + 1 = 6$

$x = 5$

30. $\log_2(x - 3) = \log_2 9$

$x - 3 = 9$

$x = 12$

31. $\log 11 = \log(x^2 + 7)$

$11 = x^2 + 7$

$x^2 = 4$

$x = \pm 2$

32. $\log(x^2 + 6x) = \log 27$

$x^2 + 6x = 27$

$x^2 + 6x - 27 = 0$

$(x + 9)(x - 3) = 0$

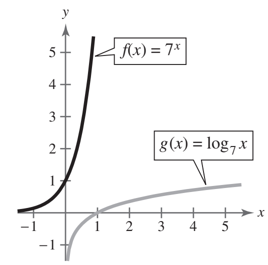
$x + 9 = 0 \rightarrow x = -9$

$x - 3 = 0 \rightarrow x = 3$

33.

x	-2	-1	0	1	2
$f(x) = 7^x$	$\frac{1}{49}$	$\frac{1}{7}$	1	7	49

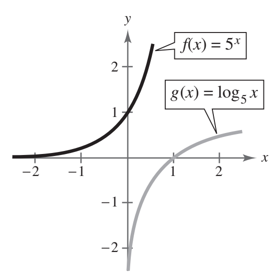
x	$\frac{1}{49}$	$\frac{1}{7}$	1	7	49
$g(x) = \log_7 x$	-2	-1	0	1	2



34.

x	-2	-1	0	1	2
$f(x) = 5^x$	$\frac{1}{25}$	$\frac{1}{5}$	1	5	25

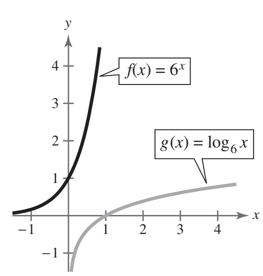
x	$\frac{1}{25}$	$\frac{1}{5}$	1	5	25
$g(x) = \log_5 x$	-2	-1	0	1	2



35.

x	-2	-1	0	1	2
$f(x) = 6^x$	$\frac{1}{36}$	$\frac{1}{6}$	1	6	36

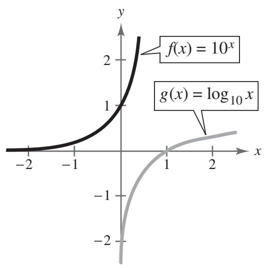
x	$\frac{1}{36}$	$\frac{1}{6}$	1	6	36
$g(x) = \log_6 x$	-2	-1	0	1	2



36.

x	-2	-1	0	1	2
$f(x) = 10^x$	$\frac{1}{100}$	$\frac{1}{10}$	1	10	100

x	$\frac{1}{100}$	$\frac{1}{10}$	1	10	100
$g(x) = \log_{10} x$	-2	-1	0	1	2



37. $f(x) = \log_3 x + 2$

Asymptote: $x = 0$

Point on graph: $(1, 2)$

Matches graph (a).

The graph of $f(x)$ is obtained from $g(x)$ by shifting the graph two units upward.

38. $f(x) = \log_3(x - 1)$

Asymptote: $x = 1$

Point on graph: $(2, 0)$

Matches graph (d).

$f(x)$ shifts $g(x)$ one unit to the right.

39. $f(x) = \log_3(1 - x) = \log_3[-(x - 1)]$

Asymptote: $x = 1$

Point on graph: $(0, 0)$

Matches graph (b).

The graph of $f(x)$ is obtained by reflecting the graph of $g(x)$ in the y-axis and shifting the graph one unit to the right.

40. $f(x) = -\log_3 x$

Asymptote: $x = 0$

Point on graph: $(1, 0)$

Matches graph (c).

$f(x)$ reflects $g(x)$ in the x-axis.

41. $f(x) = \log_4 x$

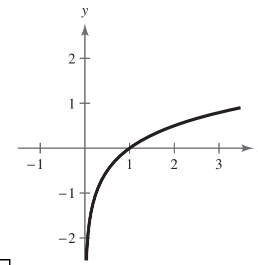
Domain: $(0, \infty)$

x-intercept: $(1, 0)$

Vertical asymptote: $x = 0$

$y = \log_4 x \Rightarrow 4^y = x$

x	$\frac{1}{4}$	1	4	2
$f(x)$	-1	0	1	$\frac{1}{2}$



42. $g(x) = \log_6 x$

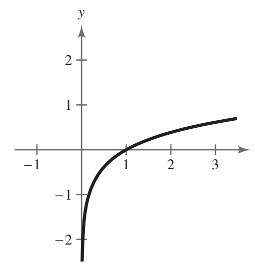
Domain: $(0, \infty)$

x-intercept: $(1, 0)$

Vertical asymptote: $x = 0$

$y = \log_6 x \Rightarrow 6^y = x$

x	$\frac{1}{6}$	1	$\sqrt{6}$	6
$g(x)$	-1	0	$\frac{1}{2}$	1



43. $y = \log_3 x + 1$

Domain: $(0, \infty)$

x-intercept:

$$\log_3 x + 1 = 0$$
$$\log_3 x = -1$$
$$3^{-1} = x$$
$$\frac{1}{3} = x$$

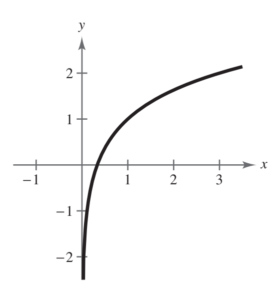
The x-intercept is $\left(\frac{1}{3}, 0\right)$.

Vertical asymptote: $x = 0$

$y = \log_3 x + 1$

$\log_3 x = y - 1 \Rightarrow 3^{y-1} = x$

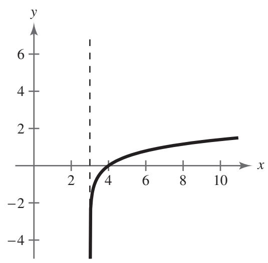
x	$\frac{1}{9}$	$\frac{1}{3}$	0	3	9
y	-1	0	1	2	3



44. $h(x) = \log_4(x - 3)$

Domain: $x - 3 > 0 \Rightarrow x > 3$

The domain is $(3, \infty)$.



x-intercept:

$$\log_4(x - 3) = 0$$
$$4^0 = x - 3$$
$$1 = x - 3$$
$$4 = x$$

The x-intercept is $(4, 0)$.

Vertical asymptote: $x - 3 = 0 \Rightarrow x = 3$

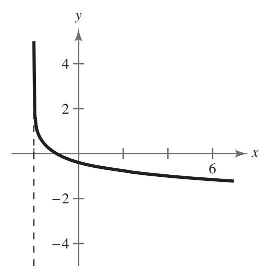
$y = \log_4(x - 3) \Rightarrow 4^y + 3 = x$

x	$3\frac{1}{4}$	4	7	19
$h(x)$	−1	0	1	2

45. $f(x) = -\log_6(x + 2)$

Domain: $x + 2 > 0 \Rightarrow x > -2$

The domain is $(-2, \infty)$.



x-intercept:

$$0 = -\log_6(x + 2)$$
$$0 = \log_6(x + 2)$$
$$6^0 = x + 2$$
$$1 = x + 2$$
$$-1 = x$$

The x-intercept is $(-1, 0)$.

Vertical asymptote: $x + 2 = 0 \Rightarrow x = -2$

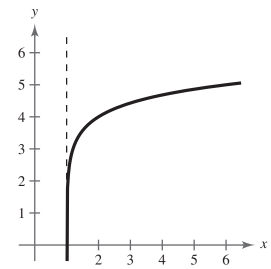
$$y = -\log_6(x + 2)$$
$$-y = \log_6(x + 2)$$
$$6^{-y} - 2 = x$$

x	4	−1	$-1\frac{5}{6}$	$-1\frac{35}{36}$
$f(x)$	−1	0	1	2

46. $y = \log_5(x - 1) + 4$

Domain: $x - 1 > 0 \Rightarrow x > 1$

The domain is $(1, \infty)$.



x-intercept:

$$\log_5(x - 1) + 4 = 0$$
$$\log_5(x - 1) = -4$$
$$5^{-4} = x - 1$$
$$\frac{1}{625} = x - 1$$
$$\frac{626}{625} = x$$

The x-intercept is $\left(\frac{626}{625}, 0\right)$.

Vertical asymptote: $x - 1 = 0 \Rightarrow x = 1$

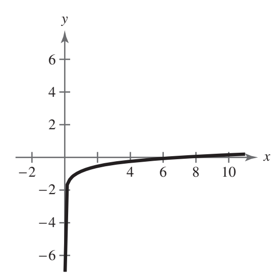
$y = \log_5(x - 1) + 4 \Rightarrow 5^{y-4} + 1 = x$

x	1.00032	1.0016	1.008	1.04	1.2
y	−1	0	1	2	3

47. $y = \log\left(\frac{x}{7}\right)$

Domain: $\frac{x}{7} > 0 \Rightarrow x > 0$

The domain is $(0, \infty)$.



x-intercept: $\log\left(\frac{x}{7}\right) = 0$

$$\frac{x}{7} = 10^0$$
$$\frac{x}{7} = 1$$
$$x = 7$$

The x-intercept is $(7, 0)$.

Vertical asymptote: $\frac{x}{7} = 0 \Rightarrow x = 0$

The vertical asymptote is the y-axis.

x	1	2	3	4	5
y	−0.85	−0.54	−0.37	−0.24	−0.15

x	6	7	8
y	−0.069	0	0.06

48. $y = \log(-2x)$

Domain: $-2x > 0 \Rightarrow x < 0$

The domain is $(-\infty, 0)$.

x-intercept: $\log(-2x) = 0$

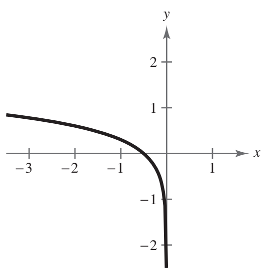
$$10^0 = -2x$$
$$1 = -2x$$
$$x = -\frac{1}{2}$$

The x-intercept is $\left(-\frac{1}{2}, 0\right)$.

Vertical asymptote: $x = 0$

$$y = \log(-2x) \Rightarrow 10^0 = -2x$$
$$10^y = -2x$$
$$-\frac{1}{2} \cdot 10^y = x$$

x	$-\frac{1}{200}$	$-\frac{1}{20}$	$-\frac{1}{2}$	-5	-50	-500
y	-2	-1	0	1	2	3



49. $\ln \frac{1}{2} = -0.693\ldots \Rightarrow e^{-0.693\ldots} = \frac{1}{2}$

50. $\ln 7 = 1.945\ldots \Rightarrow e^{1.945\ldots} = 7$

51. $\ln 250 = 5.521\ldots \Rightarrow e^{5.521\ldots} = 250$

52. $\ln 1 = 0 \Rightarrow e^0 = 1$

53. $e^2 = 7.3890\ldots \Rightarrow \ln 7.3890\ldots = 2$

54. $e^{-3/4} = 0.4723\ldots \Rightarrow \ln 0.4723\ldots = -\frac{3}{4}$

55. $e^{-4x} = \frac{1}{2} \Rightarrow \ln \frac{1}{2} = -4x$

56. $e^{2x} = 3 \Rightarrow \ln 3 = 2x$

57. $f(x) = \ln x$

$f(18.42) = \ln 18.42 \approx 2.913$

58. $f(x) = 3 \ln x$

$f(0.74) = 3 \ln 0.74 \approx -0.903$

59. $g(x) = 8 \ln x$

$g\left(\sqrt{5}\right) = 8 \ln \sqrt{5} \approx 6.438$

60. $g(x) = -\ln x$

$g\left(\frac{1}{2}\right) = -\ln \frac{1}{2} \approx 0.693$

61. $e^{\ln 4} = 4$

62. $\ln\left(\frac{1}{e^2}\right) = \ln e^{-2} = -2$

63. $2.5 \ln 1 = 2.5(0) = 0$

64. $\frac{\ln e}{\pi} = \frac{1}{\pi}$

65. $\ln e^{\ln e} = \ln e^1 = 1$

66. $e^{\ln(1/e)} = e^{\ln e^{-1}} = e^{-1} = \frac{1}{e}$

67. $f(x) = \ln(x - 4)$

Domain: $x - 4 > 0 \Rightarrow x > 4$

The domain is $(4, \infty)$.

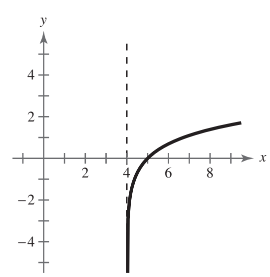
x-intercept: $0 = \ln(x - 4)$

$$e^0 = x - 4$$
$$5 = x$$

The x-intercept is $(5, 0)$.

Vertical asymptote: $x - 4 = 0 \Rightarrow x = 4$

x	4.5	5	6	7
$f(x)$	−0.69	0	0.69	1.10



68. $h(x) = \ln(x + 5)$

Domain: $x + 5 > 0 \Rightarrow x > -5$

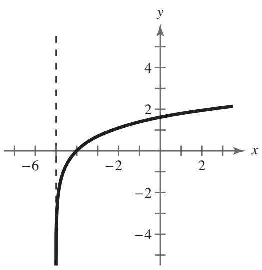
The domain is $(-5, \infty)$.

x-intercept:
$$0 = \ln(x + 5)$$
$$e^0 = x + 5$$
$$-4 = x$$

The x-intercept is $(-4, 0)$.

Vertical asymptote: $x + 5 = 0 \Rightarrow x = -5$

x	–4.5	–4	–3	–2
$h(x)$	–0.69	0	0.69	1.10



69. $g(x) = \ln(-x)$

Domain: $-x > 0 \Rightarrow x < 0$

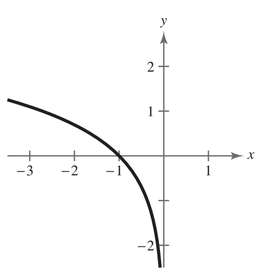
The domain is $(-\infty, 0)$.

x-intercept:
$$0 = \ln(-x)$$
$$e^0 = -x$$
$$-1 = x$$

The x-intercept is $(-1, 0)$.

Vertical asymptote: $-x = 0 \Rightarrow x = 0$

x	–0.5	–1	–2	–3
$g(x)$	–0.69	0	0.69	1.10



70. $f(x) = \ln(3 - x)$

Domain: $3 - x > 0 \Rightarrow x < 3$

The domain is $(-\infty, 3)$.

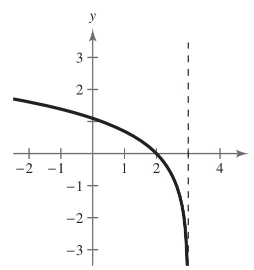
x-intercept:
$$\ln(3 - x) = 0$$
$$e^0 = 3 - x$$
$$1 = 3 - x$$
$$2 = x$$

The x-intercept is $(2, 0)$.

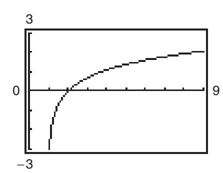
Vertical asymptote: $3 - x = 0 \Rightarrow x = 3$

$y = \ln(3 - x) \Rightarrow 3 - e^y = x$

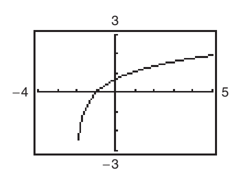
x	2.95	2.86	2.63	2	0.28
$f(x)$	–3	–2	–1	0	1



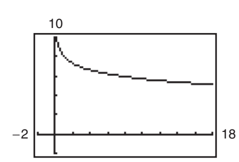
71. $f(x) = \ln(x - 1)$



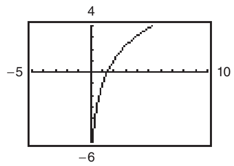
72. $f(x) = \ln(x + 2)$



73. $f(x) = -\ln x + 8$



74. $f(x) = 3 \ln x - 1$



75. $\ln(x + 4) = \ln 12$

$x + 4 = 12$

$x = 8$

76. $\ln(x - 7) = \ln 7$

$x - 7 = 7$

$x = 14$

77. $\ln(x^2 - x) = \ln 6$

$x^2 - x = 6$

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$x = -2$ or $x = 3$

78. $\ln(x^2 - 2) = \ln 23$

$x^2 - 2 = 23$

$x^2 = 25$

$x = \pm 5$

79. $t = 16.625 \ln\left(\dfrac{x}{x - 750}\right),\ x > 750$

(a) When $x = \$897.72$:

$$t = 16.625 \ln\left(\frac{897.72}{897.72 - 750}\right) \approx 30 \text{ years}$$

When $x = \$1659.24$:

$$t = 16.625 \ln\left(\frac{1659.24}{1659.24 - 750}\right) \approx 10 \text{ years}$$

(b) Total amounts:

$(897.72)(12)(30) = \$323{,}179.20 \approx \$323{,}179$

$(1659.24)(12)(10) = \$199{,}108.80 \approx \$199{,}109$

Interest charges:

$323{,}179.20 - 150{,}000 = \$173{,}179.20 \approx \$173{,}179$

$199{,}108.80 - 150{,}000 = \$49{,}108.80 \approx \$49{,}109$

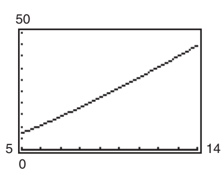
(c) The vertical asymptote is $x = 750$. The closer the payment is to \$750 per month, the longer the length of the mortgage will be. Also, the monthly payment must be greater than \$750.

80. (a) $P = -3.42 + 1.297t \ln t$

2008: $P(8) = -3.42 + 1.297(8) \ln 8 \approx 18.16\%$

2012: $P(12) = -3.42 + 1.297(12) \ln 12 \approx 35.26\%$

(b)



(c) Answers will vary. *Sample answer*: Yes, it can predict the percent for 2020,

$P(20) = -3.42 + 1.297(20) \ln 20 \approx 74.29\%$ is reasonable. However, for 2030,

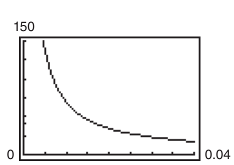
$P(30) = -3.42 + 1.297(30) \ln 30 \approx 128.92\%$ is not possible.

81. $t = \dfrac{\ln 2}{r}$

(a)

r	0.005	0.010	0.015	0.020	0.025	0.030
t	138.6	69.3	46.2	34.7	27.7	23.1

(b)



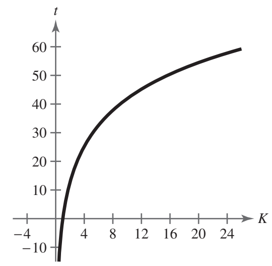
82. $t = \dfrac{\ln K}{0.055}$

(a)

K	1	2	4	6	8	10	12
t	0	12.60	25.21	32.57	37.81	41.87	45.18

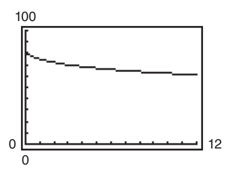
The number of years required to multiply the original investment by K increases with K. However, the larger the value of K, the fewer the years required to increase the value of the investment by an additional multiple of the original investment.

(b)



83. $f(t) = 80 - 17\log(t + 1), 0 \le t \le 12$

(a)



(b) $f(0) = 80 - 17\log 1 = 80.0$

(c) $f(4) = 80 - 17\log 5 \approx 68.1$

(d) $f(10) = 80 - 17\log 11 \approx 62.3$

84. $\beta = 10\log\left(\dfrac{I}{10^{-12}}\right)$

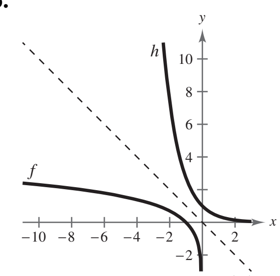
(a) $\beta = 10\log\left(\dfrac{1}{10^{-12}}\right) = 10\log\left(10^{12}\right) = 120$ decibels

(b) $\beta = 10\log\left(\dfrac{10^{-2}}{10^{-12}}\right) = 10\log\left(10^{10}\right) = 100$ decibels

(c) No, the difference is due to the logarithmic relationship between intensity and number of decibels.

85. False. Reflecting $g(x)$ about the line $y = x$ will determine the graph of $f(x)$.

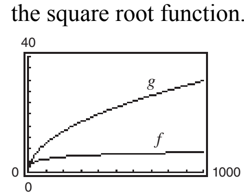
86.



True. The graph of $f(x) = \ln(-x)$ is a reflection of $h(x) = e^{-x}$ in the line $y = -x$.

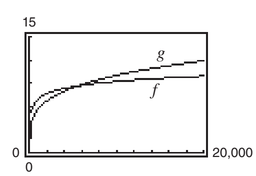
87. (a) $f(x) = \ln x, g(x) = \sqrt{x}$

The natural log function grows at a slower rate than the square root function.



(b) $f(x) = \ln x, g(x) = \sqrt[4]{x}$

The natural log function grows at a slower rate than the fourth root function.



88. (a) The function $f(x) = 3^x$ matches graph m, and $g(x) = \log_3 x$ matches graph n.

(b) Since f and g are inverse functions, the point (a, b) on the graph of f corresponds to the point (b, a) on the graph of g. So, if $f(a) = b$, then $g(b) = a$.

89.

x	1	2	8
y	0	1	3

y is not an exponential function of x, but it is a logarithmic function of x, $y = \log_2 x$.

90.

x	1	2	5
y	2	4	32

y is not a logarithmic function of x, but it is an exponential function of x, $y = 2^x$.

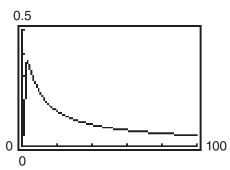
91. $f(x) = \dfrac{\ln x}{x}$

(a)

x	1	5	10	10^2	10^4	10^6
$f(x)$	0	0.322	0.230	0.046	0.00092	0.0000138

(b) As $x \to \infty$, $f(x) \to 0$.

(c)



92. $y = \log_a x \Rightarrow a^y = x$, so, for example, if $a = -2$, there is no value of y for which $(-2)^y = -4$. If $a = 1$, then every power of a is equal to 1, so x could only be 1. So, $\log_a x$ is defined only for $0 < a < 1$ and $a > 1$.

Section 3.3 Properties of Logarithms

1. change-of-base

2. $\dfrac{\log x}{\log a} = \dfrac{\ln x}{\ln a}$

3. $\dfrac{1}{\log_b a}$

4. (a) $\ln(uv) = \ln u + \ln v$

This is the Product Property

(b) $\log_a u^n = n \log_a u$

This is the Power Property.

(c) $\ln\left(\dfrac{u}{v}\right) = \ln u - \ln v$

This is the Quotient Property.

5. (a) $\log_5 16 = \dfrac{\log 16}{\log 5}$

(b) $\log_5 16 = \dfrac{\ln 16}{\ln 5}$

6. (a) $\log_{1/5} 4 = \dfrac{\log 4}{\log(1/5)}$

(b) $\log_{1/5} 4 = \dfrac{\ln 4}{\ln(1/5)}$

7. (a) $\log_x \dfrac{3}{10} = \dfrac{\log(3/10)}{\log x}$

(b) $\log_x \dfrac{3}{10} = \dfrac{\ln(3/10)}{\ln x}$

8. (a) $\log_{2.6} x = \dfrac{\log x}{\log 2.6}$

(b) $\log_{2.6} x = \dfrac{\ln x}{\ln 2.6}$

9. $\log_3 17 = \dfrac{\log 17}{\log 3} = \dfrac{\ln 17}{\ln 3} \approx 2.579$

10. $\log_{0.4} 12 = \dfrac{\log 12}{\log 0.4} = \dfrac{\ln 12}{\ln 0.4} \approx -2.712$

11. $\log_{\pi} 0.5 = \dfrac{\log 0.5}{\log \pi} = \dfrac{\ln 0.5}{\ln \pi} \approx -0.606$

12. $\log_{2/3} 0.125 = \dfrac{\log 0.125}{\log(2/3)} = \dfrac{\ln 0.125}{\ln(2/3)} \approx 5.129$

13. $\log_3 35 = \log_3 (5 \cdot 7)$
$= \log_3 5 + \log_3 7$

14. $\log_3\left(\frac{5}{7}\right) = \log_3 5 - \log_3 7$

15. $\log_3\left(\frac{7}{25}\right) = \log_3 7 - \log_3 25$
$= \log_3 7 - \log_3 5^2$
$= \log_3 7 - 2\log_3 5$

16. $\log_3 175 = \log_3 (7 \cdot 25)$
$= \log_3 7 + \log_3 5^2$
$= \log_3 7 + 2\log_3 5$

17. $\log_3\left(\frac{21}{5}\right) = \log_3 21 - \log_3 5$
$= \log_3 (3 \cdot 7) - \log_3 5$
$= \log_3 3 + \log_3 7 - \log_3 5$
$= 1 + \log_3 7 - \log_3 5$

18. $\log_3\left(\frac{45}{49}\right) = \log_3 45 - \log_3 49$
$= \log_3 (5 \cdot 9) - \log_3 49$
$= \log_3 5 + \log_3 9 - \log_3 49$
$= \log_3 5 + \log_3 3^2 - \log_3 7^2$
$= \log_3 5 + 2\log_3 3 - 2\log_3 7$
$= \log_3 5 + 2 - 2\log_3 7$

19. $\log_3 9 = 2\log_3 3 = 2$

20. $\log_5 \frac{1}{125} = \log_5 5^{-3} = -3\log_5 5 = -3(1) = -3$

21. $\log_6 \sqrt[3]{\frac{1}{6}} = \log_6\left(\frac{1}{6}\right)^{1/3} = \frac{1}{3}\log_6\left(\frac{1}{6}\right) = \frac{1}{3}\log_6 6^{-1} = \frac{1}{3}(-1) = -\frac{1}{3}$

22. $\log_2 \sqrt[4]{8} = \frac{1}{4}\log_2 2^3 = \frac{3}{4}\log_2 2 = \frac{3}{4}(1) = \frac{3}{4}$

23. $\log_2(-2)$ is undefined. -2 is not in the domain of $\log_2 x$.

24. $\log_3(-27)$ is undefined. -27 is not in the domain of $\log_3 x$.

25. $\ln \sqrt[4]{e^3} = \ln e^{3/4}$
$= \frac{3}{4}\ln e$
$= \frac{3}{4}(1)$
$= \frac{3}{4}$

26. $\ln \dfrac{1}{\sqrt{e}} = \ln 1 - \ln \sqrt{e}$
$= 0 - \dfrac{1}{2}\ln e$
$= 0 - \dfrac{1}{2}(1)$
$= -\dfrac{1}{2}$

27. $\ln e^2 + \ln e^5 = 2 + 5 = 7$

28. $2\ln e^6 - \ln e^5 = \ln e^{12} - \ln e^5$
$= \ln \dfrac{e^{12}}{e^5}$
$= \ln e^7$
$= 7$

29. $\log_5 75 - \log_5 3 = \log_5 \frac{75}{3}$
$= \log_5 25$
$= \log_5 5^2$
$= 2\log_5 5$
$= 2$

30. $\log_4 2 + \log_4 32 = \log_4 4^{1/2} + \log_4 4^{5/2}$
$= \frac{1}{2}\log_4 4 + \frac{5}{2}\log_4 4$
$= \frac{1}{2}(1) + \frac{5}{2}(1)$
$= 3$

31. $\log_4 8 = \log_4 (4 \cdot 2) = \log_4 4 + \log_4 2 = \log_4 4 + \log_4 4^{1/2} = 1 + \frac{1}{2} = \frac{3}{2}$

32. $\log_8 16 = \log_8 (8 \cdot 2) = \log_8 8 + \log_8 2 = \log_8 8 + \log_8 8^{1/3} = 1 + \frac{1}{3} = \frac{4}{3}$

33. $\log_b 10 = \log_b 2 \cdot 5$
$= \log_b 2 + \log_b 5$
$\approx 0.3562 + 0.8271$
$= 1.1833$

34. $\log_b \frac{2}{3} = \log_b 2 - \log_b 3$
$\approx 0.3562 - 0.5646$
$= -0.2084$

35. $\log_b 0.04 = \log_b \frac{4}{100} = \log_b \frac{1}{25}$
$= \log_b 1 - \log_b 25$
$= \log_b 1 - \log_b 5^2$
$= 0 - 2\log_b 5$
$\approx -2(0.8271)$
$= -1.6542$

36. $\log_b \sqrt{2} = \log_b 2^{1/2}$
$= \frac{1}{2}\log_b 2$
$\approx \frac{1}{2}(0.3562)$
$= 0.1781$

37. $\log_b 45 = \log_b 9 \cdot 5$
$= \log_b 9 + \log_b 5$
$= \log_b 3^2 + \log_b 5$
$= 2\log_b 3 + \log_b 5$
$\approx 2(0.5646) + 0.8271$
$= 1.9563$

38. $\log_b 3b^2 = \log_b 3 + \log_b b^2$
$= \log_b 3 + 2\log_b b$
$= \log_b 3 + 2(1)$
$\approx 0.5646 + 2$
$= 2.5646$

39. $\log_b (2b)^{-2} = -2\log_b 2b$
$= -2(\log_b 2 + \log_b b)$
$\approx -2(0.3562 + 1)$
$= -2.7124$

40. $\log_b \sqrt[3]{3b} = \log_b (3b)^{1/3}$
$= \frac{1}{3}\log_b (3b)$
$= \frac{1}{3}(\log_b 3 + \log_b b)$
$\approx \frac{1}{3}(0.5646 + 1)$
$= \frac{1}{3}(1.5646)$
≈ 0.5215

41. $\ln 7x = \ln 7 + \ln x$

42. $\log_3 13z = \log_3 13 + \log_3 z$

43. $\log_8 x^4 = 4\log_8 x$

44. $\ln(xy)^3 = 3\ln(xy)$
$= 3(\ln x + \ln y)$
$= 3\ln x + 3\ln y$

45. $\log_5 \dfrac{5}{x} = \log_5 5 - \log_5 x$
$= 1 - \log_5 x$

46. $\log_6 \left(\frac{w^2}{v}\right) = \log_6 w^2 - \log_6 v$
$= 2\log_6 w - \log_6 v$

47. $\ln\sqrt{z} = \ln z^{1/2} = \frac{1}{2}\ln z$

48. $\ln\sqrt[3]{t} = \ln t^{1/3} = \frac{1}{3}\ln t$

49. $\ln xyz^2 = \ln x + \ln y + \ln z^2$
$= \ln x + \ln y + 2\ln z$

50. $\log_4 (11b^2c) = \log_4 11 + \log_4 b^2 + \log_4 c$
$= \log_4 11 + 2\log_4 b + \log_4 c$

51. $\ln z(z-1)^2 = \ln z + \ln(z-1)^2$
$= \ln z + 2\ln(z-1),\ z > 1$

52. $\ln\left(\dfrac{x^2-1}{x^3}\right) = \ln(x^2-1) - \ln x^3$
$= \ln[(x+1)(x-1)] - \ln x^3$
$= \ln(x+1) + \ln(x-1) - 3\ln x$

53. $\log_2\left(\dfrac{\sqrt{a^2-4}}{7}\right) = \log_2 \sqrt{a^2-4} - \log_2 7$

$= \log_2 (a^2-4)^{1/2} - \log_2 7$

$= \frac{1}{2}\log_2 (a^2-4) - \log_2 7$

$= \frac{1}{2}\log_2 [(a-2)(a+2)] - \log_2 7$

$= \frac{1}{2}[\log_2 (a-2) + \log_2 (a+2)] - \log_2 7$

$= \frac{1}{2}\log_2 (a-2) + \frac{1}{2}\log_2 (a+2) - \log_2 7$

54. $\ln\left(\dfrac{3}{\sqrt{x^2+1}}\right) = \ln 3 - \ln\sqrt{x^2+1}$

$= \ln 3 - \ln(x^2+1)^{1/2}$

$= \ln 3 - \frac{1}{2}\ln(x^2+1)$

55. $\log_5\left(\dfrac{x^2}{y^2z^3}\right) = \log_5 x^2 - \log_5 y^2z^3$

$= \log_5 x^2 - (\log_5 y^2 + \log_5 z^3)$

$= 2\log_5 x - 2\log_5 y - 3\log_5 z$

56. $\log\dfrac{xy^4}{z^5} = \log xy^4 - \log z^5$

$= \log x + \log y^4 - \log z^5$

$= \log x + 4\log y - 5\log z$

57. $\ln\sqrt[3]{\dfrac{yz}{x^2}} = \ln\left(\dfrac{yz}{x^2}\right)^{1/3}$

$= \dfrac{1}{3}\ln\left(\dfrac{yz}{x^2}\right)$

$= \dfrac{1}{3}[\ln(yz) - \ln x^2]$

$= \dfrac{1}{3}[\ln(yz) - 2\ln x]$

$= \dfrac{1}{3}[\ln y + \ln z - 2\ln x]$

$= \dfrac{1}{3}\ln y + \dfrac{1}{3}\ln z - \dfrac{2}{3}\ln x$

58. $\log_2 x^4\sqrt{\dfrac{y}{z^3}} = \log_2 x^4 + \log_2\sqrt{\dfrac{y}{z^3}}$

$= \log_2 x^4 + \dfrac{1}{2}\log_2\dfrac{y}{z^3}$

$= \log_2 x^4 + \dfrac{1}{2}[\log_2 y - \log_2 z^3]$

$= 4\log_2 x + \dfrac{1}{2}\log_2 y - \dfrac{3}{2}\log_2 z$

59. $\ln\sqrt[4]{x^3(x^2+3)} = \frac{1}{4}\ln x^3(x^2+3)$

$= \frac{1}{4}[\ln x^3 + \ln(x^2+3)]$

$= \frac{1}{4}[3\ln x + \ln(x^2+3)]$

$= \frac{3}{4}\ln x + \frac{1}{4}\ln(x^2+3)$

60. $\ln\sqrt{x^2(x+2)} = \ln[x^2(x+2)]^{1/2}$

$= \ln[x(x+2)^{1/2}]$

$= \ln x + \ln(x+2)^{1/2}$

$= \ln x + \frac{1}{2}\ln(x+2)$

61. $\ln 3 + \ln x = \ln(3x)$

62. $\log_5 8 - \log_5 t = \log_5 \dfrac{8}{t}$

63. $\frac{2}{3}\log_7(z-2) = \log_7(z-2)^{2/3}$

64. $-4\ln 3x = \ln(3x)^{-4}$

$= \ln\dfrac{1}{(3x)^4}$

$= \ln\dfrac{1}{81x^4}$

65. $\log_3 5x - 4\log_3 x = \log_3 5x - \log_3 x^4$

$= \log_3\left(\dfrac{5x}{x^4}\right)$

$= \log_3\left(\dfrac{5}{x^3}\right)$

66. $2\log_2 x + 4\log_2 y = \log_2 x^2 + \log_2 y^4 = \log_2 x^2y^4$

67. $\log x + 2\log(x+1) = \log x + \log(x+1)^2$

$= \log[x(x+1)^2]$

68. $2 \ln 8 - 5 \ln(z - 4) = \ln 8^2 - \ln(z - 4)^5$
$= \ln 64 - \ln(z - 4)^5$
$= \ln\left[\frac{64}{(z - 4)^5}\right]$

69. $\log x - 2 \log y + 3 \log z = \log x - \log y^2 + \log z^3$
$= \log \frac{x}{y^2} + \log z^3$
$= \log \frac{xz^3}{y^2}$

70. $3 \log_3 x + \frac{1}{4} \log_3 y - 4 \log_3 z = \log_3 x^3 + \log_3 y^{1/4} - \log_3 z^4$
$= \log_3 \left(x^3 \sqrt[4]{y}\right) - \log_3 z^4$
$= \log_3 \left(\frac{x^3 \sqrt[4]{y}}{z^4}\right)$

71. $\ln x - \left[\ln(x + 1) + \ln(x - 1)\right] = \ln x - \ln(x + 1)(x - 1) = \ln \frac{x}{(x + 1)(x - 1)}$

72. $4\left[\ln z + \ln(z + 5)\right] - 2 \ln(z - 5) = 4\left[\ln z(z + 5)\right] - \ln(z - 5)^2$
$= \ln\left[z(z + 5)\right]^4 - \ln(z - 5)^2$
$= \ln \frac{z^4(z + 5)^4}{(z - 5)^2}$

73. $\frac{1}{2}\left[2 \ln(x + 3) + \ln x - \ln\left(x^2 - 1\right)\right] = \frac{1}{2}\left[\ln(x + 3)^2 + \ln x - \ln\left(x^2 - 1\right)\right]$
$= \frac{1}{2}\left[\ln\left[x(x + 3)^2\right] - \ln\left(x^2 - 1\right)\right]$
$= \frac{1}{2}\left[\ln\left(\frac{x(x + 3)^2}{x^2 - 1}\right)\right]$
$= \frac{1}{2} \ln\left[\frac{x(x + 3)^2}{x^2 - 1}\right]$
$= \ln \sqrt{\frac{x(x + 3)^2}{x^2 - 1}}$

74. $2\left[3 \ln x - \ln(x + 1) - \ln(x - 1)\right] = 2\left[\ln x^3 - \ln(x + 1) - \ln(x - 1)\right]$
$= 2\left[\ln x^3 - \left[\ln(x + 1) + \ln(x - 1)\right]\right]$
$= 2\left[\ln x^3 - \left[\ln(x + 1)(x - 1)\right]\right]$
$= 2 \ln \frac{x^3}{x^2 - 1}$
$= \ln\left(\frac{x^3}{x^2 - 1}\right)^2$

75. $\frac{1}{3}\left[\log_8 y + 2 \log_8(y + 4)\right] - \log_8(y - 1) = \frac{1}{3}\left[\log_8 y + \log_8(y + 4)^2\right] - \log_8(y - 1)$
$= \frac{1}{3} \log_8 y(y + 4)^2 - \log_8(y - 1)$
$= \log_8 \sqrt[3]{y(y + 4)^2} - \log_8(y - 1)$
$= \log_8\left(\frac{\sqrt[3]{y(y + 4)^2}}{y - 1}\right)$

76. $\frac{1}{2}\left[\log_4(x+1) + 2\log_4(x-1)\right] + 6\log_4 x = \frac{1}{2}\left[\log_4(x+1) + \log_4(x-1)^2\right] + \log_4 x^6$

$= \frac{1}{2}\left[\log_4(x+1)(x-1)^2\right] + \log_4 x^6$

$= \log_4\left[\sqrt{x+1}(x-1)\right] + \log_4 x^6$

$= \log_4\left[x^6(x-1)\sqrt{x+1}\right]$

77. $\log_2 \frac{32}{4} = \log_2 32 - \log_2 4 \neq \frac{\log_2 32}{\log_2 4}$

The second and third expressions are equal by Property 2.

78. $\log_7 \sqrt{70} = \frac{1}{2}\log_7 70 = \frac{1}{2}[\log_7 7 + \log_7 10]$

$= \frac{1}{2}[1 + \log_7 10]$

$= \frac{1}{2} + \frac{1}{2}\log_7 10$

$= \frac{1}{2} + \log_7 \sqrt{10}$ by Property 1 and Property 3

79. $\beta = 10\log\left(\frac{I}{10^{-12}}\right) = 10\left[\log I - \log 10^{-12}\right] = 10\left[\log I + 12\right] = 120 + 10\log I$

When $I = 10^{-6}$:

$\beta = 120 + 10\log 10^{-6} = 120 + 10(-6) = 60$ decibels

80. $\beta = 10\log\left(\frac{I}{10^{-12}}\right)$

Difference

$= 10\log\left(\frac{1.26\times10^{-7}}{10^{-12}}\right) - 10\log\left(\frac{3.16\times10^{-10}}{10^{-12}}\right)$

$= 10\left[\log\left(1.26\times10^5\right) - \log\left(3.16\times10^2\right)\right]$

$= 10\left[\log\left(\frac{1.26\times10^5}{3.16\times10^2}\right)\right]$

$\approx 10\log\left(0.3987\times10^3\right)$

$= 10\log(398.7)$

≈ 26 dB

81. $\beta = 10\log\left(\frac{I}{10^{-12}}\right)$

Difference $= 10\log\left(\frac{10^{-4}}{10^{-12}}\right) - 10\log\left(\frac{10^{-11}}{10^{-12}}\right)$

$= 10\left[\log 10^8 - \log 10\right]$

$= 10(8-1)$

$= 10(7)$

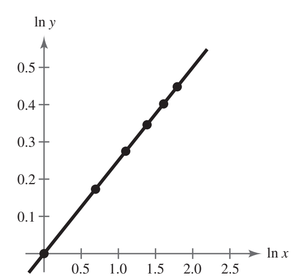
$= 70$ dB

82. $\beta = 120 + 10\log(2I) = 120 + 10(\log 2 + \log I) = (120 + 10\log I) + 10\log 2$

With both stereos playing, the music is $10\log 2 \approx 3$ decibels louder.

83.

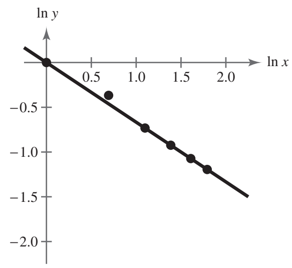
x	1	2	3	4	5	6
y	1.000	1.189	1.316	1.414	1.495	1.565
$\ln x$	0	0.693	1.099	1.386	1.609	1.792
$\ln y$	0	0.173	0.275	0.346	0.402	0.448



The slope of the line is $\frac{1}{4}$. So, $\ln y = \frac{1}{4}\ln x$

84.

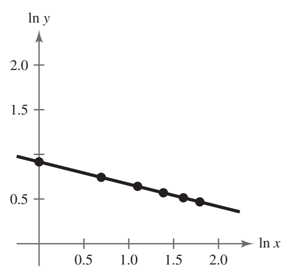
x	1	2	3	4	5	6
y	1.000	0.630	0.481	0.397	0.342	0.303
$\ln x$	0	0.693	1.099	1.386	1.609	1.792
$\ln y$	0	-0.367	-0.732	-0.924	-1.073	-1.195



The slope of the line is $-\frac{2}{3}$. So, $\ln y = -\frac{2}{3} \ln x$.

85.

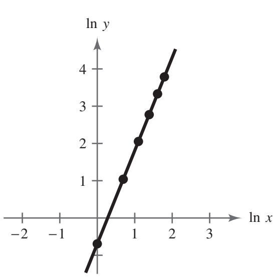
x	1	2	3	4	5	6
y	2.500	2.102	1.900	1.768	1.672	1.597
$\ln x$	0	0.693	1.099	1.386	1.609	1.792
$\ln y$	0.916	0.743	0.642	0.570	0.514	0.468



The slope of the line is $-\frac{1}{4}$. So, $\ln y = -\frac{1}{4} \ln x + \ln \frac{5}{2}$.

86.

x	1	2	3	4	5	6
y	0.500	2.828	7.794	16.000	27.951	44.091
$\ln x$	0	0.693	1.099	1.386	1.609	1.792
$\ln y$	-0.693	1.040	2.053	2.773	3.330	3.786



The slope of the line is $\frac{5}{2}$. So, $\ln y = \frac{5}{2} \ln x - \ln 2$.

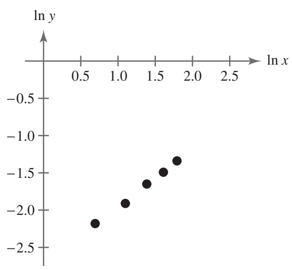
87.

Weight, x	25	35	50	75	500	1000
Galloping Speed, y	191.5	182.7	173.8	164.2	125.9	114.2
$\ln x$	3.219	3.555	3.912	4.317	6.215	6.908
$\ln y$	5.255	5.208	5.158	5.101	4.835	4.738

$\ln y = -0.14 \ln x + 5.7$

88. Take the natural logarithm of each of the x- and y-values.

x	y	$\ln x$	$\ln y$
2	0.113	0.6931	−2.1804
3	0.148	1.0986	−1.9105
4	0.192	1.3863	−1.6503
5	0.225	1.6094	−1.4917
6	0.262	1.7918	−1.3394



By plotting the points from the table, you can see that the points appear to lie on a line.

Use the points $(0.6931, -2.1804)$ and $(1.7918, -1.3394)$ to find the slope of the line.

$$m = \frac{-1.3394 - (-2.1804)}{1.7918 - 0.6931} \approx 0.7655$$

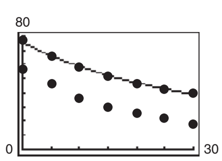
Use point-slope form where $Y = \ln y$ and $X = \ln x$.

$$Y - (-2.1804) = 0.7655(X - 0.6931)$$
$$Y + 2.1804 = 0.7655X - 0.5306$$

So, $\ln y = 0.7655 \ln x - 2.711$

Using the linear regression feature of a graphing utility yields:
$\ln y = 0.772 \ln x - 2.731$

89. (a)



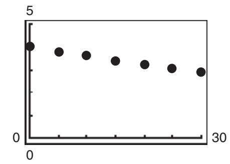
(b) $T - 21 = 54.4(0.964)^t$

$T = 54.4(0.964)^t + 21$

See graph in (a).

(c)

t (in minutes)	T (°C)	$T - 21$ (°C)	$\ln(T - 21)$	$1/(T - 21)$
0	78	57	4.043	0.0175
5	66	45	3.807	0.0222
10	57.5	36.5	3.597	0.0274
15	51.2	30.2	3.408	0.0331
20	46.3	25.3	3.231	0.0395
25	42.5	21.5	3.068	0.0465
30	39.6	18.6	2.923	0.0538



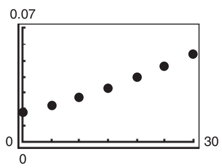
$\ln(T - 21) = -0.037t + 4$

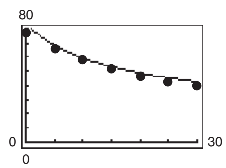
$T = e^{-0.037t+3.997} + 21$

This graph is identical to T in (b).

(d) $\dfrac{1}{T - 21} = 0.0012t + 0.016$

$T = \dfrac{1}{0.001t + 0.016} + 21$





90. Answers will vary. Sample Answer: If $y = ab^x$, then $\ln y = \ln(ab^x) = \ln a + x \ln b$, which is linear. If $y = \dfrac{1}{cx + d}$, then $\dfrac{1}{y} = cx + d$.

91. $f(x) = \ln x$

False, $f(0) \neq 0$ because 0 is not in the domain of $f(x)$.

$f(1) = \ln 1 = 0$

92. $f(ax) = f(a) + f(x), a > 0, x > 0$

True, because $f(ax) = \ln ax = \ln a + \ln x = f(a) + f(x)$ (property 1).

93. False.

$f(x) - f(2) = \ln x - \ln 2 = \ln \dfrac{x}{2} \neq \ln(x - 2)$

94. False.

$\sqrt{f(x)} = \sqrt{\ln x}$ can't be simplified further.

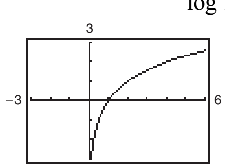
$f(\sqrt{x}) = \ln\sqrt{x} = \ln x^{1/2} = \frac{1}{2} \ln x = \frac{1}{2} f(x)$

95. False.

$f(u) = 2f(v) \Rightarrow \ln u = 2 \ln v \Rightarrow \ln u = \ln v^2 \Rightarrow u = v^2$

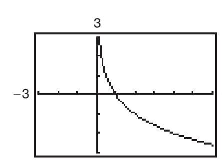
96. True. If $f(x) < 0$, then $0 < x < 1$.

97. $f(x) = \log_2 x = \dfrac{\log x}{\log 2} = \dfrac{\ln x}{\ln 2}$



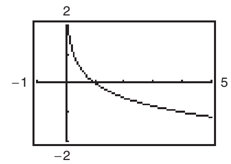
98. $f(x) = \log_{1/2} x$

$= \dfrac{\log x}{\log(1/2)} = \dfrac{\ln x}{\ln(1/2)}$



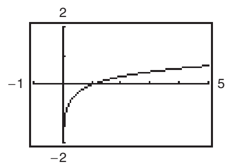
99. $f(x) = \log_{1/4} x$

$= \dfrac{\log x}{\log(1/4)} = \dfrac{\ln x}{\ln(1/4)}$



100. $f(x) = \log_{11.8} x$

$= \dfrac{\log x}{\log 11.8} = \dfrac{\ln x}{\ln 11.8}$



101. The power property cannot be used because $\ln e$ is raised to the second power, not just e.

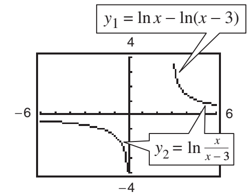
A correct statement is $(\ln e)^2 = (1)^2 = 1$.

102. $\log_2 8 = \log_2 (4 + 4) \neq \log_2 4 + \log_2 4$.

A correct statement is

$\log_2 8 = \log_2 2^3 = 3 \log_2 2 = 3(1) = 3$.

103.



The graphing utility does not show the functions with the same domain. The domain of $y_1 = \ln x - \ln(x - 3)$ is $(3, \infty)$ and the domain of $y_2 = \ln \dfrac{x}{x - 3}$ is $(-\infty, 0) \cup (3, \infty)$.

104. The function $y = \ln x$ matches graph B since the points $(1, 0)$ and $(e, 1)$ are located on the graph.

The function $y = \ln x^2, x > 0$ matches graph D since the point $(1, 0)$ is located on the graph and the graph increases at a greater rate than $y = \ln x$.

The function $y = \ln 2x$ matches graph C since the point $\left(\frac{1}{2}, 0\right)$ is located on the graph.

The function $y = \ln 2$ matches graph A since it is a constant function, represented by a horizontal line.

105. $\ln 2 \approx 0.6931, \ln 3 \approx 1.0986, \ln 5 \approx 1.6094$

$\ln 1 = 0$

$\ln 2 \approx 0.6931$

$\ln 3 \approx 1.0986$

$\ln 4 = \ln(2 \cdot 2) = \ln 2 + \ln 2 \approx 0.6931 + 0.6931 = 1.3862$

$\ln 5 \approx 1.6094$

$\ln 6 = \ln(2 \cdot 3) = \ln 2 + \ln 3 \approx 0.6931 + 1.0986 = 1.7917$

$\ln 8 = \ln 2^3 = 3 \ln 2 \approx 3(0.6931) = 2.0793$

$\ln 9 = \ln 3^2 = 2 \ln 3 \approx 2(1.0986) = 2.1972$

$\ln 10 = \ln(5 \cdot 2) = \ln 5 + \ln 2 \approx 1.6094 + 0.6931 = 2.3025$

$\ln 12 = \ln(2^2 \cdot 3) = \ln 2^2 + \ln 3 = 2 \ln 2 + \ln 3 \approx 2(0.6931) + 1.0986 = 2.4848$

$\ln 15 = \ln(5 \cdot 3) = \ln 5 + \ln 3 \approx 1.6094 + 1.0986 = 2.7080$

$\ln 16 = \ln 2^4 = 4 \ln 2 \approx 4(0.6931) = 2.7724$

$\ln 18 = \ln(3^2 \cdot 2) = \ln 3^2 + \ln 2 = 2 \ln 3 + \ln 2 \approx 2(1.0986) + 0.6931 = 2.8903$

$\ln 20 = \ln(5 \cdot 2^2) = \ln 5 + \ln 2^2 = \ln 5 + 2 \ln 2 \approx 1.6094 + 2(0.6931) = 2.9956$

Section 3.4 Exponential and Logarithmic Equations

1. (a) $x = y$

(b) $x = y$

(c) x

(d) x

2. extraneous

3. $4^{2x-7} = 64$

(a) $x = 5$

$4^{2(5)-7} = 4^3 = 64$

Yes, $x = 5$ *is* a solution.

(b) $x = 2$

$4^{2(2)-7} = 4^{-3} = \frac{1}{64} \neq 64$

No, $x = 2$ *is not* a solution.

(c) $x = \frac{1}{2}(\log_4 64 + 7)$

$4^{2(1/2(\log_4 64+7))-7} = 64$

$4^{(\log_4 64+7)-7} = 64$

$4^{(3+7)-7} = 64$

$4^3 = 64$

Yes, $x = \frac{1}{2}(\log_4 64 + 7)$ *is* a solution.

4. $4e^{x-1} = 60$

(a) $x = 1 + \ln 15$

$4e^{(1+\ln 15)-1} = 4e^{\ln 15} = 4(15) = 60$

Yes, $x = 1 + \ln 15$ *is* a solution.

(b) $x = \ln 1.708$

$4e^{\ln 1.708-1} = 4e^{\ln 1.708}e^{-1} = 4(1.708)e^{-1} = \frac{6.832}{e} \neq 60$

No, $x = \ln 1.708$ *is not* a solution.

(c) $x = \ln 16$

$4e^{\ln 16-1} = 4e^{\ln 16}e^{-1} = 4(16)e^{-1} = \frac{64}{e} \neq 60$

No, $x = \ln 16$ *is not* a solution.

5. $\log_2(x + 3) = 10$

(a) $x = 1021$

$\log_2(1021 + 3) = \log_2(1024)$

Because $2^{10} = 1024$, $x = 1021$ *is* a solution.

(b) $x = 17$

$\log_2(17 + 3) = \log_2(20)$

Because $2^{10} \neq 20$, $x = 17$ *is not* a solution.

(c) $x = 10^2 - 3 = 97$

$\log_2(97 + 3) = \log_2(100)$

Because $2^{10} \neq 100$, $10^2 - 3$ *is not* a solution.

6. $\ln(2x + 3) = 5.8$

(a) $x = \frac{1}{2}(-3 + \ln 5.8)$

$\ln\left[2\left(\frac{1}{2}\right)(-3 + \ln 5.8) + 3\right] = \ln(\ln 5.8) \neq 5.8$

No, $x = \frac{1}{2}(-3 + \ln 5.8)$ *is not* a solution.

(b) $x = \frac{1}{2}(-3 + e^{5.8})$

$\ln\left[2\left(\frac{1}{2}\right)(-3 + e^{5.8}) + 3\right] = \ln(e^{5.8}) = 5.8$

Yes, $x = \frac{1}{2}(-3 + e^{5.8})$ *is* a solution.

(c) $x \approx 163.650$

$\ln\left[2(163.650) + 3\right] = \ln 330.3 \approx 5.8$

Yes, $x \approx 163.650$ *is* an approximate solution.

7. $4^x = 16$

$4^x = 4^2$

$x = 2$

8. $\left(\frac{1}{2}\right)^x = 32$

$2^{-x} = 2^5$

$-x = 5$

$x = -5$

9. $\ln x - \ln 2 = 0$

$\ln x = \ln 2$

$x = 2$

10. $\log x - \log 10 = 0$

$\log x - 1 = 0$

$\log x = 1$

$10^{\log x} = 10$

$x = 10$

11. $e^x = 2$

$\ln e^x = \ln 2$

$x = \ln 2$

$x \approx 0.693$

12. $e^x = \frac{1}{3}$
$\ln e^x = \ln\left(\frac{1}{3}\right)$
$x = \ln\left(\frac{1}{3}\right) \approx -1.099$

13. $\ln x = -1$
$e^{\ln x} = e^{-1}$
$x = e^{-1}$
$x \approx 0.368$

14. $\log x = -2$
$10^{\log x} = 10^{-2}$
$x = 10^{-2}$
$x = \frac{1}{100} = 0.01$

15. $\log_4 x = 3$
$4^{\log_4 x} = 4^3$
$x = 4^3$
$x = 64$

16. $\log_5 x = \frac{1}{2}$
$5^{\log_5 x} = 5^{1/2}$
$x = \sqrt{5} \approx 2.236$

17. $f(x) = g(x)$
$2^x = 8$
$2^x = 2^3$
$x = 3$
Point of intersection:
$(3, 8)$

18. $f(x) = g(x)$
$\log_3 x = 2$
$x = 3^2$
$x = 9$
Point of intersection:
$(9, 2)$

19. $e^x = e^{x^2-2}$
$x = x^2 - 2$
$0 = x^2 - x - 2$
$0 = (x + 1)(x - 2)$
$x = -1, x = 2$

20. $e^{x^2-3} = e^{x-2}$
$x^2 - 3 = x - 2$
$x^2 - x - 1 = 0$
By the Quadratic Formula
$x \approx 1.618, x \approx -0.618.$

21. $4(3^x) = 20$
$3^x = 5$
$\log_3 3^x = \log_3 5$
$x = \log_3 5 = \frac{\log 5}{\log 3}$ or $\frac{\ln 5}{\ln 3}$
$x \approx 1.465$

22. $4e^x = 91$
$e^x = \frac{91}{4}$
$\ln e^x = \ln \frac{91}{4}$
$x = \ln \frac{91}{4} \approx 3.125$

23. $e^x - 8 = 31$
$e^x = 39$
$\ln e^x = \ln 39$
$x = \ln 39 \approx 3.664$

24. $5^x + 8 = 26$
$5^x = 18$
$x = \log_5 18$
$x = \frac{\ln 18}{\ln 5}$
$x \approx 1.796$

25. $3^{2x} = 80$
$\ln 3^{2x} = \ln 80$
$2x \ln 3 = \ln 80$
$x = \frac{\ln 80}{2 \ln 3} \approx 1.994$

26. $4^{-3t} = 0.10$
$\ln 4^{-3t} = \ln 0.10$
$(-3t) \ln 4 = \ln 0.10$
$-3t = \frac{\ln 0.10}{\ln 4}$
$t = -\frac{\ln 0.10}{3 \ln 4} \approx 0.554$

27.
$$\begin{aligned} 3^{2-x} &= 400 \\ \ln 3^{2-x} &= \ln 400 \\ (2 - x)\ln 3 &= \ln 400 \\ 2\ln 3 - x\ln 3 &= \ln 400 \\ -x\ln 3 &= \ln 400 - 2\ln 3 \\ x\ln 3 &= 2\ln 3 - \ln 400 \\ x &= \frac{2\ln 3 - \ln 400}{\ln 3} \\ x &= 2 - \frac{\ln 400}{\ln 3} \approx -3.454 \end{aligned}$$

28.
$$\begin{aligned} 7^{-3-x} &= 242 \\ \ln 7^{-3-x} &= \ln 242 \\ (-3 - x)\ln 7 &= \ln 242 \\ -3\ln 7 - x\ln 7 &= \ln 242 \\ -x\ln 7 &= \ln 242 + 3\ln 7 \\ x\ln 7 &= -3\ln 7 - \ln 242 \\ x &= \frac{-3\ln 7 - \ln 242}{\ln 7} \\ x &= -3 - \frac{\ln 242}{\ln 7} \approx -5.821 \end{aligned}$$

29.
$$\begin{aligned} 8(10^{3x}) &= 12 \\ 10^{3x} &= \tfrac{12}{8} \\ \log 10^{3x} &= \log\left(\tfrac{3}{2}\right) \\ 3x &= \log\left(\tfrac{3}{2}\right) \\ x &= \tfrac{1}{3}\log\left(\tfrac{3}{2}\right) \\ x &\approx 0.059 \end{aligned}$$

30.
$$\begin{aligned} 8(3^{6-x}) &= 40 \\ 3^{6-x} &= 5 \\ \ln 3^{6-x} &= \ln 5 \\ (6 - x)\ln 3 &= \ln 5 \\ 6 - x &= \frac{\ln 5}{\ln 3} \\ -x &= \frac{\ln 5}{\ln 3} - 6 \\ x &= 6 - \frac{\ln 5}{\ln 3} \approx 4.535 \end{aligned}$$

31.
$$\begin{aligned} e^{3x} &= 12 \\ 3x &= \ln 12 \\ x &= \frac{\ln 12}{3} \approx 0.828 \end{aligned}$$

32.
$$\begin{aligned} 500e^{-2x} &= 125 \\ e^{-2x} &= \tfrac{1}{4} \\ \ln e^{-2x} &= \ln\tfrac{1}{4} \\ -2x &= \ln\tfrac{1}{4} \\ x &= -\tfrac{1}{2}\ln\tfrac{1}{4} \\ x &\approx 0.693 \end{aligned}$$

33.
$$\begin{aligned} 7 - 2e^{x} &= 5 \\ -2e^{x} &= -2 \\ e^{x} &= 1 \\ x &= \ln 1 = 0 \end{aligned}$$

34.
$$\begin{aligned} -14 + 3e^{x} &= 11 \\ 3e^{x} &= 25 \\ e^{x} &= \tfrac{25}{3} \\ \ln e^{x} &= \ln\tfrac{25}{3} \\ x &= \ln\tfrac{25}{3} \\ x &\approx 2.120 \end{aligned}$$

35.
$$\begin{aligned} 6(2^{3x-1}) - 7 &= 9 \\ 6(2^{3x-1}) &= 16 \\ 2^{3x-1} &= \frac{8}{3} \\ \log_2 2^{3x-1} &= \log_2\left(\frac{8}{3}\right) \\ 3x - 1 &= \log_2\left(\frac{8}{3}\right) = \frac{\log(8/3)}{\log 2} \text{ or } \frac{\ln(8/3)}{\ln 2} \\ x &= \frac{1}{3}\left[\frac{\log(8/3)}{\log 2} + 1\right] \approx 0.805 \end{aligned}$$

36.
$$\begin{aligned} 8(4^{6-2x}) + 13 &= 41 \\ 8(4^{6-2x}) &= 28 \\ 4^{6-2x} &= 3.5 \\ 6 - 2x &= \log_4 3.5 \\ 6 - 2x &= \frac{\ln 3.5}{\ln 4} \\ -2x &= -6 + \frac{\ln 3.5}{\ln 4} \\ x &= 3 - \frac{\ln 3.5}{2\ln 4} \approx 2.548 \end{aligned}$$

37.
$$\begin{aligned} 3^x &= 2^{x-1} \\ \ln 3^x &= \ln 2^{x-1} \\ x \ln 3 &= (x-1)\ln 2 \\ x\ln 3 &= x\ln 2 - \ln 2 \\ x\ln 3 - x\ln 2 &= -\ln 2 \\ x(\ln 3 - \ln 2) &= -\ln 2 \\ x &= \frac{\ln 2}{\ln 2 - \ln 3} \approx -1.710 \end{aligned}$$

38.
$$\begin{aligned} e^{x+1} &= 2^{x+2} \\ \ln e^{x+1} &= \ln 2^{x+2} \\ x+1 &= (x+2)\ln 2 \\ x+1 &= x\ln 2 + 2\ln 2 \\ x - x\ln 2 &= 2\ln 2 - 1 \\ x(1-\ln 2) &= 2\ln 2 - 1 \\ x &= \frac{2\ln 2 - 1}{1 - \ln 2} \approx 1.259 \end{aligned}$$

39.
$$\begin{aligned} 4^x &= 5^{x^2} \\ \ln 4^x &= \ln 5^{x^2} \\ x\ln 4 &= x^2 \ln 5 \\ x^2\ln 5 - x\ln 4 &= 0 \\ x(x\ln 5 - \ln 4) &= 0 \end{aligned}$$

$x = 0$

$x\ln 5 - \ln 4 = 0 \Rightarrow x = \frac{\ln 4}{\ln 5} \approx 0.861$

40.
$$\begin{aligned} 3^{x^2} &= 7^{6-x} \\ \ln 3^{x^2} &= \ln 7^{6-x} \\ x^2\ln 3 &= (6-x)\ln 7 \\ x^2\ln 3 &= 6\ln 7 - x\ln 7 \\ x^2\ln 3 + x\ln 7 - 6\ln 7 &= 0 \end{aligned}$$

Use Quadratic Formula:

$$x = \frac{-\ln 7 \pm \sqrt{(\ln 7)^2 - 4(\ln 3)(-6\ln 7)}}{2(\ln 3)}$$

$$x = \frac{-\ln 7 \pm \sqrt{(\ln 7)^2 + 24(\ln 3)(\ln 7)}}{2\ln 3} \approx -4.264, 2.493$$

41.
$$\begin{aligned} e^{2x} - 4e^x - 5 &= 0 \\ (e^x + 1)(e^x - 5) &= 0 \end{aligned}$$

$e^x = -1$ or $e^x = 5$

(No solution) $x = \ln 5 \approx 1.609$

42.
$$\begin{aligned} e^{2x} - 5e^x + 6 &= 0 \\ (e^x - 2)(e^x - 3) &= 0 \end{aligned}$$

$e^x = 2$ or $e^x = 3$

$x = \ln 2 \approx 0.693$ or $x = \ln 3 \approx 1.099$

43.
$$\begin{aligned} \frac{1}{1-e^x} &= 5 \\ 1 &= 5(1-e^x) \\ \frac{1}{5} &= 1 - e^x \\ \frac{1}{5} - 1 &= -e^x \\ -\frac{4}{5} &= -e^x \\ \frac{4}{5} &= e^x \\ \ln\frac{4}{5} &= \ln e^x \\ \ln\frac{4}{5} &= x \\ x &\approx -0.223 \end{aligned}$$

44.
$$\begin{aligned} \frac{100}{1+e^{2x}} &= 1 \\ 100 &= 1 + e^{2x} \\ 99 &= e^{2x} \\ \ln 99 &= \ln e^{2x} \\ \ln 99 &= 2x \\ \frac{1}{2}\ln 99 &= x \\ x &\approx 2.298 \end{aligned}$$

45.
$$\begin{aligned} \left(1 + \frac{0.065}{365}\right)^{365t} &= 4 \\ \ln\left(1 + \frac{0.065}{365}\right)^{365t} &= \ln 4 \\ 365t\ln\left(1 + \frac{0.065}{365}\right) &= \ln 4 \\ t &= \frac{\ln 4}{365\ln\left(1 + \frac{0.065}{365}\right)} \approx 21.330 \end{aligned}$$

46. $\left(1 + \dfrac{0.10}{12}\right)^{12t} = 2$

$\ln\left(1 + \dfrac{0.10}{12}\right)^{12t} = \ln 2$

$12t \ln\left(1 + \dfrac{0.10}{12}\right) = \ln 2$

$t = \dfrac{\ln 2}{12 \ln\left(1 + \dfrac{0.10}{12}\right)} \approx 6.960$

47. $\ln x = -3$

$x = e^{-3} \approx 0.050$

48. $\ln x - 7 = 0$

$\ln x = 7$

$x = e^7 \approx 1096.633$

49. $2.1 = \ln 6x$

$e^{2.1} = 6x$

$\dfrac{e^{2.1}}{6} = x$

$1.361 \approx x$

50. $\log 3z = 2$

$10^{\log 3z} = 10^2$

$3z = 100$

$z = \frac{100}{3} \approx 33.333$

51. $3 - 4 \ln x = 11$

$-4 \ln x = 8$

$\ln x = -2$

$x = e^{-2} = \dfrac{1}{e^2} \approx 0.135$

52. $3 + 8 \ln x = 7$

$8 \ln x = 4$

$\ln x = \dfrac{1}{2}$

$x = e^{1/2} = \sqrt{e} \approx 1.649$

53. $6 \log_3(0.5x) = 11$

$\log_3(0.5x) = \frac{11}{6}$

$3^{\log_3(0.5x)} = 3^{11/6}$

$0.5x = 3^{11/6}$

$x = 2\left(3^{11/6}\right) \approx 14.988$

54. $4 \log(x - 6) = 11$

$\log(x - 6) = \frac{11}{4}$

$10^{\log(x-6)} = 10^{11/4}$

$x - 6 = 10^{11/4}$

$x = 10^{11/4} + 6 \approx 568.341$

55. $\ln x - \ln(x + 1) = 2$

$\ln\left(\dfrac{x}{x + 1}\right) = 2$

$\dfrac{x}{x + 1} = e^2$

$x = e^2(x + 1)$

$x = e^2 x + e^2$

$x - e^2 x = e^2$

$x(1 - e^2) = e^2$

$x = \dfrac{e^2}{1 - e^2} \approx -1.157$

This negative value is extraneous. The equation has no solution.

56. $\ln x + \ln(x + 1) = 1$

$\ln[x(x + 1)] = 1$

$e^{\ln[x(x+1)]} = e^1$

$x(x + 1) = e^1$

$x^2 + x - e = 0$

$x = \dfrac{-1 \pm \sqrt{1 + 4e}}{2}$

The only solution is $x = \dfrac{-1 + \sqrt{1 + 4e}}{2} \approx 1.223.$

57. $\ln(x + 5) = \ln(x - 1) - \ln(x + 1)$

$\ln(x + 5) = \ln\left(\dfrac{x - 1}{x + 1}\right)$

$x + 5 = \dfrac{x - 1}{x + 1}$

$(x + 5)(x + 1) = x - 1$

$x^2 + 6x + 5 = x - 1$

$x^2 + 5x + 6 = 0$

$(x + 2)(x + 3) = 0$

$x = -2 \quad \text{or} \quad x = -3$

Both of these solutions are extraneous, so the equation has no solution.

58.
$$\ln(x+1) - \ln(x-2) = \ln x$$
$$\ln\left(\frac{x+1}{x-2}\right) = \ln x$$
$$\frac{x+1}{x-2} = x$$
$$x + 1 = x^2 - 2x$$
$$0 = x^2 - 3x - 1$$
$$\frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2(1)} = x$$
$$\frac{3 \pm \sqrt{13}}{2} = x$$

The negative value is extraneous. The only solution is $x = \frac{3 + \sqrt{13}}{2} \approx 3.303$.

59.
$$\log(3x + 4) = \log(x - 10)$$
$$3x + 4 = x - 10$$
$$2x = -14$$
$$x = -7$$

The negative value is extraneous.
The equation has no solution.

60.
$$\log_2 x + \log_2(x+2) = \log_2(x+6)$$
$$\log_2[x(x+2)] = \log_2(x+6)$$
$$x(x+2) = x + 6$$
$$x^2 + x - 6 = 0$$
$$(x+3)(x-2) = 0$$
$$x = -3 \text{ or } x = 2$$

The value $x = -3$ is extraneous. The only solution is $x = 2$.

61.
$$\log_4 x - \log_4(x-1) = \frac{1}{2}$$
$$\log_4\left(\frac{x}{x-1}\right) = \frac{1}{2}$$
$$4^{\log_4[x/(x-1)]} = 4^{1/2}$$
$$\frac{x}{x-1} = 4^{1/2}$$
$$x = 2(x-1)$$
$$x = 2x - 2$$
$$-x = -2$$
$$x = 2$$

62.
$$\log 8x - \log\left(1 + \sqrt{x}\right) = 2$$
$$\log\frac{8x}{1+\sqrt{x}} = 2$$
$$\frac{8x}{1+\sqrt{x}} = 10^2$$
$$8x = 100\left(1 + \sqrt{x}\right)$$
$$2x = 25\left(1 + \sqrt{x}\right) = 25 + 25\sqrt{x}$$
$$2x - 25 = 25\sqrt{x}$$
$$(2x - 25)^2 = \left(25\sqrt{x}\right)^2$$
$$4x^2 - 100x + 625 = 625x$$
$$4x^2 - 725x + 625 = 0$$
$$x = \frac{725 \pm \sqrt{725^2 - 4(4)(625)}}{2(4)} = \frac{725 \pm \sqrt{515{,}625}}{8} = \frac{25\left(29 \pm 5\sqrt{33}\right)}{8}$$

$x \approx 0.866$ (extraneous) or $x \approx 180.384$

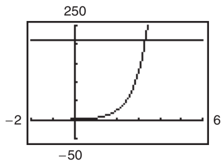
The only solution is $x = \frac{25\left(29 + 5\sqrt{33}\right)}{8} \approx 180.384$.

63. $f(x) = 5^x - 212$

Algebraically:

$$5^x = 212$$
$$\ln 5^x = \ln 212$$
$$x \ln 5 = \ln 212$$
$$x = \frac{\ln 212}{\ln 5}$$
$$x \approx 3.328$$

The zero is $x \approx 3.328$.

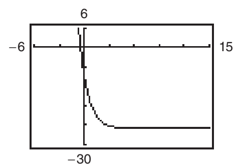


64. $g(x) = 6e^{1-x} - 25$

Algebraically:

$$6e^{1-x} = 25$$
$$e^{1-x} = \frac{25}{6}$$
$$1 - x = \ln\left(\frac{25}{6}\right)$$
$$x = 1 - \ln\left(\frac{25}{6}\right)$$
$$x \approx -0.427$$

The zero is $x \approx -0.427$.

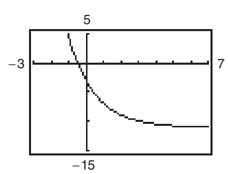


65. $g(x) = 8e^{-2x/3} - 11$

Algebraically:

$$8e^{-2x/3} = 11$$
$$e^{-2x/3} = 1.375$$
$$-\frac{2x}{3} = \ln 1.375$$
$$x = -1.5 \ln 1.375$$
$$x \approx -0.478$$

The zero is $x \approx -0.478$.

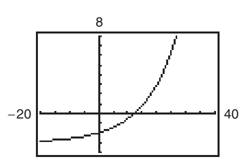


66. $g(t) = e^{0.09t} - 3$

Algebraically:

$$e^{0.09t} = 3$$
$$0.09t = \ln 3$$
$$t = \frac{\ln 3}{0.09}$$
$$t \approx 12.207$$

The zero is $t \approx 12.207$.

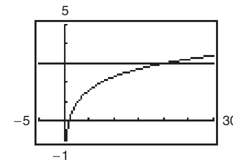


67. $y_1 = 3$

$y_2 = \ln x$

From the graph,

$x \approx 20.086$ when $y = 3$.



Algebraically:

$$3 - \ln x = 0$$
$$\ln x = 3$$
$$x = e^3 \approx 20.086$$

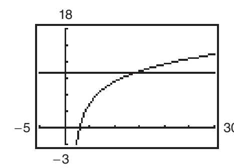
68. $y_1 = 4\ln(x - 2)$

$y_2 = 10$

From the graph, $x \approx 14.182$ when $y = 10$.

Algebraically:

$$10 - 4\ln(x - 2) = 0$$
$$-4\ln(x - 2) = -10$$
$$\ln(x - 2) = 2.5$$
$$e^{\ln(x-2)} = e^{2.5}$$
$$x - 2 = e^{2.5}$$
$$x = e^{2.5} + 2$$
$$x \approx 14.182$$



The solution is $x \approx 14.182$.

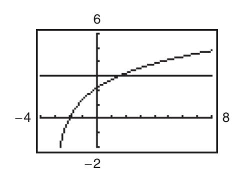
69. $y_1 = 2\ln(x + 3)$

$y_2 = 3$

From the graph, $x \approx 1.482$ when $y = 3$.

Algebraically:

$$2\ln(x + 3) = 3$$
$$\ln(x + 3) = \tfrac{3}{2}$$
$$x + 3 = e^{3/2}$$
$$x = e^{3/2} - 3 \approx 1.482$$



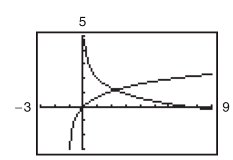
70. $y_1 = \ln(x + 1)$

$y_2 = 2 - \ln x$

From the graph, $x \approx 2.264$ when $y \approx 1.183$.

Algebraically:

$$\ln(x + 1) = 2 - \ln x$$
$$\ln(x + 1) + \ln x = 2$$
$$\ln\left[x(x + 1)\right] = 2$$
$$x(x + 1) = e^2$$
$$x^2 + x - e^2 = 0$$
$$x = \frac{-1 \pm \sqrt{1 + 4e^2}}{2}$$



The negative value is extraneous. The only solution is

$$x = \frac{-1 + \sqrt{1 + 4e^2}}{2} \approx 2.264.$$

71. (a)
$$\begin{aligned} r &= 0.025 \\ A &= Pe^{rt} \\ 5000 &= 2500e^{0.025t} \\ 2 &= e^{0.025t} \\ \ln 2 &= 0.025t \\ \frac{\ln 2}{0.025} &= t \\ t &\approx 27.73 \text{ years} \end{aligned}$$

(b)
$$\begin{aligned} r &= 0.025 \\ A &= Pe^{rt} \\ 7500 &= 2500e^{0.025t} \\ 3 &= e^{0.025t} \\ \ln 3 &= 0.025t \\ \frac{\ln 3}{0.025} &= t \\ t &\approx 43.94 \text{ years} \end{aligned}$$

72. (a)
$$\begin{aligned} r &= 0.0375 \\ A &= Pe^{rt} \\ 5000 &= 2500e^{0.0375t} \\ 2 &= e^{0.0375t} \\ \ln 2 &= 0.0375t \\ \frac{\ln 2}{0.0375} &= t \\ t &\approx 18.48 \text{ years} \end{aligned}$$

(b)
$$\begin{aligned} r &= 0.0375 \\ A &= Pe^{rt} \\ 7500 &= 2500e^{0.0375t} \\ 3 &= e^{0.0375t} \\ \ln 3 &= 0.0375t \\ \frac{\ln 3}{0.0375} &= t \\ t &\approx 29.30 \text{ years} \end{aligned}$$

73.
$$\begin{aligned} 2x^2e^{2x} + 2xe^{2x} &= 0 \\ (2x^2 + 2x)e^{2x} &= 0 \\ 2x^2 + 2x &= 0 \quad (\text{because } e^{2x} \neq 0) \\ 2x(x+1) &= 0 \\ x &= 0, -1 \end{aligned}$$

74.
$$\begin{aligned} -x^2e^{-x} + 2xe^{-x} &= 0 \\ (-x^2 + 2x)e^{-x} &= 0 \\ -x^2 + 2x &= 0 \quad (\text{because } e^{-x} \neq 0) \\ -x(x-2) &= 0 \\ x &= 0, 2 \end{aligned}$$

75.
$$\begin{aligned} -xe^{-x} + e^{-x} &= 0 \\ (-x+1)e^{-x} &= 0 \\ -x + 1 &= 0 \quad (\text{because } e^{-x} \neq 0) \\ x &= 1 \end{aligned}$$

76.
$$\begin{aligned} e^{-2x} - 2xe^{-2x} &= 0 \\ (1-2x)e^{-2x} &= 0 \\ 1 - 2x &= 0 \quad (\text{because } e^{-2x} \neq 0) \\ x &= \tfrac{1}{2} \end{aligned}$$

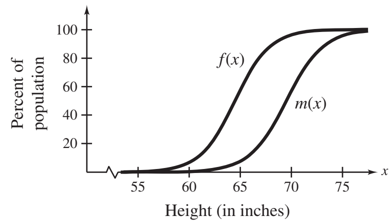
77.
$$\begin{aligned} \frac{1 + \ln x}{2} &= 0 \\ 1 + \ln x &= 0 \\ \ln x &= -1 \\ x &= e^{-1} = \frac{1}{e} \approx 0.368 \end{aligned}$$

78.
$$\begin{aligned} \frac{1 - \ln x}{x^2} &= 0 \\ 1 - \ln x &= 0 \quad (\text{because } x > 0) \\ \ln x &= 1 \\ x &= e \approx 2.718 \end{aligned}$$

79.
$$\begin{aligned} 2x\ln x + x &= 0 \\ x(2\ln x + 1) &= 0 \\ 2\ln x + 1 &= 0 \quad (\text{because } x > 0) \\ \ln x &= -\tfrac{1}{2} \\ x &= e^{-1/2} \approx 0.607 \end{aligned}$$

80.
$$\begin{aligned} 2x\ln\left(\frac{1}{x}\right) - x &= 0 \\ x\left(2\ln\left(\frac{1}{x}\right) - 1\right) &= 0 \\ 2\ln\left(\frac{1}{x}\right) - 1 &= 0 \quad (\text{because } x > 0) \\ \ln\left(\frac{1}{x}\right) &= \frac{1}{2} \\ \frac{1}{x} &= e^{1/2} \\ x &= e^{-1/2} \approx 0.607 \end{aligned}$$

81. (a)



From the graph you see horizontal asymptotes at $y = 0$ and $y = 100$.

These represent the lower and upper percent bounds; the range falls between 0% and 100%.

(b) Males:

$$50 = \frac{100}{1 + e^{-0.5536(x-69.51)}}$$

$$1 + e^{-0.5536(x-69.51)} = 2$$

$$e^{-0.5536(x-69.51)} = 1$$

$$-0.5536(x - 69.51) = \ln 1$$

$$-0.5536(x - 69.51) = 0$$

$$x = 69.51$$

The average height of an American male is 69.51 inches.

Females:

$$50 = \frac{100}{1 + e^{-0.5834(x-64.49)}}$$

$$1 + e^{-0.5834(x-64.49)} = 2$$

$$e^{-0.5834(x-64.49)} = 1$$

$$-0.5834(x - 64.49) = \ln 1$$

$$-0.5834(x - 64.49) = 0$$

$$x = 64.49$$

The average height of an American female is 64.49 inches.

82. (a) Let $p = 169$, and solve for x.

$$p = 5000\left(1 - \frac{4}{4 + e^{-0.002x}}\right)$$

$$169 = 5000\left(1 - \frac{4}{4 + e^{-0.002x}}\right)$$

$$\frac{169}{5000} = 1 - \frac{4}{4 + e^{-0.002x}}$$

$$-0.9662 = -\frac{4}{4 + e^{-0.002x}}$$

$$4 + e^{-0.002x} \approx 4.1399$$

$$e^{-0.002x} \approx 0.1399$$

$$-0.002x \approx \ln 0.1399$$

$$-0.002x \approx -1.9668$$

$$x \approx 983$$

When the price is \$169, the demand is 983 phones.

(b) Let $p = 299$ and solve for x.

$$299 = 5000\left(1 - \frac{4}{4 + e^{-0.002x}}\right)$$

$$\frac{299}{5000} = 1 - \frac{4}{4 + e^{-0.002x}}$$

$$-0.9402 = -\frac{4}{4 + e^{-0.002x}}$$

$$4 + e^{-0.002x} \approx 4.2544$$

$$e^{-0.002x} \approx 0.2544$$

$$-0.002x \approx \ln 0.2544$$

$$-0.002x \approx -1.3688$$

$$x \approx 684$$

When the price is \$299, the demand is 684 phones.

83. $N = 5.5 \cdot 10^{0.23x}$

When $N = 78$:

$$78 = 5.5 \cdot 10^{0.23x}$$

$$\frac{78}{5.5} = 10^{0.23x}$$

$$\log_{10} \frac{78}{5.5} = 0.23x$$

$$x = \frac{\log_{10}(78/5.5)}{0.23} \approx 5.008 \text{ years}$$

The beaver population will reach 78 in about 5 years.

84. $N = 3500\left(10^{-0.12x}\right)$

When $N = 22$:

$$22 = 3500\left(10^{-0.12x}\right)$$
$$\frac{22}{3500} = 10^{-0.12x}$$
$$\log_{10}\frac{22}{3500} = -0.12x$$
$$x = -\frac{\log_{10}(22/3500)}{0.12} \approx 18.347 \text{ inches}$$

85. $P = 75 \ln t + 540$

Let $P = 720$

$$720 = 75 \ln t + 540$$
$$180 = 75 \ln t$$
$$\frac{180}{75} = \ln t$$
$$\ln t = 2.4$$
$$t = e^{2.4} \approx 11.02 \text{ or } 2011$$

86. $P = 81 \ln t + 807$

Let $P = 965$

$$965 = 81 \ln t + 807$$
$$158 = 81 \ln t$$
$$\frac{158}{81} = \ln t$$
$$t = e^{158/81} \approx 7.03 \text{ or } 2007$$

87. $T = 20 + 60e^{-0.06m}$

Let $T = 70$

$$70 = 20 + 60e^{-0.06m}$$
$$50 = 60e^{-0.06m}$$
$$\tfrac{5}{6} = e^{-0.06m}$$
$$\ln\tfrac{5}{6} = -0.06m$$
$$m = -\tfrac{1}{0.06}\ln\tfrac{5}{6}$$
$$m \approx 3.039 \text{ minutes}$$

88. $T = 20 + 140e^{-0.68h}$

(a) From the graph, you see a horizontal asymptote at $T = 20$.

This horizontal asymptote represents the room temperature.

(b)
$$100 = 20 + 140e^{-0.68h}$$
$$80 = 140e^{-0.68h}$$
$$\frac{4}{7} = e^{-0.68h}$$
$$\ln\left(\frac{4}{7}\right) = \ln e^{-0.68h}$$
$$\ln\left(\frac{4}{7}\right) = -0.68h$$
$$\frac{\ln(4/7)}{-0.68} = h$$
$$h \approx 0.823 \text{ hour} \approx 49.4 \text{ minutes}$$

89. $\log_a(uv) = \log_a u + \log_a v$

True by Property 1 in Section 3.3.

90. $\log_a(u + v) = (\log_a u)(\log_a v)$

False.

$2.04 \approx \log_{10}(10 + 100) \neq (\log_{10}10)(\log_{10}100) = 2$

91. $\log_a(u - v) = \log_a u - \log_a v$

False.

$$1.95 = \log(100 - 10)$$
$$\neq \log 100 - \log 10 = 1$$

92. $\log_a\left(\frac{u}{v}\right) = \log_a u - \log_a v$

True by Property 2 in Section 3.3.

93. Yes, a logarithmic equation can have more than one extraneous solution. See Exercise 57.

94. The domain of the term $\log_3(x - 8)$ is $x > 8$. So, the domain of the entire function is also $x > 8$. Therefore, $x = 9$ is the only solution because $x = -1$ is extraneous.

95. $A = Pe^{rt}$

(a) $A = (2P)e^{rt} = 2\left(Pe^{rt}\right)$ This doubles your money.

(b) $A = Pe^{(2r)t} = Pe^{rt}e^{rt} = e^{rt}\left(Pe^{rt}\right)$

(c) $A = Pe^{r(2t)} = Pe^{rt}e^{rt} = e^{rt}\left(Pe^{rt}\right)$

Doubling the interest rate yields the same result as doubling the number of years.

If $2 > e^{rt}$ (i.e., $rt < \ln 2$), then doubling your investment would yield the most money. If $rt > \ln 2$, then doubling either the interest rate or the number of years would yield more money.

96. Yes.

Time to Double	Time to Quadruple
$2P = Pe^{rt}$	$4P = Pe^{rt}$
$2 = e^{rt}$	$4 = e^{rt}$
$\ln 2 = rt$	$\ln 4 = rt$
$\dfrac{\ln 2}{r} = t$	$\dfrac{2 \ln 2}{r} = t$

So, the time to quadruple is twice as long as the time to double.

97. (a) $P = 1000, r = 0.07,$ compounded annually, $n = 1$

Effective yield: $A = P\left(1 + \frac{r}{n}\right)^{nt} = 1000\left(1 + \frac{0.07}{1}\right)^{1} = \$1070$

$\frac{1070 - 1000}{1000} = 7\%$

The effective yield is 7%.

Balance after 5 years: $A = P\left(1 + \frac{r}{n}\right)^{nt} = 1000\left(1 + \frac{0.07}{1}\right)^{1(5)} \approx \$1402.55$

(b) $P = 1000, r = 0.07,$ compounded continuously

Effective yield: $A = Pe^{rt} = 1000e^{0.07(1)} \approx \$1072.51$

$\frac{1072.51 - 1000}{1000} = 7.25\%$

The effective yield is about 7.25%.

Balance after 5 years: $A = Pe^{rt} = 1000e^{0.07(5)} \approx \$1419.07$

(c) $P = 1000, r = 0.07,$ compounded quarterly, $n = 4$

Effective yield: $A = P\left(1 + \frac{r}{n}\right)^{nt} = 1000\left(1 + \frac{0.07}{4}\right)^{4(1)} \approx \$1071.86$

$\frac{1071.86 - 1000}{1000} = 7.19\%$

The effective yield is about 7.19%.

Balance after 5 years: $A = P\left(1 + \frac{r}{n}\right)^{nt} = 1000\left(1 + \frac{0.07}{4}\right)^{4(5)} \approx \$1414.78$

(d) $P = 1000, r = 0.0725,$ compounded quarterly, $n = 4$

Effective yield: $A = P\left(1 + \frac{r}{n}\right)^{nt} = 1000\left(1 + \frac{0.0725}{4}\right)^{4(1)} \approx \$1074.50$

$\frac{1074.50 - 1000}{1000} \approx 7.45\%$

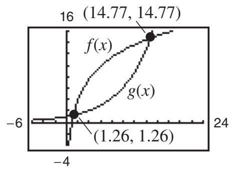
The effective yield is about 7.45%.

Balance after 5 years: $A = P\left(1 + \frac{r}{n}\right)^{nt} = 1000\left(1 + \frac{0.0725}{4}\right)^{4(5)} \approx \$1432.26$

Savings plan (d) has the greatest effective yield and the highest balance after 5 years.

98. $f(x) = \log_a x, g(x) = a^x, a > 1$

(a) $a = 1.2$



The curves intersect twice: $(1.258, 1.258)$ and $(14.767, 14.767)$

(b) If $f(x) = \log_a x = a^x = g(x)$ intersect exactly once, then

$x = \log_a x = a^x \Rightarrow a = x^{1/x}$.

The graphs of $y = x^{1/x}$ and $y = a$ intersect once for $a = e^{1/e} \approx 1.445$. Then

$\log_a x = x \Rightarrow \left(e^{1/e}\right)^x = x \Rightarrow e^{x/e} = x \Rightarrow x = e.$

For $a = e^{1/e}$, then curves intersect once at (e, e).

(c) For $1 < a < e^{1/e}$ the curves intersect twice. For $a > e^{1/e}$, the curves do not intersect.

Section 3.5 Exponential and Logarithmic Models

1. $y = ae^{bx}; y = ae^{-bx}$

2. $y = a + b \ln x; y = a + b \log x$

3. normally distributed

4. $y = \dfrac{a}{1 + be^{-rx}}$

5. (a)
$$A = Pe^{rt}$$
$$\frac{A}{e^{rt}} = P$$

(b)
$$A = Pe^{rt}$$
$$\frac{A}{P} = e^{rt}$$
$$\ln \frac{A}{P} = \ln e^{rt}$$
$$\ln \frac{A}{P} = rt$$
$$\frac{\ln(A/P)}{r} = t$$

6. (a)
$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$\frac{A}{\left(1 + \frac{r}{n}\right)^{nt}} = P$$
$$A\left(\frac{1}{\left(\frac{r + n}{n}\right)^{nt}}\right) = P$$
$$A\left(\frac{n}{r + n}\right)^{nt} = P$$

(b)
$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$\frac{A}{P} = \left(1 + \frac{r}{n}\right)^{nt}$$
$$\ln \frac{A}{P} = \ln\left(1 + \frac{r}{n}\right)^{nt}$$
$$\ln \frac{A}{P} = nt \ln\left(1 + \frac{r}{n}\right)$$
$$\frac{\ln \frac{A}{P}}{\ln\left(1 + \frac{r}{n}\right)} = nt$$
$$\frac{\ln \frac{A}{P}}{n \cdot \ln\left(\frac{r + n}{n}\right)} = t$$

7. Because $A = 1000e^{0.035t}$, the time to double is given by $2000 = 1000e^{0.035t}$ and you have

$$2 = e^{0.035t}$$
$$\ln 2 = \ln e^{0.035t}$$
$$\ln 2 = 0.035t$$
$$t = \frac{\ln 2}{0.035} \approx 19.8 \text{ years.}$$

Amount after 10 years: $A = 1000e^{0.35} \approx \$1419.07$

8. Because $A = 750e^{0.105t}$, the time to double is given by $1500 = 750e^{0.105t}$, and you have

$$1500 = 750e^{0.105t}$$
$$2 = e^{0.105t}$$
$$\ln 2 = \ln e^{0.105t}$$
$$\ln 2 = 0.105t$$
$$t = \frac{\ln 2}{0.105} \approx 6.60 \text{ years.}$$

Amount after 10 years: $A = 750e^{0.105(10)} \approx \$2143.24$

9. Because $A = 750e^{rt}$ and $A = 1500$ when $t = 7.75$, you have

$$1500 = 750e^{7.75r}$$
$$2 = e^{7.75r}$$
$$\ln 2 = \ln e^{7.75r}$$
$$\ln 2 = 7.75r$$
$$r = \frac{\ln 2}{7.75} \approx 0.089438 = 8.9438\%.$$

Amount after 10 years: $A = 750e^{0.089438(10)} \approx \$1834.37$

10. Because $A = 500e^{rt}$ and $A = \$1505.00$ when $t = 10$, you have

$$1505.00 = 500e^{10r}$$
$$r = \frac{\ln(1505.00/500)}{10} \approx 0.110 = 11.0\%.$$

The time to double is given by

$$1000 = 500e^{0.110t}$$
$$t = \frac{\ln 2}{0.110} \approx 6.3 \text{ years.}$$

11. Because $A = Pe^{0.045t}$ and $A = 10{,}000.00$ when $t = 10$, you have

$$10{,}000.00 = Pe^{0.045(10)}$$
$$\frac{10{,}000.00}{e^{0.045(10)}} = P \approx \$6376.28.$$

The time to double is given by

$$t = \frac{\ln 2}{0.045} \approx 15.40 \text{ years.}$$

12. Because $A = Pe^{rt}$ and the time to double is 12 years, you have $2P = Pe^{12r}$.

$$2P = Pe^{12r}$$
$$2 = e^{12r}$$
$$\ln 2 = \ln e^{12r}$$
$$\ln 2 = 12r$$
$$\tfrac{1}{12}\ln 2 = r$$
$$0.057762 \approx r$$
$$r \approx 5.7762\%$$

Amount after 10 years:

$$2000 = Pe^{(0.057762)(10)}$$
$$2000 = Pe^{0.57762}$$
$$\frac{2000}{e^{0.57762}} = P$$
$$1122.465 \approx P$$

13. $A = 500{,}000, r = 0.05, n = 12, t = 10$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$500{,}000 = P\left(1 + \frac{0.05}{12}\right)^{12(10)}$$
$$P = \frac{500{,}000}{\left(1 + \frac{0.05}{12}\right)^{12(10)}}$$
$$\approx \$303{,}580.52$$

14. $A = 500{,}000, r = 0.035, n = 12, t = 15$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$500{,}000 = P\left(1 + \frac{0.035}{12}\right)^{12(15)}$$
$$P = \frac{500{,}000}{\left(1 + \frac{0.035}{12}\right)^{12(15)}}$$
$$\approx \$296{,}003.78$$

15. $P = 1000, r = 0.1, A = 2000$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$2000 = 1000\left(1 + \frac{0.1}{n}\right)^{nt}$$

$$2 = \left(1 + \frac{0.1}{n}\right)^{nt}$$

(a) $n = 1$

$$(1 + 0.1)^t = 2$$

$$(1.1)^t = 2$$

$$\ln(1.1)^t = \ln 2$$

$$t \ln 1.1 = \ln 2$$

$$t = \frac{\ln 2}{\ln 1.1} \approx 7.27 \text{ years}$$

(b) $n = 12$

$$\left(1 + \frac{0.1}{12}\right)^{12t} = 2$$

$$\ln\left(\frac{12.1}{12}\right)^{12t} = \ln 2$$

$$12t \ln\left(\frac{12.1}{12}\right) = \ln 2$$

$$12t = \frac{\ln 2}{\ln(12.1/12)}$$

$$t = \frac{\ln 2}{12 \ln(12.1/12)} \approx 6.96 \text{ years}$$

(c) $n = 365$

$$\left(1 + \frac{0.1}{365}\right)^{365t} = 2$$

$$\ln\left(\frac{365.1}{365}\right)^{365t} = \ln 2$$

$$365t \ln\left(\frac{365.1}{365}\right) = \ln 2$$

$$365t = \frac{\ln 2}{\ln(365.1/365)}$$

$$t = \frac{\ln 2}{365 \ln(365.1/365)} \approx 6.93 \text{ years}$$

(d) Compounded continuously

$$A = Pe^{rt}$$

$$2000 = 1000e^{0.1t}$$

$$2 = e^{0.1t}$$

$$\ln 2 = \ln e^{0.1t}$$

$$0.1t = \ln 2$$

$$t = \frac{\ln 2}{0.1} \approx 6.93 \text{ years}$$

16. $P = 1000, r = 0.065, A = 2000$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$2000 = 1000\left(1 + \frac{0.065}{n}\right)^{nt}$$

$$2 = \left(1 + \frac{0.065}{n}\right)^{nt}$$

(a) $n = 1$

$$(1 + 0.065)^t = 2$$

$$(1.065)^t = 2$$

$$\ln(1.065)^t = \ln 2$$

$$t \ln(1.065) = \ln 2$$

$$t = \frac{\ln 2}{\ln 1.065} \approx 11.01 \text{ years}$$

(b) $n = 12$

$$\left(1 + \frac{0.065}{12}\right)^{12t} = 2$$

$$\ln\left(\frac{12.065}{12}\right)^{12t} = \ln 2$$

$$12t \ln\left(\frac{12.065}{12}\right) = \ln 2$$

$$12t = \frac{\ln 2}{\ln(12.065/12)}$$

$$t = \frac{\ln 2}{12 \ln(12.065/12)} \approx 10.69 \text{ years}$$

(c) $n = 365$

$$\left(1 + \frac{0.065}{365}\right)^{365t} = 2$$

$$\ln\left(\frac{365.065}{365}\right)^{365t} = \ln 2$$

$$365t \ln\left(\frac{365.065}{365}\right) = \ln 2$$

$$365t = \frac{\ln 2}{\ln(365.065/365)}$$

$$t = \frac{\ln 2}{365 \ln(365.065/365)} \approx 10.66 \text{ years}$$

(d) Compounded continuously

$$A = Pe^{rt}$$

$$2000 = 1000e^{0.065t}$$

$$2 = e^{0.065t}$$

$$\ln 2 = \ln e^{0.065t}$$

$$0.065t = \ln 2$$

$$t = \frac{\ln 2}{0.065} \approx 10.66 \text{ years}$$

17. (a)

$$3P = Pe^{rt}$$

$$3 = e^{rt}$$

$$\ln 3 = rt$$

$$\frac{\ln 3}{r} = t$$

r	2%	4%	6%	8%	10%	12%
$t = \dfrac{\ln 3}{r}$ (years)	54.93	27.47	18.31	13.73	10.99	9.16

(b)

$$3P = P(1 + r)^t$$

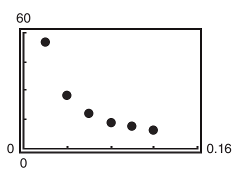
$$3 = (1 + r)^t$$

$$\ln 3 = \ln (1 + r)^t$$

$$\frac{\ln 3}{\ln (1 + r)} = t$$

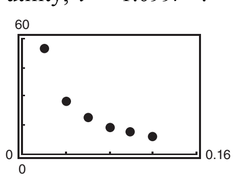
r	2%	4%	6%	8%	10%	12%
$t = \dfrac{\ln 3}{\ln (1 + r)}$ (years)	55.48	28.01	18.85	14.27	11.53	9.69

18. (a)



Using the power regression feature of a graphing utility, $t = 1.099r^{-1}$.

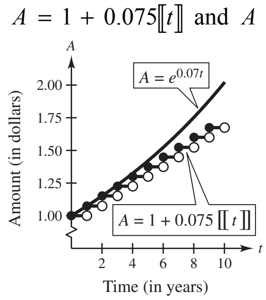
(b)



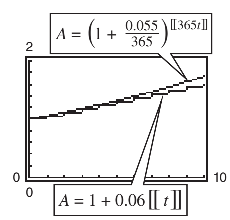
Using the power regression feature of a graphing utility, $t = 1.222r^{-1}$.

19. Continuous compounding results in faster growth.

$A = 1 + 0.075[[t]]$ and $A = e^{0.07t}$



20.



From the graph, $5\frac{1}{2}\%$ compounded daily grows faster than 6% simple interest.

21. $a = 10, y = \frac{1}{2}(10) = 5, t = 1599$

$$y = ae^{-bt}$$

$$5 = 10e^{-b(1599)}$$

$$0.5 = e^{-1599b}$$

$$\ln 0.5 = \ln e^{-1599b}$$

$$\ln 0.5 = -1599b$$

$$b = -\frac{\ln 0.5}{1599}$$

Given an initial quantity of 10 grams, after 1000 years, you have

$$y = 10e^{-[-(\ln 0.5)/1599](1000)} \approx 6.48 \text{ grams.}$$

22. $a = 6.5, y = \frac{1}{2}(6.5) = 3.25, t = 5715$

$$\begin{aligned} y &= ae^{-bt} \\ 3.25 &= 6.5e^{-b(5715)} \\ 0.5 &= e^{-5715b} \\ \ln 0.5 &= \ln e^{-5715b} \\ \ln 0.5 &= -5715b \\ b &= -\frac{\ln 0.5}{5715} \end{aligned}$$

Given an initial quantity of 6.5 grams, after 1000 years, you have

$y = 6.5e^{-[-(\ln 0.5)/5715](1000)} \approx 5.76$ grams.

23. $y = 2, a = 2(2) = 4, t = 5715$

$$\begin{aligned} y &= ae^{-bt} \\ 2 &= 4e^{-b(5715)} \\ 0.5 &= e^{-5715b} \\ \ln 0.5 &= \ln e^{-5715b} \\ \ln 0.5 &= -5715b \\ b &= -\frac{\ln 0.5}{5715} \end{aligned}$$

Given 2 grams after 1000 years, the initial amount is

$2 = ae^{-[-(\ln 0.5)/5715](1000)}$

$a \approx 2.26$ grams.

24. $y = 0.4, a = 2(0.4) = 0.8, t = 24{,}100$

$$\begin{aligned} y &= ae^{-bt} \\ 0.4 &= 0.8e^{-b(24{,}100)} \\ 0.5 &= e^{-24{,}100b} \\ \ln 0.5 &= \ln e^{-24{,}100b} \\ \ln 0.5 &= -24{,}100b \\ b &= -\frac{\ln 0.5}{24{,}100} \end{aligned}$$

Given 0.4 gram after 1000 years, the initial amount is

$0.4 = ae^{-[-(\ln 0.5)/24{,}100](1000)}$

$a \approx 0.41$ gram.

25.

$$\begin{aligned} y &= ae^{bx} \\ 1 &= ae^{b(0)} \Rightarrow 1 = a \\ 10 &= e^{b(3)} \\ \ln 10 &= 3b \\ \frac{\ln 10}{3} &= b \Rightarrow b \approx 0.7675 \end{aligned}$$

So, $y = e^{0.7675x}$.

26.

$$\begin{aligned} y &= ae^{bx} \\ \frac{1}{2} &= ae^{b(0)} \Rightarrow a = \frac{1}{2} \\ 5 &= \frac{1}{2}e^{b(4)} \\ 10 &= e^{4b} \\ \ln 10 &= \ln e^{4b} \\ \ln 10 &= 4b \\ \frac{\ln 10}{4} &= b \Rightarrow b \approx 0.5756 \end{aligned}$$

So, $y = \frac{1}{2}e^{0.5756x}$.

27.

$$\begin{aligned} y &= ae^{bx} \\ 5 &= ae^{b(0)} \Rightarrow 5 = a \\ 1 &= 5e^{b(4)} \\ \frac{1}{5} &= e^{4b} \\ \ln\left(\frac{1}{5}\right) &= 4b \\ \frac{\ln(1/5)}{4} &= b \Rightarrow b \approx -0.4024 \end{aligned}$$

So, $y = 5e^{-0.4024x}$.

28.

$$\begin{aligned} y &= ae^{bx} \\ 1 &= ae^{b(0)} \Rightarrow 1 = a \\ \frac{1}{4} &= e^{b(3)} \\ \ln\left(\frac{1}{4}\right) &= \ln e^{3b} \\ \ln\left(\frac{1}{4}\right) &= 3b \\ \frac{\ln(1/4)}{3} &= b \Rightarrow b \approx -0.4621 \end{aligned}$$

So, $y = e^{-0.4621x}$.

29. (a) $P = 76.6e^{0.0313t}$

Year	1980	1990	2000	2010
P	104.752	143.251	195.899	267.896
Population	104,752	143,251	195,899	267,896

(b) Let $P = 360$, and solve for t.

$$360 = 76.6e^{0.0313t}$$

$$\frac{360}{76.6} = e^{0.0313t}$$

$$\ln\left(\frac{360}{76.6}\right) = 0.0313t$$

$$\frac{1}{0.0313}\ln\left(\frac{360}{76.6}\right) = t$$

$$49.4 \approx t$$

According to the model, the population will reach 360,000 in 2019.

(c) No; As t increases, the population increases rapidly.

30. (a) *Bulgaria*: $(15, 7.2), (25, 6.7)$

Let $y = ae^{bt}$ so,

$7.2 = ae^{15b}$ and $6.7 = ae^{25b}$.

$$\frac{7.2}{e^{15b}} = a \Rightarrow 6.7 = \frac{7.2}{e^{15b}}e^{25b}$$

$$\frac{6.7}{7.2} = e^{10b}$$

$$\ln\left(\frac{6.7}{7.2}\right) = 10b$$

$$\frac{1}{10}\ln\left(\frac{6.7}{7.2}\right) = b$$

$$-0.00720 \approx b$$

Since $b \approx -0.00720$,

$$a = \frac{7.2}{e^{(15)(-0.00720)}} \approx 8.0$$

So, $y = 8.0e^{-0.00720t}$.

In 2035, when $t = 35$,

$y = 8.0e^{-0.00720(35)} \approx 6.2$ million people.

Canada: $(15, 35.1), (25, 37.6)$

Let $y = ae^{bt}$ so,

$35.1 = ae^{15b}$ and $37.6 = ae^{25b}$.

$$\frac{35.1}{e^{15b}} = a \Rightarrow 37.6 = \frac{35.1}{e^{15b}}e^{25b}$$

$$\frac{37.6}{35.1} = e^{10b}$$

$$\ln\left(\frac{37.6}{35.1}\right) = 10b$$

$$\frac{1}{10}\ln\left(\frac{37.6}{35.1}\right) = b$$

$$0.00688 \approx b$$

Since $b \approx 0.00688$,

$$a = \frac{35.1}{e^{(15)(0.00688)}} \approx 31.7$$

So, $y = 31.7e^{0.00688t}$.

In 2035, when $t = 35$,

$y = 31.7e^{0.00688(35)} \approx 40.3$ million people.

China: $(15, 1367.5), (25, 1407.0)$

Let $y = ae^{bt}$ so,

$1367.5 = ae^{15b}$ and $1407.0 = ae^{25b}$.

$$\frac{1367.5}{e^{15b}} = a \Rightarrow 1407.0 = \frac{1367.5}{e^{15b}}e^{25b}$$

$$\frac{1407.0}{1367.5} = e^{10b}$$

$$\ln\left(\frac{1407.0}{1367.5}\right) = 10b$$

$$\frac{1}{10}\ln\left(\frac{1407.0}{1367.5}\right) = b$$

$$0.00285 \approx b$$

Since $b \approx 0.00285$,

$$a = \frac{1367.5}{e^{(15)(0.00285)}} \approx 1310.3.$$

So, $y = 1310.3e^{0.00285t}$.

In 2035, when $t = 35$,

$y = 1310.3e^{0.00285(35)} \approx 1447.7$ million people.

United Kingdom: $(15, 64.1), (25, 67.2)$

Let $y = ae^{bt}$ so,

$64.1 = ae^{15b}$ and $67.2 = ae^{25b}$.

$$\frac{64.1}{e^{15b}} = a \Rightarrow 67.2 = \frac{64.1}{e^{15b}}e^{25b}$$
$$\frac{67.2}{64.1} = e^{10b}$$
$$\ln\left(\frac{67.2}{64.1}\right) = 10b$$
$$\frac{1}{10}\ln\left(\frac{67.2}{64.1}\right) = b$$
$$0.00472 \approx b$$

Since $b \approx 0.00472$,

$$a = \frac{64.1}{e^{(15)(0.00472)}} \approx 59.7.$$

So, $y = 59.7e^{0.00472t}$.

In 2035, when $t = 35$,

$y = 59.7e^{0.00472(35)} \approx 70.4$ million peole.

United States: $(15, 321.4), (25, 347.3)$

Let $y = ae^{bt}$ so,

$321.4 = ae^{15b}$ and $347.3 = ae^{25b}$

$$\frac{321.4}{e^{15b}} = a \Rightarrow 347.3 = \frac{321.4}{e^{15b}}e^{25b}$$
$$\frac{347.3}{321.4} = e^{10b}$$
$$\ln\left(\frac{347.3}{321.4}\right) = 10b$$
$$\frac{1}{10}\ln\left(\frac{347.3}{321.4}\right) = b$$
$$0.00775 \approx b$$

Since $b \approx 0.00775$,

$$a = \frac{321.4}{e^{(15)(0.00775)}} \approx 286.1.$$

So, $y = 286.1e^{0.00775t}$.

In 2035, when $t = 35$,

$y = 286.1e^{(0.00775)(35)} \approx 375.2$ million people.

(b) b; The greater the rate of growth, the greater the value of b.

31. $y = 4080e^{kt}$

When $t = 3$, $y = 10{,}000$:

$$10{,}000 = 4080e^{k(3)}$$
$$\frac{10{,}000}{4080} = e^{3k}$$
$$\ln\left(\frac{10{,}000}{4080}\right) = 3k$$
$$k = \frac{\ln(10{,}000/4080)}{3} \approx 0.2988$$

When $t = 24$: $y = 4080e^{0.2988(24)} \approx 5{,}309{,}734$ hits

32. (a) $P = 150.9e^{kt}$

When $t = 5$, $P = 163.075$:

$$163.075 = 150.9e^{5k}$$
$$\frac{163.075}{150.9} = e^{5k}$$
$$\ln\left(\frac{163.075}{150.9}\right) = 5k$$
$$\frac{1}{5}\ln\left(\frac{163.075}{150.9}\right) = k$$
$$0.0155 \approx k$$

Since $k > 0$, the population is increasing.

(b) In 2020, when $t = 20$, $P = 150.9e^{(0.0155)(20)} \approx 205.741$ thousand people.

In 2025, when $t = 25$, $P = 150.9^{(0.0155)(25)} \approx 222.320$ thousand people.

The populations are reasonable if it continues to increase at the same rate from the year 2020 through 2025.

(c) Let $P = 200$ and solve for t

$$200 = 150.9e^{0.0155t}$$
$$\frac{200}{150.9} = e^{0.0155t}$$
$$\ln\left(\frac{200}{150.9}\right) = 0.0155t$$
$$\frac{1}{0.0155}\ln\left(\frac{200}{150.9}\right) = t$$
$$18.2 \approx t$$

The population will reach 200,000 during the year 2018.

33. $y = ae^{bt}$

When $t = 3$, $y = 100$: $100 = ae^{3b}$, $\frac{100}{e^{3b}} = a$

When $t = 5$, $y = 400$: $400 = ae^{5b}$

Substitute $\frac{100}{e^{3b}}$ for a in the equation on the right.

$$400 = \frac{100}{e^{3b}}e^{5b}$$
$$400 = 100e^{2b}$$
$$4 = e^{2b}$$
$$\ln 4 = 2b$$
$$\ln 2^2 = 2b$$
$$2\ln 2 = 2b$$
$$\ln 2 = b$$
$$a = \frac{100}{e^{3b}} = \frac{100}{e^{3\ln 2}} = \frac{100}{e^{\ln 2^3}} = \frac{100}{2^3} = \frac{100}{8} = 12.5$$

$y = 12.5e^{(\ln 2)t}$

After 6 hours, there are $y = 12.5e^{(\ln 2)(6)} = 800$ bacteria.

34. $y = 250e^{bt}$

After 1 hour: $y = 250e^{b}$

After 10 hours: $y = 250e^{10b}$

$$\text{Population after 10 hours} = 2(\text{population after 1 hour})$$
$$250e^{10b} = 2(250e^{b})$$
$$250e^{10b} = 500e^{b}$$
$$e^{10b} = 2e^{b}$$
$$e^{9b} = 2$$
$$9b = \ln 2$$
$$b = \frac{\ln 2}{9}$$

$y = 250e^{[(\ln 2)/9]t}$

After 6 hours, there are

$y = 250e^{[(\ln 2)/9]6} \approx 397$ bacteria.

35. $(0, 575), (2, 275)$

(a) $m = \dfrac{275 - 575}{2 - 0} = -150$

$V = -150t + 575$

(b) Since $V = 575$, when

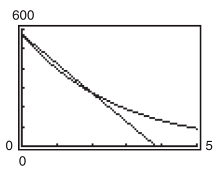
$t = 0, 575 = ae^{(b)(0)} \rightarrow a = 575$

Then $275 = 575e^{k(2)}$

$\ln\left(\dfrac{275}{575}\right) = 2k \Rightarrow k \approx -0.3688$

$V = 575e^{-0.3688t}$

(c)



The exponential model depreciates faster in the first two years.

(d)

t	1	3
$V = -150t + 575$	\$425	\$125
$V = 575e^{-0.3688t}$	\$397.65	\$190.18

(e) Answers will vary. Sample Answer: The slope of the linear model means that the laptop depreciates \$150 per year, then loses all value late in the third year. The exponential model depreciates faster in the first three years but maintains value longer.

36. $N = 30\left(1 - e^{kt}\right)$

(a)
$$N = 19, t = 20$$
$$19 = 30\left(1 - e^{20k}\right)$$
$$30e^{20k} = 11$$
$$e^{20k} = \frac{11}{30}$$
$$\ln e^{20k} = \ln\left(\frac{11}{30}\right)$$
$$20k = \ln\left(\frac{11}{30}\right)$$
$$k = -0.050$$
$$\text{So, } N = 30\left(1 - e^{-0.050t}\right).$$

(b)
$$N = 25$$
$$25 = 30\left(1 - e^{-0.050t}\right)$$
$$\frac{5}{30} = e^{-0.050t}$$
$$\ln\left(\frac{5}{30}\right) = \ln e^{-0.050t}$$
$$\ln\left(\frac{5}{30}\right) = -0.050t$$
$$t = \frac{\ln(5/30)}{-0.050} = 36 \text{ days}$$

37. $R = \dfrac{1}{10^{12}}e^{-t/8223}$

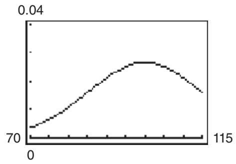
$$R = \frac{1}{8^{14}}$$
$$\frac{1}{10^{12}}e^{-t/8223} = \frac{1}{8^{14}}$$
$$e^{-t/8223} = \frac{10^{12}}{8^{14}}$$
$$-\frac{t}{8223} = \ln\left(\frac{10^{12}}{8^{14}}\right)$$
$$t = -8223 \ln\left(\frac{10^{12}}{8^{14}}\right) \approx 12{,}180 \text{ years old}$$

38. $\dfrac{1}{10^{12}}e^{-t/8223} = \dfrac{1}{13^{11}}$

$$e^{-t/8223} = \frac{10^{12}}{13^{11}}$$
$$-\frac{t}{8223} = \ln\left(\frac{10^{12}}{13^{11}}\right)$$
$$t = -8223 \ln\left(\frac{10^{12}}{13^{11}}\right) \approx 4797 \text{ years old}$$

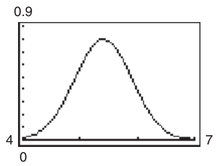
39. $y = 0.0266e^{-(x-100)^2/450}$, $70 \le x \le 116$

(a)



(b) The average IQ score of an adult student is 100.

40. (a)



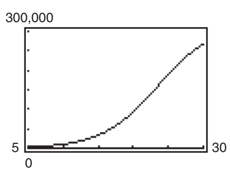
(b) The average number of hours per week a student uses the tutor center is 5.4.

41. (a) 1998: $t = 18$, $y = \dfrac{320{,}110}{1 + 374e^{-0.252(18)}} \approx 63{,}992$ sites

2003: $t = 23$, $y = \dfrac{320{,}110}{1 + 374e^{-0.252(23)}} \approx 149{,}805$ sites

2006: $t = 26$, $y = \dfrac{320{,}110}{1 + 374e^{-0.252(26)}} \approx 208{,}705$ sites

(b)



(c) When $y = 270{,}000$, $t \approx 30.2$. So, the number of cell sites will reach 270,000 in the year 2010.

(d) Let $y = 270{,}000$ and solve for t.

$$270{,}000 = \frac{320{,}110}{1 + 374e^{-0.252t}}$$
$$1 + 374e^{-0.252t} = \frac{320{,}110}{270{,}000}$$
$$374e^{-0.252t} = 0.1855926$$
$$e^{-0.252t} \approx 0.000496237$$
$$-0.252t \approx \ln(0.000496237)$$
$$t \approx 30.2$$

The number of cell sites will reach 270,000 during the year 2010.

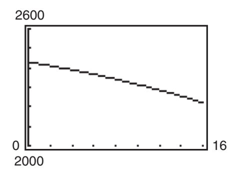
42. (a) 2000: $t = 0$, $P = \dfrac{2632}{1 + 0.083e^{(0.050)(0)}}$
$\approx 2{,}430.286$ thousand
$= 2{,}430{,}286$ people

2005: $t = 5$, $P = \dfrac{2632}{1 + 0.083e^{(0.050)(5)}}$
≈ 2378.512 thousand
$= 2{,}378{,}512$ people

2010: $t = 10$, $P = \dfrac{2632}{1 + 0.083e^{(0.050)(10)}}$
$\approx 2{,}315.182$ thousand
$= 2{,}315{,}182$ people

2015: $t = 15$, $P = \dfrac{2632}{1 + 0.083e^{(0.050)(15)}}$
$\approx 2{,}238.645$ thousand
$= 2{,}238{,}645$ people

(b)



(c) When $P = 2200$, $t \approx 17.2$. So, the population will have reached 2.2 million or $P = 2200$ thousand in 2017.

(d) Let $P = 2200$ and solve for t.

$$2200 = \frac{2632}{1 + 0.083e^{0.050t}}$$
$$1 + 0.083e^{0.050t} = \frac{2632}{2200}$$
$$0.083e^{0.050t} \approx 0.196364$$
$$e^{0.050t} \approx 2.3658270$$
$$0.050t \approx \ln(2.3658270)$$
$$t \approx 17.2$$

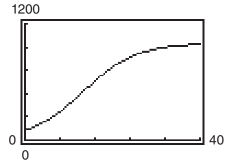
The population will reach 2.2 million during 2017.

43. $p(t) = \dfrac{1000}{1 + 9e^{-0.1656t}}$

(a) $p(5) = \dfrac{1000}{1 + 9e^{-0.1656(5)}} \approx 203$ animals

(b)
$$500 = \frac{1000}{1 + 9e^{-0.1656t}}$$
$$1 + 9e^{-0.1656t} = 2$$
$$9e^{-0.1656t} = 1$$
$$e^{-0.1656t} = \frac{1}{9}$$
$$t = -\frac{\ln(1/9)}{0.1656} \approx 13 \text{ months}$$

(c)



The horizontal asymptotes are $p = 0$ and $p = 1000$.

The asymptote with the larger p-value, $p = 1000$, indicates that the population size will approach 1000 as time increases.

44. (a) In the year 2020 $(t = 10)$, the number of units sold is about 40,000 units.

(b) In 2014, when $t = 4$ and $S = 300{,}000$, you have the following.

$$300{,}000 = \frac{500{,}000}{1 + 0.1e^{4k}}$$
$$1 + 0.1e^{4k} = \frac{5}{3}$$
$$0.1e^{4k} = \frac{2}{3}$$
$$e^{14} = \frac{20}{3}$$
$$4k = \ln\left(\frac{20}{3}\right)$$
$$k = \frac{1}{4}\ln\left(\frac{20}{3}\right)$$
$$k \approx 0.4743$$

So, the model is $S = \dfrac{500{,}000}{1 + 0.1e^{0.4743t}}$.

(c) In 2020, when

$t = 10,\ S = \dfrac{500{,}000}{1 + 0.1e^{(0.4743)(10)}} \approx 40{,}071$ units sold.

The algebraic result of approximately 40,071 units is similar to the graphical result.

45. $R = \log \dfrac{I}{I_0} = \log I$ because $I_0 = 1$.

(a)
$$R = 7.6$$
$$7.6 = \log I$$
$$10^{7.6} = 10^{\log I}$$
$$39{,}810{,}717 \approx I$$

(b)
$$R = 5.6$$
$$5.6 = \log I$$
$$10^{5.6} = 10^{\log I}$$
$$10^{5.6} = I$$
$$398{,}107 \approx I$$

(c)
$$R = 6.6$$
$$6.6 = \log I$$
$$10^{6.6} = 10^{\log I}$$
$$3{,}981{,}072 \approx I$$

46. $R = \log \dfrac{I}{I_0} = \log I$ because $I_0 = 1$.

(a) $R = \log 199{,}500{,}000 \approx 8.30$

(b) $R = \log 48{,}275{,}000 \approx 7.68$

(c) $R = \log 17{,}000 \approx 4.23$

47. $\beta = 10 \log \dfrac{I}{I_0}$ where $I_0 = 10^{-12}$ watt/m^2.

(a) $\beta = 10 \log \dfrac{10^{-10}}{10^{-12}} = 10 \log 10^2 = 20$ decibels

(b) $\beta = 10 \log \dfrac{10^{-5}}{10^{-12}} = 10 \log 10^7 = 70$ decibels

(c) $\beta = 10 \log \dfrac{10^{-8}}{10^{-12}} = 10 \log 10^4 = 40$ decibels

(d) $\beta = 10 \log \dfrac{10^{-3}}{10^{-12}} = 10 \log 10^9 = 90$ decibels

48. $\beta(I) = 10 \log \dfrac{I}{I_0}$ where $I_0 = 10^{-12}$ watt/m^2.

(a) $\beta(10^{-11}) = 10 \log \dfrac{10^{-11}}{10^{-12}} = 10 \log 10^1 = 10$ decibels

(b) $\beta(10^{2}) = 10 \log \dfrac{10^{2}}{10^{-12}} = 10 \log 10^{14} = 140$ decibels

(c) $\beta(10^{-4}) = 10 \log \dfrac{10^{-4}}{10^{-12}} = 10 \log 10^{8} = 80$ decibels

(d) $\beta(10^{-2}) = 10 \log \dfrac{10^{-6}}{10^{-12}} = 10 \log 10^{6} = 60$ decibels

49.

$$\beta = 10 \log \frac{I}{I_0}$$
$$\frac{\beta}{10} = \log \frac{I}{I_0}$$
$$10^{\beta/10} = 10^{\log I/I_0}$$
$$10^{\beta/10} = \frac{I}{I_0}$$
$$I = I_0 10^{\beta/10}$$
$$\%\text{ decrease} = \frac{I_0 10^{9.3} - I_0 10^{8.0}}{I_0 10^{9.3}} \times 100 \approx 95\%$$

50.

$$\beta = 10 \log_{10} \frac{I}{I_0}$$
$$10^{\beta/10} = \frac{I}{I_0}$$
$$I = I_0 10^{\beta/10}$$
$$\%\text{ decrease} = \frac{I_0 10^{8.8} - I_0 10^{7.2}}{I_0 10^{8.8}} \times 100 \approx 97\%$$

51. $\text{pH} = -\log[H^+]$

$-\log(2.3 \times 10^{-5}) \approx 4.64$

52. $\text{pH} = -\log[H^+]$

$-\log[1.13 \times 10^{-5}] \approx 4.95$

53.

$$5.8 = -\log[H^+]$$
$$-5.8 = \log[H^+]$$
$$10^{-5.8} = 10^{\log[H^+]}$$
$$10^{-5.8} = [H^+]$$
$$[H^+] \approx 1.58 \times 10^{-6} \text{ moles per liter}$$

54.

$$3.2 = -\log[H^+]$$
$$10^{-3.2} = [H^+]$$
$$[H^+] \approx 6.3 \times 10^{-4} \text{ moles per liter}$$

55.

$$2.9 = -\log[H^+]$$
$$-2.9 = \log[H^+]$$
$$[H^+] = 10^{-2.9} \text{ for the apple juice}$$
$$8.0 = -\log[H^+]$$
$$-8.0 = \log[H^+]$$
$$[H^+] = 10^{-8} \text{ for the drinking water}$$

$\dfrac{10^{-2.9}}{10^{-8}} = 10^{5.1}$ times the hydrogen ion concentration of drinking water

56.

$$\text{pH} - 1 = -\log[H^+]$$
$$-(\text{pH} - 1) = \log[H^+]$$
$$10^{-(\text{pH}-1)} = [H^+]$$
$$10^{-\text{pH}+1} = [H^+]$$
$$10^{-\text{pH}} \cdot 10 = [H^+]$$

The hydrogen ion concentration is increased by a factor of 10.

57. $t = -10 \ln \dfrac{T - 70}{98.6 - 70}$

At 9:00 A.M. you have:

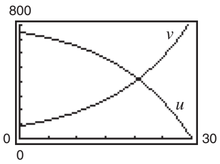
$t = -10 \ln \dfrac{85.7 - 70}{98.6 - 70} \approx 6$ hours

From this you can conclude that the person died at 3:00 A.M.

58. Interest: $u = M - \left(M - \dfrac{Pr}{12}\right)\left(1 + \dfrac{r}{12}\right)^{12t}$

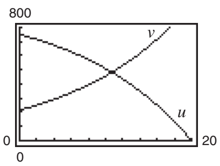
Principal: $v = \left(M - \dfrac{Pr}{12}\right)\left(1 + \dfrac{r}{12}\right)^{12t}$

(a) $P = 120{,}000, t = 30, r = 0.075, M = 839.06$



(b) In the early years of the mortgage, the majority of the monthly payment goes toward interest. The principal and interest are nearly equal when $t \approx 21$ years.

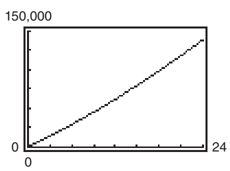
(c) $P = 120{,}000, t = 20, r = 0.075, M = 966.71$



The interest is still the majority of the monthly payment in the early years. Now, the principal and interest are nearly equal when $t \approx 11$ years.

59. $u = 120{,}000\left[\dfrac{0.075t}{1 - \left(\dfrac{1}{1 + 0.075/12}\right)^{12t}} - 1\right]$

(a)



(b) From the graph, $u = \$120{,}000$ when $t \approx 21$ years. It would take approximately 37.6 years to pay \$240,000 in interest. Yes, it is possible to pay twice as much in interest charges as the size of the mortgage. It is especially likely when the interest rates are higher.

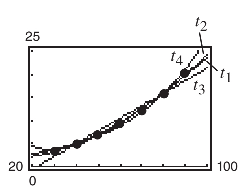
60. $t_1 = 40.757 + 0.556s - 15.817 \ln s$

$t_2 = 1.2259 + 0.0023s^2$

(a) Linear model: $t_3 = 0.2729s - 6.0143$

Exponential model: $t_4 = 1.5385e^{0.02913s}$ or $t_4 = 1.5385(1.0296)^s$

(b)



(c)

s	30	40	50	60	70	80	90
t_1	3.6	4.6	6.7	9.4	12.5	15.9	19.6
t_2	3.3	4.9	7.0	9.5	12.5	15.9	19.9
t_3	2.2	4.9	7.6	10.4	13.1	15.8	18.5
t_4	3.7	4.9	6.6	8.8	11.8	15.8	21.2

Note: Table values will vary slightly depending on the model used for t_4.

(d) Model t_1: $S_1 = |3.4 - 3.6| + |5 - 4.6| + |7 - 6.7| + |9.3 - 9.4| + |12 - 12.5| + |15.8 - 15.9| + |20 - 19.6| = 2.0$

Model t_2: $S_2 = |3.4 - 3.3| + |5 - 4.9| + |7 - 7| + |9.3 - 9.5| + |12 - 12.5| + |15.8 - 15.9| + |20 - 19.9| = 1.1$

Model t_3: $S_3 = |3.4 - 2.2| + |5 - 4.9| + |7 - 7.6| + |9.3 - 10.4| + |12 - 13.1| + |15.8 - 15.8| + |20 - 18.5| = 5.6$

Model t_4: $S_4 = |3.4 - 3.7| + |5 - 4.9| + |7 - 6.6| + |9.3 - 8.8| + |12 - 11.8| + |15.8 - 15.8| + |20 - 21.2| = 2.7$

The quadratic model, t_2, best fits the data.

61. False. The domain can be the set of real numbers for a logistic growth function.

62. False. A logistic growth function never has an x-intercept.

63. False. The graph of $f(x)$ is the graph of $g(x)$ shifted upward five units.

64. True. Powers of e are always positive, so if $a > 0$, a Gaussian model will always be greater than 0, and if $a < 0$, a Gaussian model will always be less than 0.

65. Answers will vary.

66. (a) The model is logarithmic because it slowly increases.

(b) The model is logistic because it initially has rapid growth and then has a declining rate of growth.

(c) The model is exponential because it rapidly decreases.

(d) The model is linear because the points are in a straight line.

(e) The model is none of the ones given because it seems to be a combination of a linear model and a quadratic model.

(f) The model is exponential because it rapidly increases.

(g) The model is quadratic because it is a parabola that is symmetric about the y-axis.

(h) The model is Gaussian because it is bell-shaped.

Review Exercises for Chapter 3

1. $f(x) = 0.3^x$

$f(1.5) = 0.3^{1.5} \approx 0.164$

2. $f(x) = 30^x$

$f(\sqrt{3}) = 30^{\sqrt{3}} \approx 361.784$

3. $f(x) = 2^x$

$f\left(\frac{2}{3}\right) = 2^{2/3} \approx 1.587$

4. $f(x) = \left(\frac{1}{2}\right)^{2x}$

$f(\pi) = \left(\frac{1}{2}\right)^{2\pi} \approx 0.013$

5. $f(x) = 7(0.2^x)$

$f(-\sqrt{11}) = 7(0.2^{-\sqrt{11}})$

≈ 1456.529

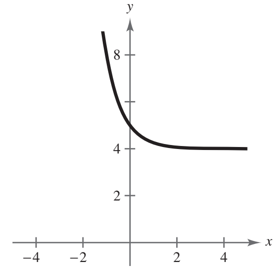
6. $f(x) = -14(5^x)$

$f(-0.8) = -14(5^{-0.8}) \approx -3.863$

7. $f(x) = 4^{-x} + 4$

Horizontal asymptote: $y = 4$

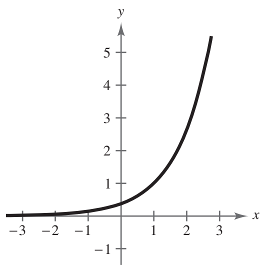
x	-1	0	1	2	3
$f(x)$	8	5	4.25	4.063	4.016



8. $f(x) = 2.65^{x-1}$

Horizontal asymptote: $y = 0$

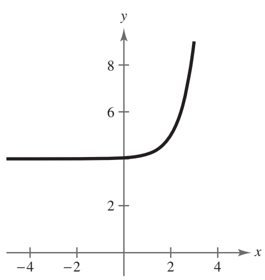
x	-3	-1	0	1	3
$f(x)$	0.020	0.142	0.377	1	7.023



9. $f(x) = 5^{x-2} + 4$

Horizontal asymptote: $y = 4$

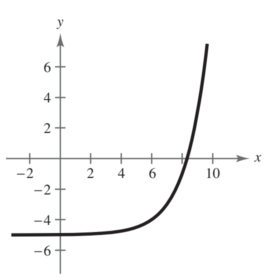
x	-1	0	1	2	3
$f(x)$	4.008	4.04	4.2	5	9



10. $f(x) = 2^{x-6} - 5$

Horizontal asymptote: $y = -5$

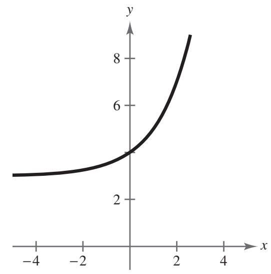
x	0	5	6	7	8	9
$f(x)$	-4.984	-4.5	-4	-3	-1	3



11. $f(x) = \left(\frac{1}{2}\right)^{-x} + 3 = 2^x + 3$

Horizontal asymptote: $y = 3$

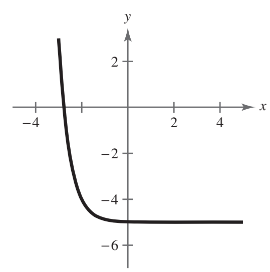
x	-2	-1	0	1	2
$f(x)$	3.25	3.5	4	5	7



12. $f(x) = \left(\frac{1}{8}\right)^{x+2} - 5$

Horizontal asymptote: $y = -5$

x	-3	-2	-1	0	2
$f(x)$	3	-4	-4.875	-4.984	-5



13.
$$\left(\frac{1}{3}\right)^{x-3} = 9$$
$$\left(\frac{1}{3}\right)^{x-3} = 3^2$$
$$\left(\frac{1}{3}\right)^{x-3} = \left(\frac{1}{3}\right)^{-2}$$
$$x - 3 = -2$$
$$x = 1$$

14.
$$3^{x+3} = \frac{1}{81}$$
$$3^{x+3} = \left(\frac{1}{3}\right)^4$$
$$3^{x+3} = 3^{-4}$$
$$x + 3 = -4$$
$$x = -7$$

15. $e^{3x-5} = e^7$

$3x - 5 = 7$

$3x = 12$

$x = 4$

16. $e^{8-2x} = e^{-3}$

$8 - 2x = -3$

$-2x = -11$

$x = \frac{11}{2}$

17. $f(x) = 5^x,\ g(x) = 5^x + 1$

Because $g(x) = f(x) + 1,$ the graph of g can be obtained by shifting the graph of f one unit upward.

18. $f(x) = 6^x,\ g(x) = 6^{x+1}$

Because $g(x) = f(x + 1),$ the graph of g can be obtained by shifting the graph of f one unit to the left.

19. $f(x) = 3^x,\ g(x) = 1 - 3^x$

Because $g(x) = 1 - f(x),$ the graph of g can be obtained by reflecting the graph of f in the x-axis and shifting the graph one unit upward. (**Note:** This is equivalent to shifting the graph of f one unit upward and then reflecting the graph in the x-axis.)

20. $f(x) = \left(\frac{1}{2}\right)^x,\ g(x) = -\left(\frac{1}{2}\right)^{x+2}$

Because $g(x) = -f(x + 2),$ the graph of g can be obtained by shifting the graph of f two units to the left and reflecting it in the x-axis.

21. $f(x) = e^x$

$f(3.4) = e^{3.4} \approx 29.964$

22. $f(x) = e^x$

$f(-2.5) = e^{-2.5} \approx 0.082$

23. $f(x) = e^x$

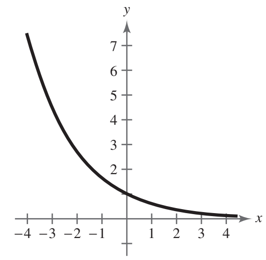
$f\left(\frac{3}{5}\right) = e^{3/5} \approx 1.822$

24. $f(x) = e^x$

$f\left(\frac{2}{7}\right) = e^{2/7} \approx 1.331$

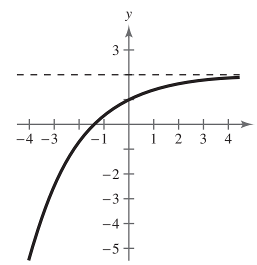
25. $h(x) = e^{-x/2}$

x	-2	-1	0	1	2
$h(x)$	2.72	1.65	1	0.61	0.37



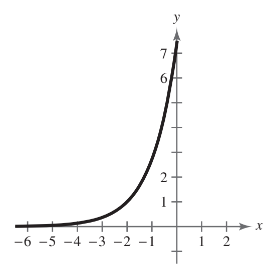
26. $h(x) = 2 - e^{-x/2}$

x	-2	-1	0	1	2
y	-0.72	0.35	1	1.39	1.63



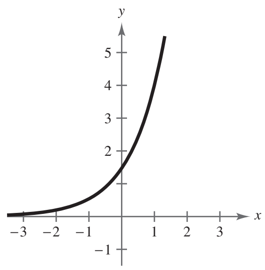
27. $f(x) = e^{x+2}$

x	-3	-2	-1	0	1
$f(x)$	0.37	1	2.72	7.39	20.09



28. $s(t) = 4e^{t-1}$

t	-2	-1	0	1	2
$s(t)$	0.20	0.54	1.47	4	10.87



29. $F(t) = 1 - e^{-t/3}$

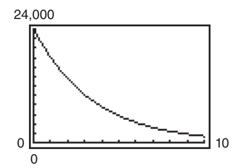
(a) $F(1) \approx 0.283$

(b) $F(2) \approx 0.487$

(c) $F(5) \approx 0.811$

30. $V(t) = 23{,}970\left(\frac{3}{4}\right)^t$

(a)



(b) $V(2) = 23{,}970\left(\frac{3}{4}\right)^2 \approx \$13{,}483.13$

(c) According to the model, the car depreciates most rapidly at the beginning. Yes, this is realistic.

(d) The x-axis is a horizontal asymptote, so the car will never have no value.

31. $P = \$5000, r = 3\%, t = 10$ years

Compounded n times per year: $A = P\left(1 + \frac{r}{n}\right)^{nt} = 5000\left(1 + \frac{0.03}{n}\right)^{10n}$

Compounded continuously: $A = Pe^{rt} = 5000e^{0.03(10)}$

n	1	2	4	12	365	Continuous
A	\$6719.58	\$6734.28	\$6741.74	\$6746.77	\$6749.21	\$6749.29

32. $P = \$4500, r = 2.5\%, t = 30$ years

Compounded n times per year: $A = P\left(1 + \frac{r}{n}\right)^{nt} = 4500\left(1 + \frac{0.025}{n}\right)^{30n}$

Compounded continuously: $A = Pe^{rt} = 4500e^{0.025(30)}$

n	1	2	4	12	365	Continuous
A	\$9439.05	\$9482.32	\$9504.29	\$9519.07	\$9526.26	\$9526.50

33. $3^3 = 27$

$\log_3 27 = 3$

34. $25^{3/2} = 125$

$\log_{25} 125 = \frac{3}{2}$

35. $e^{0.8} = 2.2255\ldots$

$\ln 2.2255\ldots = 0.8$

36. $e^0 = 1$

$\ln 1 = 0$

37. $f(x) = \log x$

$f(1000) = \log 1000$

$= \log 10^3 = 3$

38. $g(x) = \log_9 x$

$g(3) = \log_9 3$

$= \log_9 9^{1/2} = \frac{1}{2}$

39. $g(x) = \log_2 x$

$g\left(\frac{1}{4}\right) = \log_2 \frac{1}{4}$

$= \log_2 2^{-2} = -2$

40. $f(x) = \log_3 x$

$f\left(\frac{1}{81}\right) = \log_3 \frac{1}{81}$

$= \log_3 3^{-4} = -4$

41. $\log_4(x + 7) = \log_4 14$

$x + 7 = 14$

$x = 7$

42. $\log_8(3x - 10) = \log_8 5$

$3x - 10 = 5$

$3x = 15$

$x = 5$

43. $\ln(x + 9) = \ln 4$

$x + 9 = 4$

$x = -5$

44. $\log(3x - 2) = \log 7$

$3x - 2 = 7$

$3x = 9$

$x = 3$

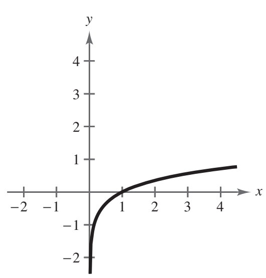
45. $g(x) = \log_7 x \Rightarrow x = 7^y$

Domain: $(0, \infty)$

x-intercept: $(1, 0)$

Vertical asymptote: $x = 0$

x	$\frac{1}{7}$	1	7	49
$g(x)$	−1	0	1	2



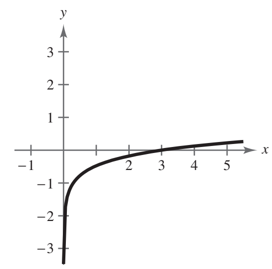
46. $f(x) = \log\left(\frac{x}{3}\right) \Rightarrow \frac{x}{3} = 10^y \Rightarrow x = 3(10^y)$

Domain: $(0, \infty)$

x-intercept: $(3, 0)$

Vertical asymptote: $x = 0$

x	0.03	0.3	3	30
$f(x)$	−2	−1	0	1



47. $f(x) = 4 - \log(x + 5)$

Domain: $(-5, \infty)$

Because

$4 - \log(x + 5) = 0 \Rightarrow \log(x + 5) = 4$

$x + 5 = 10^4$

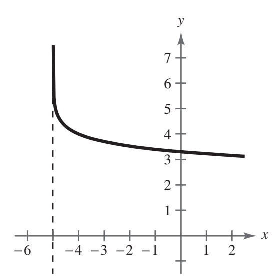
$x = 10^4 - 5$

$= 9995.$

x-intercept: $(9995, 0)$

Vertical asymptote: $x = -5$

x	−4	−3	−2	−1	0	1
$f(x)$	4	3.70	3.52	3.40	3.30	3.22



48. $f(x) = \log(x - 3) + 1$

Domain: $(3, \infty)$

$\log(x - 3) + 1 = 0$

$\log(x - 3) = -1$

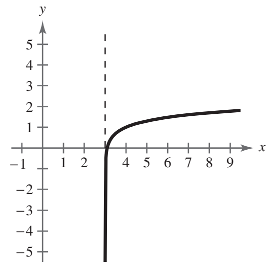
$x - 3 = 10^{-1}$

$x = 3.1$

x-intercept: $(3.1, 0)$

Vertical asymptote: $x = 3$

x	4	5	6	7	8
$f(x)$	1	1.3	1.5	1.6	1.7



49. $f(22.6) = \ln 22.6 \approx 3.118$

50. $f(e^{-12}) = \ln e^{-12} = -12$

51. $f(\sqrt{e}) = \frac{1}{2} \ln \sqrt{e} = 0.25$

52. $f(0.98) = 5 \ln 0.98 \approx -0.101$

53. $f(x) = \ln x + 6 = 6 + \ln x$

Domain: $(0, \infty)$

$\ln x + 6 = 0$

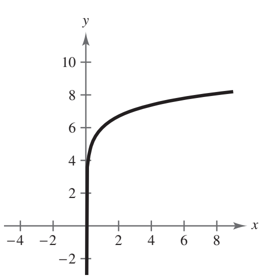
$\ln x = -6$

$x = e^{-6}$

x-intercept: $(e^{-6}, 0)$

Vertical asymptote: $x = 0$

x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	3
$f(x)$	4.613	5.037	6	6.693	7.098



54. $f(x) = \ln x - 5 = -5 + \ln x$

Domain: $(0, \infty)$

$\ln x - 5 = 0$

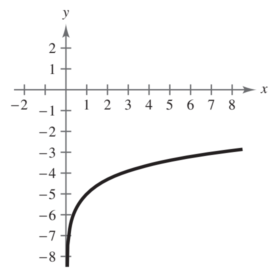
$\ln x = 5$

$x = e^5$

x-intercept: $(e^5, 0)$

Vertical asymptote: $x = 0$

x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	3
$f(x)$	−6.386	−5.693	−5	−4.307	−3.901



55. $f(x) = \ln(x - 6)$

Domain: $(6, \infty)$

$\ln(x - 6) = 0$

$x - 6 = e^0$

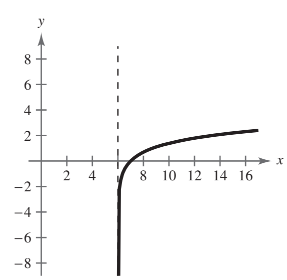
$x - 6 = 1$

$x = 7$

x-intercept: $(7, 0)$

Vertical asymptote: $x = 6$

x	6.5	7	8	9	10
$f(x)$	−0.693	0	0.693	1.099	1.386



56. $f(x) = \ln(x + 4)$

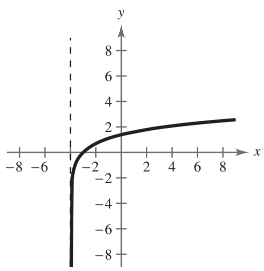
Domain: $(-4, \infty)$

$$\begin{aligned} \ln(x+4) &= 0 \\ x + 4 &= e^0 \\ x + 4 &= 1 \\ x &= -3 \end{aligned}$$

x-intercept: $(-3, 0)$

Vertical asymptote: $x = -4$

x	-3.5	-3	-2	-1	0
$f(x)$	-0.693	0	0.693	1.099	1.386



57. $M = m - 5\log\left(\dfrac{d}{10}\right)$

Let $m = 2.08$ and $M = 1.3$ and solve for d.

$$\begin{aligned} 1.3 &= 2.08 - 5\log\left(\frac{d}{10}\right) \\ -0.78 &= -5\log\left(\frac{d}{10}\right) \\ 0.156 &= \log\left(\frac{d}{10}\right) \\ 10^{0.156} &= 10^{\log(d/10)} \\ 10^{0.156} &= \frac{d}{10} \\ 10 \cdot 10^{0.156} &= d \\ d &= 10^{1.156} \approx 14.32 \text{ parsecs} \end{aligned}$$

58. $s = 25 - \dfrac{13\ln(10/12)}{\ln 3}$

≈ 27.16 miles

59. (a) $\log_2 6 = \dfrac{\log 6}{\log 2} \approx 2.585$

(b) $\log_2 6 = \dfrac{\ln 6}{\ln 2} \approx 2.585$

60. (a) $\log_{12} 200 = \dfrac{\log 200}{\log 12} \approx 2.132$

(b) $\log_{12} 200 = \dfrac{\ln 200}{\ln 12} \approx 2.132$

61. (a) $\log_{1/2} 5 = \dfrac{\log 5}{\log(1/2)} \approx -2.322$

(b) $\log_{1/2} 5 = \dfrac{\ln 5}{\ln(1/2)} \approx -2.322$

62. (a) $\log_4 0.75 = \dfrac{\log 0.75}{\log 4} \approx -0.208$

(b) $\log_4 0.75 = \dfrac{\ln 0.75}{\ln 4} \approx -0.208$

63. $\log_2 \frac{5}{3} = \log_2 5 - \log_2 3$

64. $$\begin{aligned} \log_2 45 &= \log_2(5 \cdot 9) \\ &= \log_2 5 + \log_2 9 \\ &= \log_2 5 + \log_2 3^2 \\ &= \log_2 5 + 2\log_2 3 \end{aligned}$$

65. $$\begin{aligned} \log_2 \tfrac{9}{5} &= \log_2 9 - \log_2 5 \\ &= \log_2 3^2 - \log_2 9 \\ &= 2\log_2 3 - \log_2 5 \end{aligned}$$

66. $$\begin{aligned} \log_2 \tfrac{20}{9} &= \log_2 20 - \log_2 9 \\ &= \log_2(4 \cdot 5) - \log_2 9 \\ &= \log_2 4 + \log_2 5 - \log_2 9 \\ &= \log_2 2^2 + \log_2 5 - \log_2 3^2 \\ &= 2 + \log_2 5 - 2\log_2 3 \end{aligned}$$

67. $$\begin{aligned} \log 7x^2 &= \log 7 + \log x^2 \\ &= \log 7 + 2\log x \end{aligned}$$

68. $$\begin{aligned} \log 11x^3 &= \log 11 + \log x^3 \\ &= \log 11 + 3\log x \end{aligned}$$

69. $$\begin{aligned} \log_3 \frac{9}{\sqrt{x}} &= \log_3 9 - \log_3 \sqrt{x} \\ &= \log_3 3^2 - \log_3 x^{1/2} \\ &= 2 - \frac{1}{2}\log_3 x \end{aligned}$$

70. $\log_7 \dfrac{\sqrt[3]{x}}{19} = \log_7 \sqrt[3]{x} - \log_7 19$

$= \log_7 x^{1/3} - \log_7 19$

$= \dfrac{1}{3}\log_7 x - \log_7 19$

71. $\ln x^2y^2z = \ln x^2 + \ln y^2 + \ln z$

$= 2\ln x + 2\ln y + \ln z$

72. $\ln\left(\dfrac{y-1}{3}\right)^2 = 2\ln\left(\dfrac{y-1}{3}\right)$

$= 2\ln(y-1) - 2\ln 3,\ y > 1$

73. $\ln 7 + \ln x = \ln(7x)$

74. $\log_2 y - \log_2 3 = \log_2\left(\dfrac{y}{3}\right)$

75. $\log x - \dfrac{1}{2}\log y = \log x - \log y^{1/2}$

$= \log\left(\dfrac{x}{\sqrt{y}}\right)$

76. $3\ln x + 2\ln(x+1) = \ln x^3 + \ln(x+1)^2$

$= \ln x^3(x+1)^2$

77. $\dfrac{1}{2}\log_3 x - 2\log_3(y+8) = \log_3 x^{1/2} - \log_3(y+8)^2$

$= \log_3 \sqrt{x} - \log_3(y+8)^2$

$= \log_3 \dfrac{\sqrt{x}}{(y+8)^2}$

78. $5\ln(x-2) - \ln(x+2) - 3\ln x = \ln(x-2)^5 - \ln(x+2) - \ln x^3$

$= \ln(x-2)^5 - \left[\ln(x+2) + \ln x^3\right]$

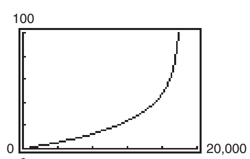
$= \ln(x-2)^5 - \ln x^3(x+2)$

$= \ln \dfrac{(x-2)^5}{x^3(x+2)}$

79. $t = 50\log \dfrac{18{,}000}{18{,}000 - h}$

(a) Domain: $0 \le h < 18{,}000$

(b)



Vertical asymptote: $h = 18{,}000$

(c) As the plane approaches its absolute ceiling, it climbs at a slower rate, so the time required increases.

(d) $50\log \dfrac{18{,}000}{18{,}000 - 4000} \approx 5.46$ minutes

80. Using a calculator gives $s = 84.66 + (-11\ln t)$.

81. $5^x = 125$

$5^x = 5^3$

$x = 3$

82. $6^x = \frac{1}{216}$

$6^x = 6^{-3}$

$x = -3$

83. $e^x = 3$

$x = \ln 3 \approx 1.099$

84. $\log x - \log 5 = 0$

$\log \dfrac{x}{5} = 0$

$10^{\log x/5} = 10^0$

$\dfrac{x}{5} = 1$

$x = 5$

85. $\ln x = 4$

$x = e^4 \approx 54.598$

86. $\ln x = -1.6$

$x = e^{-1.6} \approx 0.202$

87. $e^{4x} = e^{x^2+3}$

$4x = x^2 + 3$

$0 = x^2 - 4x + 3$

$0 = (x-1)(x-3)$

$x = 1, x = 3$

88.
$$\begin{aligned} e^{3x} &= 25 \\ \ln e^{3x} &= \ln 25 \\ 3x &= \ln 25 \\ x &= \frac{\ln 25}{3} \approx 1.073 \end{aligned}$$

89.
$$\begin{aligned} 2^x - 3 &= 29 \\ 2^x &= 32 \\ 2^x &= 2^5 \\ x &= 5 \end{aligned}$$

90.
$$\begin{aligned} e^{2x} - 6e^x + 8 &= 0 \\ (e^x - 2)(e^x - 4) &= 0 \end{aligned}$$
$$\begin{aligned} e^x &= 2 \quad \text{or} \quad e^x = 4 \\ x &= \ln 2 \qquad\quad x = \ln 4 \\ x &\approx 0.693 \qquad x \approx 1.386 \end{aligned}$$

91.
$$\begin{aligned} \ln 3x &= 8.2 \\ e^{\ln 3x} &= e^{8.2} \\ 3x &= e^{8.2} \\ x &= \frac{e^{8.2}}{3} \approx 1213.650 \end{aligned}$$

92.
$$\begin{aligned} 4 \ln 3x &= 15 \\ \ln 3x &= \frac{15}{4} \\ 3x &= e^{15/4} \\ x &= \frac{e^{15/4}}{3} \approx 14.174 \end{aligned}$$

93.
$$\begin{aligned} \ln x + \ln(x - 3) &= 1 \\ \ln[x(x - 3)] &= 1 \\ \ln(x^2 - 3x) &= 1 \\ e^{\ln(x^2 - 3x)} &= e^1 \\ x^2 - 3x - e &= 0 \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-e)}}{2(1)} \\ x &= \frac{3 \pm \sqrt{9 + 4e}}{2} \\ x &= \frac{3 + \sqrt{9 + 4e}}{2} \approx 3.729 \end{aligned}$$

$x = \dfrac{3 - \sqrt{9 + 4e}}{2}$ is extraneous since the domain of the $\ln x$ term is $x > 0$.

94.
$$\begin{aligned} \ln(x + 2) - \ln x &= 2 \\ \ln\left(\frac{x + 2}{x}\right) &= 2 \\ e^{\ln(x+2/x)} &= e^2 \\ \frac{x + 2}{x} &= e^2 \\ x + 2 &= e^2 x \\ 2 &= e^2 x - x \\ 2 &= x(e^2 - 1) \\ x &= \frac{2}{e^2 - 1} \approx 0.313 \end{aligned}$$

95. $\log_8(x-1) = \log_8(x-2) - \log_8(x+2)$

$$\log_8(x-1) = \log_8\left(\frac{x-2}{x+2}\right)$$

$$x - 1 = \frac{x-2}{x+2}$$

$$(x-1)(x+2) = x - 2$$

$$x^2 + x - 2 = x - 2$$

$$x^2 = 0$$

$$x = 0$$

Because $x = 0$ is not in the domain of $\log_8(x-1)$ or of $\log_8(x-2)$, it is an extraneous solution. The equation has no solution.

96. $\log_6(x+2) - \log_6 x = \log_6(x+5)$

$$\log_6\left(\frac{x+2}{x}\right) = \log_6(x+5)$$

$$\frac{x+2}{x} = x + 5$$

$$x + 2 = x^2 + 5x$$

$$0 = x^2 + 4x - 2$$

$$x = -2 \pm \sqrt{6}, \text{ Quadratic Formula}$$

Only $x = -2 + \sqrt{6} \approx 0.449$ is a valid solution.

97. $\log(1-x) = -1$

$$1 - x = 10^{-1}$$

$$1 - \tfrac{1}{10} = x$$

$$x = 0.900$$

98. $\log(-x-4) = 2$

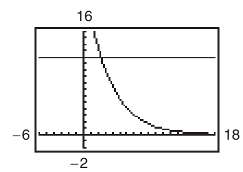
$$-x - 4 = 10^2$$

$$-x = 100 + 4$$

$$x = -104$$

99. $25e^{-0.3x} = 12$

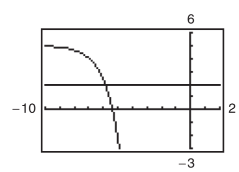
Graph $y_1 = 25e^{-0.3x}$ and $y_2 = 12$.



The graphs intersect at $x \approx 2.447$.

100. $2 = 5 - e^{x+7}$

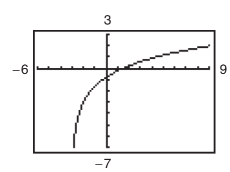
Graph $y_1 = 2$ and $y_2 = 5 - e^{x+7}$.



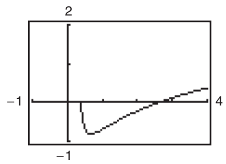
The graphs intersect at $x \approx -5.901$.

101. $2\ln(x+3) - 3 = 0$

Graph $y_1 = 2\ln(x+3) - 3$.



The x-intercept is at $x \approx 1.482$.

102. 

The x-intercepts are at $x \approx 2.618, 0.382$.

$$2 \ln x - \ln(3x - 1) = 0$$
$$\ln x^2 - \ln(3x - 1) = 0$$
$$\ln\left(\frac{x^2}{3x - 1}\right) = 0$$
$$e^{\ln(x^2/3x-1)} = e^0$$
$$\frac{x^2}{3x - 1} = 1$$
$$x^2 = 3x - 1$$
$$x^2 - 3x + 1 = 0$$
$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$$
$$x = \frac{3 \pm \sqrt{5}}{2} \approx 2.618, 0.382$$

103. $P = 8500, A = 3(8500) = 25{,}500, r = 1.5\%$

$$A = Pe^{rt}$$
$$25{,}500 = 8500e^{0.015t}$$
$$3 = e^{0.015t}$$
$$\ln 3 = 0.015t$$
$$t = \frac{\ln 3}{0.015} \approx 73.2 \text{ years}$$

104.
$$S = 93 \log(d) + 65$$
$$283 = 93 \log(d) + 65$$
$$218 = 93 \log(d)$$
$$\log(d) = \tfrac{218}{93}$$
$$d = 10^{(218/93)} \approx 221 \text{ miles}$$

105. $y = 3e^{-2x/3}$

Exponential decay model

Matches graph (e).

106. $y = 4e^{2x/3}$

Exponential growth model

Matches graph (b).

107. $y = \ln(x + 3)$

Logarithmic model

Vertical asymptote: $x = -3$

Graph includes $(-2, 0)$

Matches graph (f).

108. $y = 7 - \log(x + 3)$

Logarithmic model

Vertical asymptote: $x = -3$

Matches graph (d).

109. $y = 2e^{-(x+4)^2/3}$

Gaussian model

Matches graph (a).

110. $y = \dfrac{6}{1 + 2e^{-2x}}$

Logistics growth model

Matches graph (c).

111. $y = ae^{bx}$

Using the point $(0, 2)$, you have

$2 = ae^{b(0)}$

$2 = ae^{0}$

$2 = a(1)$

$2 = a$

Then, using the point $(4, 3)$, you have

$3 = 2e^{b(4)}$

$3 = 2e^{4b}$

$\frac{3}{2} = e^{4b}$

$\ln \frac{3}{2} = 4b$

$\frac{1}{4}\ln\left(\frac{3}{2}\right) = b$

So, $y = 2e^{\frac{1}{4}\ln\left(\frac{3}{2}\right)x}$

or

$y = 2e^{0.1014x}$

112. $N_0 = 2000$ and $N_3 = 1400$, so $N = 2000e^{kt}$ and $1400 = 2000e^{3k}$.

$\frac{7}{10} = e^{3k}$

$3k = \ln\left(\frac{7}{10}\right)$

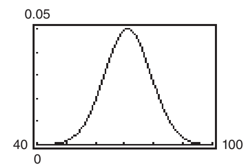
$k = \frac{\ln(7/10)}{3} = -0.11889$

The population one year ago:

$N(4) = 2000e^{-0.11889(4)} = 1243$ bats

113. $y = 0.0499e^{-(x-71)^2/128}$, $40 \le x \le 100$

Graph $y_1 = 0.0499e^{-(x-71)^2/128}$.



The average test score is 71.

114. $N = \frac{157}{1 + 5.4e^{-0.12t}}$

(a) When $N = 50$:

$$50 = \frac{157}{1 + 5.4e^{-0.12t}}$$

$$1 + 5.4e^{-0.12t} = \frac{157}{50}$$

$$5.4e^{-0.12t} = \frac{107}{50}$$

$$e^{-0.12t} = \frac{107}{270}$$

$$-0.12t = \ln \frac{107}{270}$$

$$t = \frac{\ln(107/270)}{-0.12} \approx 7.7 \text{ weeks}$$

(b) When $N = 75$:

$$75 = \frac{157}{1 + 5.4e^{-0.12t}}$$

$$1 + 5.4e^{-0.12t} = \frac{157}{75}$$

$$5.4e^{-0.12t} = \frac{82}{75}$$

$$e^{-0.12t} = \frac{82}{405}$$

$$-0.12t = \ln \frac{82}{405}$$

$$t = \frac{\ln(82/405)}{-0.12} \approx 13.3 \text{ weeks}$$

115. $\beta = 10 \log\left(\frac{I}{10^{-12}}\right)$

$\frac{\beta}{10} = \log\left(\frac{I}{10^{-12}}\right)$

$10^{\beta/10} = \frac{I}{10^{-12}}$

$I = 10^{\beta/10-12}$

(a) $\beta = 60$

$I = 10^{60/10-12}$

$= 10^{-6}$ watt/m^2

(b) $\beta = 135$

$I = 10^{135/10-12}$

$= 10^{1.5}$

$= 10\sqrt{10}$ watts/m^2

(c) $\beta = 1$

$I = 10^{1/10-12}$

$= 10^{\frac{1}{10}} \times 10^{-12}$

$\approx 1.259 \times 10^{-12}$ watt/m^2

116. $y = e^{kt}$

When k is positive, the graph is increasing because $e^{kt} \to 0$ as $x \to -\infty$.

When k is negative, the graph is decreasing because $e^{kt} \to 0$ as $x \to \infty$.

117. True. By the inverse properties, $\log_b b^{2x} = 2x$.

118. False. $\ln x + \ln y = \ln(xy) \neq \ln(x + y)$

Problem Solving for Chapter 3

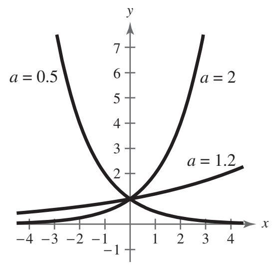
1. $y = a^x$

$y_1 = 0.5^x$

$y_2 = 1.2^x$

$y_3 = 2.0^x$

$y_4 = x$



The curves $y = 0.5^x$ and $y = 1.2^x$ cross the line $y = x$. From checking the graphs it appears that $y = x$ will cross $y = a^x$ for $0 \le a \le 1.44$.

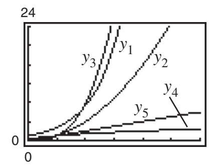
2. $y_1 = e^x$

$y_2 = x^2$

$y_3 = x^3$

$y_4 = \sqrt{x}$

$y_5 = |x|$



The function that increases at the fastest rate for "large" values of x is $y_1 = e^x$. (**Note:** One of the intersection points of $y = e^x$ and $y = x^3$ is approximately $(4.536, 93)$ and past this point $e^x > x^3$. This is not shown on the graph above.)

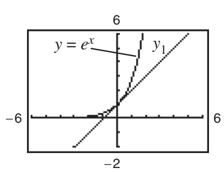
3. The exponential function, $y = e^x$, increases at a faster rate than the polynomial $y = x^n$.

4. It usually implies rapid growth.

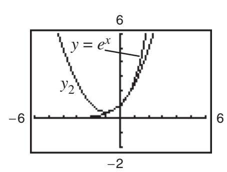
5. (a) $f(u + v) = a^{u+v} = a^u \cdot a^v = f(u) \cdot f(v)$ (b) $f(2x) = a^{2x} = (a^x)^2 = [f(x)]^2$

6.
$$\begin{aligned}[f(x)]^2 - [g(x)]^2 &= \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2 \\ &= \left(\frac{e^{2x} + 2 + e^{-2x}}{4}\right) - \left(\frac{e^{2x} - 2 + e^{-2x}}{4}\right) \\ &= \frac{4}{4} \\ &= 1\end{aligned}$$

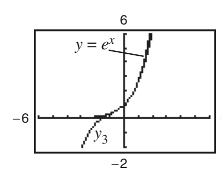
7. (a)



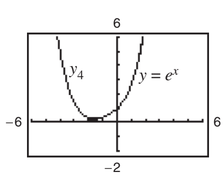
(b)



(c)



8. $y_4 = 1 + \dfrac{x}{1!} + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \dfrac{x^4}{4!}$



As more terms are added, the polynomial approaches e^x.

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots$$

9.
$$\begin{aligned} f(x) &= e^x - e^{-x} \\ y &= e^x - e^{-x} \\ x &= e^y - e^{-y} \\ x &= \frac{e^{2y} - 1}{e^y} \\ xe^y &= e^{2y} - 1 \\ e^{2y} - xe^y - 1 &= 0 \\ e^y &= \frac{x \pm \sqrt{x^2 + 4}}{2} \quad \text{Quadratic Formula} \end{aligned}$$

Choosing the positive quantity for e^y you have

$y = \ln\left(\frac{x + \sqrt{x^2 + 4}}{2}\right)$. So,

$f^{-1}(x) = \ln\left(\frac{x + \sqrt{x^2 + 4}}{2}\right)$.

10. $f(x) = \frac{a^x + 1}{a^x - 1}, a > 0, a \neq 1$

$$\begin{aligned} x &= \frac{a^y + 1}{a^y - 1} \\ x(a^y - 1) &= a^y + 1 \\ xa^y - a^y &= x + 1 \\ a^y(x - 1) &= x + 1 \\ a^y &= \frac{x + 1}{x - 1} \end{aligned}$$

$$y = \log_a\left(\frac{x + 1}{x - 1}\right) = \frac{\ln\left(\frac{x + 1}{x - 1}\right)}{\ln a} = f^{-1}(x)$$

11. Answer (c). $y = 6\left(1 - e^{-x^2/2}\right)$

The graph passes through (0, 0) and neither (a) nor (b) pass through the origin. Also, the graph has y-axis symmetry and a horizontal asymptote at $y = 6$.

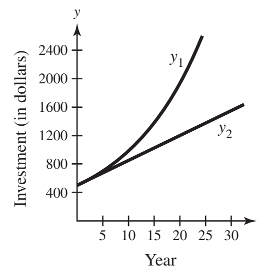
12. (a) The steeper curve represents the investment earning compound interest, because compound interest earns more than simple interest. With simple interest there is no compounding so the growth is linear.

(b) Compound interest formula:

$$y_1 = 500\left(1 + \frac{0.07}{1}\right)^{(1)t} = 500(1.07)^t$$

Simple interest formula:

$$y_2 = Prt + P = 500(0.07)t + 500 = 35t + 500$$



(c) You should choose compound interest since the earnings would be higher.

13. $y_1 = c_1\left(\frac{1}{2}\right)^{t/k_1}$ and $y_2 = c_2\left(\frac{1}{2}\right)^{t/k_2}$

$$\begin{aligned} c_1\left(\frac{1}{2}\right)^{t/k_1} &= c_2\left(\frac{1}{2}\right)^{t/k_2} \\ \frac{c_1}{c_2} &= \left(\frac{1}{2}\right)^{(t/k_2 - t/k_1)} \\ \ln\left(\frac{c_1}{c_2}\right) &= \left(\frac{t}{k_2} - \frac{t}{k_1}\right)\ln\left(\frac{1}{2}\right) \\ \ln c_1 - \ln c_2 &= t\left(\frac{1}{k_2} - \frac{1}{k_1}\right)\ln\left(\frac{1}{2}\right) \\ t &= \frac{\ln c_1 - \ln c_2}{\left[(1/k_2) - (1/k_1)\right]\ln(1/2)} \end{aligned}$$

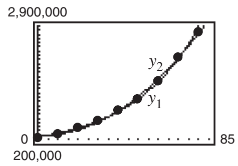
14. $B = B_0a^{kt}$ through $(0, 500)$ and $(2, 200)$

$$\begin{aligned} B_0 &= 500 \\ 200 &= 500a^{k(2)} \\ \frac{2}{5} &= a^{2k} \\ \log_a\left(\frac{2}{5}\right) &= 2k \\ \frac{1}{2}\log_a\left(\frac{2}{5}\right) &= k \\ B &= 500a^{[(1/2)\log_a(2/5)]t} \\ &= 500\left[a^{\log_a(2/5)}\right]^{t/2} \\ &= 500\left(\frac{2}{5}\right)^{t/2} \end{aligned}$$

15. (a) $y_1 \approx 252{,}606(1.0310)^t$

(b) $y_2 \approx 400.88t^2 - 1464.6t + 291{,}782$

(c)



(d) The exponential model is a better fit for the data, but neither would be reliable to predict the population of the United States in 2020. The exponential model approaches infinity rapidly.

16. Let $\log_a x = m$ and $\log_{a/b} x = n$. Then $x = a^m$ and $x = (a/b)^n$.

$$a^m = \left(\frac{a}{b}\right)^n$$

$$a^{m/n} = \frac{a}{b}$$

$$a^{m/n-1} = \frac{1}{b}$$

$$\log_a \frac{1}{b} = \frac{m}{n} - 1$$

$$1 + \log_a \frac{1}{b} = \frac{m}{n}$$

$$1 + \log_a \frac{1}{b} = \frac{\log_a x}{\log_{a/b} x}$$

17.

$$(\ln x)^2 = \ln x^2$$

$$(\ln x)^2 - 2\ln x = 0$$

$$\ln x(\ln x - 2) = 0$$

$$\ln x = 0 \text{ or } \ln x = 2$$

$$x = 1 \text{ or } x = e^2$$

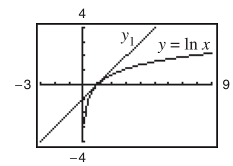
18. $y = \ln x$

$y_1 = x - 1$

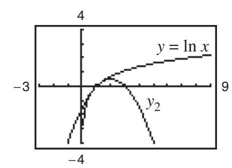
$y_2 = (x - 1) - \frac{1}{2}(x - 1)^2$

$y_3 = (x - 1) - \frac{1}{2}(x - 1)^2 + \frac{1}{3}(x - 1)^3$

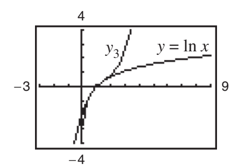
(a)



(b)



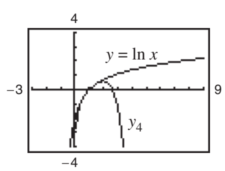
(c)



19. $y_4 = (x - 1) - \frac{1}{2}(x - 1)^2 + \frac{1}{3}(x - 1)^3 - \frac{1}{4}(x - 1)^4$

The pattern implies that

$\ln x = (x - 1) - \frac{1}{2}(x - 1)^2 + \frac{1}{3}(x - 1)^3 - \frac{1}{4}(x - 1)^4 + \ldots.$



20.

$$y = ab^x \qquad\qquad y = ax^b$$

$$\ln y = \ln(ab^x) \qquad\qquad \ln y = \ln(ax^b)$$

$$\ln y = \ln a + \ln b^x \qquad\qquad \ln y = \ln a + \ln x^b$$

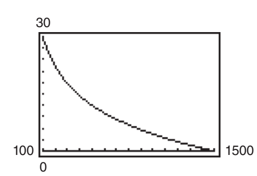
$$\ln y = \ln a + x\ln b \qquad\qquad \ln y = \ln a + b\ln x$$

$$\ln y = (\ln b)x + \ln a \qquad\qquad \ln y = b\ln x + \ln a$$

Slope: $m = \ln b$ — Slope: $m = b$

y-intercept: $(0, \ln a)$ — y-intercept: $(0, \ln a)$

21. $y = 80.4 - 11\ln x$



$y(300) = 80.4 - 11\ln 300 \approx 17.7 \text{ ft}^3/\text{min}$

22. (a) $\frac{450}{30} = 15$ cubic feet per minute

(b) $$15 = 80.4 - 11 \ln x$$
$$11 \ln x = 65.4$$
$$\ln x = \frac{65.4}{11}$$
$$x = e^{65.4/11}$$
$x \approx 382$ cubic feet of air space per child.

(c) Total air space required:
$382(30) = 11{,}460$ cubic feet

Let x = floor space in square feet and $h = 30$ feet.

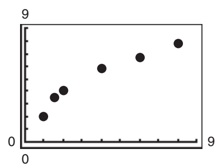
$V = xh$

$11{,}460 = x(30)$

$x = 382$

If the ceiling height is 30 feet, the minimum number of square feet of floor space required is 382 square feet.

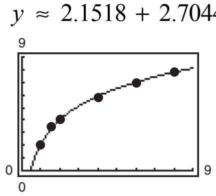
23. (a)



(b) The data could best be modeled by a logarithmic model.

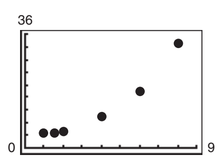
(c) The shape of the curve looks much more logarithmic than linear or exponential.

(d) $y \approx 2.1518 + 2.7044 \ln x$



(e) The model is a good fit to the actual data.

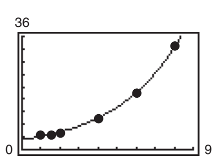
24. (a)



(b) The data could best be modeled by an exponential model.

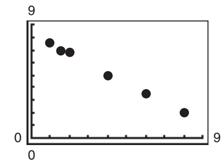
(c) The data scatter plot looks exponential.

(d) $y \approx 3.114(1.341)^x$



(e) The model graph hits every point of the scatter plot.

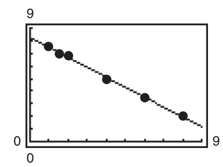
25. (a)



(b) The data could best be modeled by a linear model.

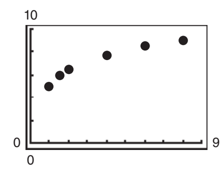
(c) The shape of the curve looks much more linear than exponential or logarithmic.

(d) $y \approx -0.7884x + 8.2566$



(e) The model is a good fit to the actual data.

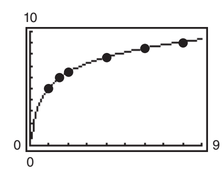
26. (a)



(b) The data could best be modeled by a logarithmic model.

(c) The data scatter plot looks logarithmic.

(d) $y \approx 5.099 + 1.92 \ln(x)$



(e) The model graph hits every point of the scatter plot.

Practice Test for Chapter 3

1. Solve for x: $x^{3/5} = 8$.

2. Solve for x: $3^{x-1} = \frac{1}{81}$.

3. Graph $f(x) = 2^{-x}$.

4. Graph $g(x) = e^x + 1$.

5. If \$5000 is invested at 9% interest, find the amount after three years if the interest is compounded
(a) monthly.
(b) quarterly.
(c) continuously.

6. Write the equation in logarithmic form: $7^{-2} = \frac{1}{49}$.

7. Solve for x: $x - 4 = \log_2 \frac{1}{64}$.

8. Given $\log_b 2 = 0.3562$ and $\log_b 5 = 0.8271$, evaluate $\log_b \sqrt[4]{8/25}$.

9. Write $5 \ln x - \frac{1}{2} \ln y + 6 \ln z$ as a single logarithm.

10. Using your calculator and the change of base formula, evaluate $\log_9 28$.

11. Use your calculator to solve for N: $\log_{10} N = 0.6646$

12. Graph $y = \log_4 x$.

13. Determine the domain of $f(x) = \log_3(x^2 - 9)$.

14. Graph $y = \ln(x - 2)$.

15. True or false: $\dfrac{\ln x}{\ln y} = \ln(x - y)$

16. Solve for x: $5^x = 41$

17. Solve for x: $x - x^2 = \log_5 \frac{1}{25}$

18. Solve for x: $\log_2 x + \log_2(x - 3) = 2$

19. Solve for x: $\dfrac{e^x + e^{-x}}{3} = 4$

20. Six thousand dollars is deposited into a fund at an annual interest rate of 13%. Find the time required for the investment to double if the interest is compounded continuously.