CHAPTER 10
Topics in Analytic Geometry

CHAPTER 10
Topics in Analytic Geometry

Section 10.1 Lines

1. inclination

2. tan θ

3. $\left|\dfrac{m_2 - m_1}{1 + m_1 m_2}\right|$

4. $\dfrac{\left|Ax_1 + By_1 + C\right|}{\sqrt{A^2 + B^2}}$.

5. $m = \tan\dfrac{\pi}{6} = \dfrac{\sqrt{3}}{3}$

6. $m = \tan\dfrac{\pi}{4} = 1$

7. $m = \tan\dfrac{3\pi}{4} = -1$

8. $m = \tan\dfrac{2\pi}{3} = -\sqrt{3}$

9. $m = \tan\dfrac{\pi}{3} = \sqrt{3}$

10. $m = \tan\dfrac{5\pi}{6} = -\dfrac{\sqrt{3}}{3}$

11. $m = \tan 0.39 \approx 0.4111$

12. $m = \tan 0.63 \approx 0.7291$

13. $m = \tan 1.27 \approx 3.2236$

14. $m = \tan 1.35 \approx 4.4552$

15. $m = \tan 1.81 \approx -4.1005$

16. $m = \tan 2.88 \approx -0.2677$

17. $m = 1$

$1 = \tan\theta$

$\theta = \dfrac{\pi}{4}\text{ radian} = 45°$

18. $m = \sqrt{3}$

$\sqrt{3} = \tan\theta$

$\theta = \tan^{-1}\sqrt{3}$

$= \dfrac{\pi}{3}\text{ radians} = 60°$

19. $m = \dfrac{2}{3}$

$\dfrac{2}{3} = \tan\theta$

$\theta = \arctan\left(\dfrac{2}{3}\right)$

$\approx 0.5880\text{ radian} \approx 33.7°$

20. $m = \dfrac{1}{4}$

$\dfrac{1}{4} = \tan\theta$

$\theta = \tan^{-1}\left(\dfrac{1}{4}\right)$

$\approx 0.2450\text{ radian} \approx 14.0°$

21. $m = -1$

$-1 = \tan\theta$

$\theta = 180° + \arctan(-1)$

$= \dfrac{3\pi}{4}\text{ radians} = 135°$

22. $m = -\sqrt{3}$

$-\sqrt{3} = \tan\theta$

$\theta = \tan^{-1}\left(-\sqrt{3}\right) + \pi$

$= \dfrac{2\pi}{3}\text{ radians} = 120°$

23. $m = -\dfrac{3}{2}$

$-\dfrac{3}{2} = \tan\theta$

$\theta = \tan^{-1}\left(-\dfrac{3}{2}\right) + \pi$

$\approx 2.1588\text{ radians} \approx 123.7°$

24. $m = -\frac{5}{9}$

$-\frac{5}{9} = \tan\theta$

$\theta = \tan^{-1}\left(-\frac{5}{9}\right) + \pi$

$\approx 2.6345 \text{ radians} \approx 150.9°$

25. $\left(\sqrt{3}, 2\right), (0, 1)$

$m = \frac{1-2}{0-\sqrt{3}} = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\frac{1}{\sqrt{3}} = \tan\theta$

$\theta = \arctan\frac{1}{\sqrt{3}}$

$= \frac{\pi}{6} \text{ radian} = 30°$

26. $\left(1, 2\sqrt{3}\right), \left(0, \sqrt{3}\right)$

$m = \frac{\sqrt{3} - 2\sqrt{3}}{0-1} = \frac{-\sqrt{3}}{-1} = \sqrt{3}$

$\sqrt{3} = \tan\theta$

$\theta = \arctan\sqrt{3} = \frac{\pi}{3} \text{ radians} = 60°$

27. $\left(-\sqrt{3}, -1\right), (0, -2)$

$m = \frac{-2-(-1)}{0-\left(-\sqrt{3}\right)} = \frac{-1}{\sqrt{3}}$

$-\frac{1}{\sqrt{3}} = \tan\theta$

$\theta = \arctan\left(-\frac{1}{\sqrt{3}}\right) = \frac{5\pi}{6} \text{ radians} = 150°$

28. $\left(3, \sqrt{3}\right), \left(6, -2\sqrt{3}\right)$

$m = \frac{-2\sqrt{3} - \sqrt{3}}{6-3} = \frac{-3\sqrt{3}}{3} = -\sqrt{3}$

$-\sqrt{3} = \tan\theta$

$\theta = \arctan\left(-\sqrt{3}\right) = \frac{2\pi}{3} \text{ radians} = 120°$

29. $(6, 1), (10, 8)$

$m = \frac{8-1}{10-6} = \frac{7}{4}$

$\frac{7}{4} = \tan\theta$

$\theta = \arctan\frac{7}{4} \approx 1.0517 \text{ radians} \approx 60.3°$

30. $(12, 8), (-4, -3)$

$m = \frac{8-(-3)}{12-(-4)} = \frac{11}{16}$

$\frac{11}{16} = \tan\theta$

$\theta = \tan^{-1}\frac{11}{16} \approx 0.6023 \text{ radian} \approx 34.5°$

31. $(-2, 20), (10, 0)$

$m = \frac{0-20}{10-(-2)} = -\frac{20}{12} = -\frac{5}{3}$

$-\frac{5}{3} = \tan\theta$

$\theta = \pi + \arctan\left(-\frac{5}{3}\right) \approx 2.1112 \text{ radians} \approx 121.0°$

32. $(0, 100), (50, 0)$

$m = \frac{100-0}{0-50} = -2$

$-2 = \tan\theta$

$\theta = \tan^{-1}(-2) + \pi \approx 2.0344 \text{ radians} \approx 116.6°$

33. $\left(\frac{1}{4}, \frac{3}{2}\right), \left(\frac{1}{3}, \frac{1}{2}\right)$

$m = \frac{\frac{1}{2} - \frac{3}{2}}{\frac{1}{3} - \frac{1}{4}} = -\frac{1}{\frac{1}{12}} = -12$

$-12 = \tan\theta$

$\theta = \arctan(-12) + \pi \approx 1.6539 \text{ radians}$

$\approx 94.8°$

34. $\left(\frac{2}{5}, -\frac{3}{4}\right), \left(-\frac{11}{10}, -\frac{1}{4}\right)$

$m = \frac{-\frac{1}{4} - \left(-\frac{3}{4}\right)}{-\frac{11}{10} - \frac{2}{5}} = -\frac{\frac{1}{2}}{-\frac{3}{2}} = -\frac{1}{3}$

$-\frac{1}{3} = \tan\theta$

$\theta = \arctan\left(-\frac{1}{3}\right) + \pi \approx 2.8198 \text{ radians}$

$\approx 161.6°$

35. $2x + 2y - 5 = 0$

$y = -x + \frac{5}{2} \Rightarrow m = -1$

$-1 = \tan\theta$

$\theta = \arctan(-1) = \frac{3\pi}{4} \text{ radians} = 135°$

36. $x - \sqrt{3}y + 1 = 0$

$$y = \frac{1}{\sqrt{3}}x + \frac{1}{\sqrt{3}} \Rightarrow m = \frac{1}{\sqrt{3}}$$

$$\frac{1}{\sqrt{3}} = \tan\theta$$

$$\theta = \arctan\frac{1}{\sqrt{3}} = \frac{\pi}{6}\text{ radian} = 30°$$

37. $3x - 3y + 1 = 0$

$$y = x + \frac{1}{3} \Rightarrow m = 1$$

$$1 = \tan\theta$$

$$\theta = \arctan 1 = \frac{\pi}{4}\text{ radian} = 45°$$

38. $\sqrt{3}x - y + 2 = 0$

$$y = \sqrt{3}x + 2 \Rightarrow m = \sqrt{3}$$

$$\sqrt{3} = \tan\theta$$

$$\theta = \arctan\sqrt{3} = \frac{\pi}{3}\text{ radians} = 60°$$

39. $x + \sqrt{3y} + 2 = 0$

$$y = -\frac{1}{\sqrt{3}}x - \frac{2}{\sqrt{3}} \Rightarrow m = -\frac{1}{\sqrt{3}}$$

$$-\frac{1}{\sqrt{3}} = \tan\theta$$

$$\theta = \arctan\left(-\frac{1}{\sqrt{3}}\right) = \frac{5\pi}{6}\text{ radians} = 150°$$

40. $-2\sqrt{3}x - 2y = 0$

$$y = -\sqrt{3}x \Rightarrow m = -\sqrt{3}$$

$$-\sqrt{3} = \tan\theta$$

$$\theta = \arctan\left(-\sqrt{3}\right) = \frac{2\pi}{3}\text{ radians} = 120°$$

41. $6x - 2y + 8 = 0$

$$y = 3x + 4 \Rightarrow m = 3$$

$$3 = \tan\theta$$

$$\theta = \arctan 3 \approx 1.2490\text{ radians} \approx 71.6°$$

42. $2x - 6y - 12 = 0$

$$y = \tfrac{1}{3}x - 2 \Rightarrow m = \tfrac{1}{3}$$

$$\tfrac{1}{3} = \tan\theta$$

$$\theta = \arctan\tfrac{1}{3} \approx 0.3218\text{ radian} \approx 18.4°$$

43. $4x + 5y - 9 = 0$

$$y = -\tfrac{4}{5}x + \tfrac{9}{5} \Rightarrow m = -\tfrac{4}{5}$$

$$-\tfrac{4}{5} = \tan\theta$$

$$\theta = \tan^{-1}\left(-\tfrac{4}{5}\right) + \pi$$

$$\approx 2.4669\text{ radians} \approx 141.3°$$

44. $5x + 3y = 0$

$$y = -\tfrac{5}{3}x \Rightarrow m = -\tfrac{5}{3}$$

$$-\tfrac{5}{3} = \tan\theta$$

$$\theta = \pi + \arctan\left(-\tfrac{5}{3}\right) \approx 2.1112\text{ radians} \approx 121.0°$$

45. $3x + y = 3 \Rightarrow y = -3x + 3 \Rightarrow m_1 = -3$

$x - y = 2 \Rightarrow y = x - 2 \Rightarrow m_2 = 1$

$$\tan\theta = \left|\frac{1 - (-3)}{1 + (-3)(1)}\right| = 2$$

$$\theta = \arctan 2 \approx 1.1071\text{ radians} \approx 63.4°$$

46. $x + 3y = 2 \Rightarrow y = -\frac{1}{3}x + \frac{2}{3} \Rightarrow m_1 = -\frac{1}{3}$

$x - 2y = -3 \Rightarrow y = \frac{1}{2}x + \frac{3}{2} \Rightarrow m_2 = \frac{1}{2}$

$$\tan\theta = \left|\frac{(1/2) - (-1/3)}{1 + (-1/3)(1/2)}\right| = 1$$

$$\theta = \tan^{-1} 1 = 45° = \frac{\pi}{4}\text{ radian}$$

47. $x - y = 0 \Rightarrow y = x \Rightarrow m_1 = 1$

$3x - 2y = -1 \Rightarrow y = \frac{3}{2}x + \frac{1}{2} \Rightarrow m_2 = \frac{3}{2}$

$$\tan\theta = \left|\frac{\frac{3}{2} - 1}{1 + \left(\frac{3}{2}\right)(1)}\right| = \frac{1}{5}$$

$$\theta = \arctan\frac{1}{5} \approx 0.1974\text{ radian} \approx 11.3°$$

48. $2x - y = 2 \Rightarrow y = 2x - 2 \Rightarrow m_1 = 2$

$4x + 3y = 24 \Rightarrow y = -\frac{4}{3}x + 8 \Rightarrow m_2 = -\frac{4}{3}$

$$\tan\theta = \left|\frac{(-4/3) - 2}{1 + (2)(-4/3)}\right| = 2$$

$$\theta = \tan^{-1} 2 \approx 63.4° \approx 1.1071\text{ radians}$$

49. $x - 2y = 7 \Rightarrow y = \frac{1}{2}x - \frac{7}{2} \Rightarrow m_1 = \frac{1}{2}$

$6x + 2y = 5 \Rightarrow y = -3x + \frac{5}{2} \Rightarrow m_2 = -3$

$$\tan\theta = \left|\frac{-3 - \frac{1}{2}}{1 + \left(\frac{1}{2}\right)(-3)}\right| = 7$$

$\theta = \arctan 7 \approx 1.4289 \text{ radians} \approx 81.9°$

50. $5x + 2y = 16 \Rightarrow y = -\frac{5}{2}x + 8 \Rightarrow m_1 = -\frac{5}{2}$

$3x - 5y = -1 \Rightarrow y = \frac{3}{5}x + \frac{1}{5} \Rightarrow m_2 = \frac{3}{5}$

$$\tan\theta = \left|\frac{(-5/2) - (3/5)}{1 + (-5/2)(3/5)}\right| = \frac{31}{5}$$

$\theta = \tan^{-1}\frac{31}{5} \approx 80.8° \approx 1.4109 \text{ radians}$

51. $x + 2y = 8 \Rightarrow y = -\frac{1}{2}x + 4 \Rightarrow m_1 = -\frac{1}{2}$

$x - 2y = 2 \Rightarrow y = \frac{1}{2}x - 1 \Rightarrow m_2 = \frac{1}{2}$

$$\tan\theta = \left|\frac{\frac{1}{2} - \left(-\frac{1}{2}\right)}{1 + \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)}\right| = \frac{4}{3}$$

$\theta = \arctan\left(\frac{4}{3}\right) \approx 0.9273 \text{ radian} \approx 53.1°$

52. $3x - 5y = 3 \Rightarrow y = \frac{3}{5}x - \frac{3}{5} \Rightarrow m_1 = \frac{3}{2}$

$3x + 5y = 12 \Rightarrow y = -\frac{3}{5}x + \frac{12}{5} \Rightarrow m_2 = -\frac{3}{5}$

$$\tan\theta = \left|\frac{(3/5) - (-3/5)}{1 + (3/5)(-3/5)}\right| = \frac{15}{8}$$

$\theta = \tan^{-1}\frac{15}{8} \approx 61.9° \approx 1.0808 \text{ radians}$

53. $0.05x - 0.03y = 0.21 \Rightarrow y = \frac{5}{3}x - 7 \Rightarrow m_1 = \frac{5}{3}$

$0.07x + 0.02y = 0.16 \Rightarrow y = -\frac{7}{2}x + 8 \Rightarrow m_2 = -\frac{7}{2}$

$$\tan\theta = \left|\frac{\left(-\frac{7}{2}\right) - \left(\frac{5}{3}\right)}{1 + \left(\frac{5}{3}\right)\left(-\frac{7}{2}\right)}\right| = \frac{31}{29}$$

$\theta = \arctan\left(\frac{31}{29}\right) \approx 0.8187 \text{ radian} \approx 46.9°$

54. $0.02x - 0.05y = -0.19 \Rightarrow y = \frac{2}{5}x + \frac{19}{5} \Rightarrow m_1 = \frac{2}{5}$

$0.03x + 0.04y = 0.52 \Rightarrow y = -\frac{3}{4}x + 13 \Rightarrow m_2 = -\frac{3}{4}$

$$\tan\theta = \left|\frac{(-3/4) - (2/5)}{1 + (2/5)(-3/4)}\right| \approx \frac{23}{14}$$

$\theta = \tan^{-1}\left(\frac{23}{14}\right) \approx 58.7° \approx 1.0240 \text{ radians}$

55. Let $A = (1, 5)$, $B = (3, 8)$, and $C = (4, 5)$.

Slope of AB: $m_1 = \dfrac{8 - 5}{3 - 1} = \dfrac{3}{2}$

Slope of BC: $m_2 = \dfrac{5 - 8}{4 - 3} = \dfrac{-3}{1} = -3$

Slope of AC: $m_3 = \dfrac{5 - 5}{4 - 1} = \dfrac{0}{3} = 0$

$$\tan A = \left|\frac{0 - \frac{3}{2}}{1 + \left(\frac{3}{2}\right)(0)}\right| = \frac{\frac{3}{2}}{1} = \frac{3}{2}$$

$$A = \arctan\left(\frac{3}{2}\right) \approx 56.3°$$

$$\tan B = \left|\frac{\frac{3}{2} - (-3)}{1 + (-3)\left(\frac{3}{2}\right)}\right| = \frac{\frac{9}{2}}{\frac{7}{2}} = \frac{9}{7}$$

$$B = \arctan\frac{9}{7} \approx 52.1°$$

$$\tan C = \left|\frac{-3 - 0}{1 + (0)(-3)}\right| = \frac{3}{1} = 3$$

$$C = \arctan 3 \approx 71.6°$$

56. Let $A = (2, 1)$, $B = (4, 4)$, and $C = (6, 2)$.

Slope of AB: $m_1 = \dfrac{1 - 4}{2 - 4} = \dfrac{3}{2}$

Slope of BC: $m_2 = \dfrac{4 - 2}{4 - 6} = -1$

Slope of AC: $m_3 = \dfrac{1 - 2}{2 - 6} = \dfrac{1}{4}$

$$\tan A = \left|\frac{\frac{1}{4} - \frac{3}{2}}{1 + \left(\frac{3}{2}\right)\left(\frac{1}{4}\right)}\right| = \frac{\frac{5}{4}}{\frac{11}{8}} = \frac{10}{11}$$

$$A = \arctan\left(\frac{10}{11}\right) \approx 42.3°$$

$$\tan B = \left|\frac{\frac{3}{2} - (-1)}{1 + (-1)\left(\frac{3}{2}\right)}\right| = \frac{\frac{5}{2}}{\frac{1}{2}} = 5$$

$$B = \arctan 5 \approx 78.7°$$

$$\tan C = \left|\frac{-1 - \frac{1}{4}}{1 + \left(\frac{1}{4}\right)(-1)}\right| = \frac{\frac{5}{4}}{\frac{3}{4}} = \frac{5}{3}$$

$$C = \arctan\left(\frac{5}{3}\right) \approx 59.0°$$

57. Let $A = (-4, -1)$, $B = (3, 2)$, and $C = (1, 0)$.

Slope of AB: $m_1 = \dfrac{-1 - 2}{-4 - 3} = \dfrac{3}{7}$

Slope of BC: $m_2 = \dfrac{2 - 0}{3 - 1} = 1$

Slope of AC: $m_3 = \dfrac{-1 - 0}{-4 - 1} = \dfrac{1}{5}$

$$\tan A = \left|\frac{\frac{1}{5} - \frac{3}{7}}{1 + \left(\frac{3}{7}\right)\left(\frac{1}{5}\right)}\right| = \frac{\frac{8}{35}}{\frac{38}{35}} = \frac{4}{9}$$

$$A = \arctan\left(\frac{4}{19}\right) \approx 11.9°$$

$$\tan B = \left|\frac{1 - \frac{3}{7}}{1 + \left(\frac{3}{7}\right)(1)}\right| = \frac{\frac{4}{7}}{\frac{10}{7}} = \frac{2}{5}$$

$$B = \arctan\left(\frac{2}{5}\right) \approx 21.8°$$

$$\begin{aligned} C &= 180° - A - B \\ &\approx 180° - 11.9° - 21.8° = 146.3° \end{aligned}$$

58. Let $A = (-3, 4)$, $B = (2, 1)$, and $C = (-2, 2)$.

Slope of AB: $m_1 = \dfrac{4 - 1}{-3 - 2} = -\dfrac{3}{5}$

Slope of BC: $m_2 = \dfrac{1 - 2}{2 - (-2)} = -\dfrac{1}{4}$

Slope of AC: $m_3 = \dfrac{4 - 2}{-3 - (-2)} = -2$

$$\tan A = \left|\frac{(-3/5) - (-2)}{1 + (-3/5)(-2)}\right| = \frac{7}{11}$$

$$A = \tan^{-1}\left(\frac{7}{11}\right) \approx 32.5°$$

$$\tan B = \left|\frac{(-3/5) - (-1/4)}{1 + (-3/5)(-1/4)}\right| = \frac{7}{23}$$

$$B = \tan^{-1}\left(\frac{7}{23}\right) \approx 16.9°$$

$$C = 180° - A - B \approx 180° - 32.5° - 16.9° = 130.6°$$

59. $(x_1, y_1) = (1, 2)$

$y = x + 2 \Rightarrow x - y + 2 = 0$

$$d = \frac{|(1)(1) + (-1)(2) + 2|}{\sqrt{1^2 + (-1)^2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \approx 0.7071$$

60. $(x_1, y_1) = (3, 1)$

$y = x + 3 \Rightarrow x - y + 3 = 0$

$$d = \frac{|(1)(3) + (-1)(1) + 3|}{\sqrt{1^2 + (-1)^2}} = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2} \approx 3.5355$$

61. $(x_1, y_1) = (2, 3)$

$y = 2x - 3 \Rightarrow 2x - y - 3 = 0$

$$d = \frac{|2(2) + (-1)(3) + (-3)|}{\sqrt{2^2 + (-1)^2}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5} \approx 0.8944$$

62. $(x_1, y_1) = (1, 5)$

$y = 4x + 5 \Rightarrow 4x - y + 5 = 0$

$$d = \frac{|4(1) + (-1)(5) + 5|}{\sqrt{4^2 + (-1)^2}} = \frac{4}{\sqrt{17}} = \frac{4\sqrt{17}}{17} \approx 0.9701$$

63. $(x_1, y_1) = (-2, 4)$

$y = -x + 6 \Rightarrow x + y - 6 = 0$

$$d = \frac{|(1)(-2) + (1)(4) + (-6)|}{\sqrt{1^2 + 1^2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2} \approx 2.8284$$

64. $(x_1, y_1) = (3, -3)$

$y = -3x - 4 \Rightarrow 3x + y + 4 = 0$

$$d = \frac{|(3)(3) + (1)(-3) + 4|}{\sqrt{3^2 + 1^2}} = \frac{10}{\sqrt{10}} = \sqrt{10} \approx 3.1623$$

65. $(x_1, y_1) = (1, -2)$

$y = 3x - 6 \Rightarrow 3x - y - 6 = 0$

$$d = \frac{|(3)(1) + (-1)(-2) + (-6)|}{\sqrt{3^2 + (-1)^2}} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10} \approx 0.3162$$

66. $(x_1, y_1) = (-3, 7)$

$y = -4x + 3 \Rightarrow 4x + y - 3 = 0$

$$d = \frac{|(4)(-3) + (1)(7) + (-3)|}{\sqrt{4^2 + 1^2}} = \frac{8}{\sqrt{17}} = \frac{8\sqrt{17}}{17} \approx 1.9403$$

67. $(x_1, y_1) = (2, 3)$

$3x + y = 1 \Rightarrow 3x + y - 1 = 0$

$$d = \frac{|3(2) + (1)(3) + (-1)|}{\sqrt{3^2 + 1^2}} = \frac{8}{\sqrt{10}} = \frac{8\sqrt{10}}{10} = \frac{4\sqrt{10}}{5} \approx 2.5298$$

68. $(x_1, y_1) = (2, 1)$

$-2x + y = 2 \Rightarrow -2x + y - 2 = 0$

$$d = \frac{|(-2)(2) + (1)(1) + (-2)|}{\sqrt{(-2)^2 + (1)^2}} = \frac{5}{\sqrt{5}} = \sqrt{5} \approx 2.2361$$

69. $(x_1, y_1) = (6, 2)$

$-3x + 4y = -5 \Rightarrow -3x + 4y + 5 = 0$

$$d = \frac{|(-3)(6) + (4)(2) + (5)|}{\sqrt{(-3)^2 + (4)^2}} = \frac{5}{\sqrt{25}} = 1$$

70. $(x_1, y_1) = (1, -4)$

$2x - 3y = -5 \Rightarrow 2x - 3y + 5 = 0$

$$d = \frac{|(2)(1) + (-3)(-4) + (5)|}{\sqrt{2^2 + (-3)^2}} = \frac{19}{\sqrt{13}} = \frac{19\sqrt{13}}{13} \approx 5.2697$$

71. $(x_1, y_1) = (-2, 4)$

$4x + 3y = 5 \Rightarrow 4x + 3y - 5 = 0$

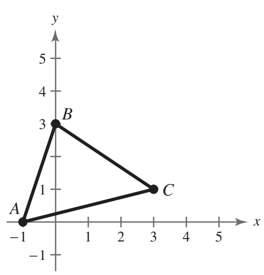
$$d = \frac{|(4)(-2) + (3)(4) + (-5)|}{\sqrt{4^2 + 3^2}} = \frac{1}{\sqrt{25}} = \frac{1}{5}$$

72. $(x_1, y_1) = (-3, -5)$

$-3x - 4y = 4 \Rightarrow 3x + 4y + 4 = 0$

$$d = \frac{|(3)(-3) + (4)(-5) + (4)|}{\sqrt{3^2 + 4^2}} = \frac{25}{\sqrt{25}} = 5$$

73. (a)



(b) Slope of the line AC: $m = \frac{1 - 0}{3 - (-1)} = \frac{1}{4}$

Equation of the line AC: $y - 0 = \frac{1}{4}(x + 1)$

$x - 4y + 1 = 0$

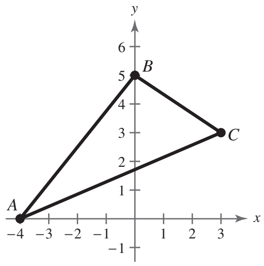
Altitude from $B = (0, 3)$: $h = \frac{|(1)(0) + (-4)(3) + (1)|}{\sqrt{1^2 + (-4)^2}} = \frac{11}{\sqrt{17}} = \frac{11\sqrt{17}}{17}$

(c) Length of the base AC: $b = \sqrt{(3 + 1)^2 + (1 - 0)^2} = \sqrt{17}$

Area of the triangle: $A = \frac{1}{2}bh$

$= \frac{1}{2}\left(\sqrt{17}\right)\left(\frac{11}{\sqrt{17}}\right) = \frac{11}{2}$ units^2

74. (a)



(b) Slope of the line of AC: $m = \dfrac{3 - 0}{3 - (-4)} = \dfrac{3}{7}$

Equation of the line AC: $y - 0 = \dfrac{3}{7}(x + 4)$

$$3x - 7y + 12 = 0$$

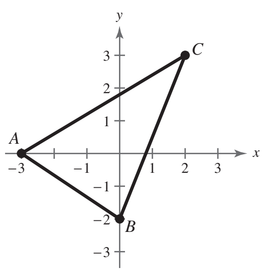
Altitude from $B = (0, 5)$: $h = \dfrac{\left|3(0) + (-7)(5) + (12)\right|}{\sqrt{3^2 + (-7)^2}} = \dfrac{23}{\sqrt{58}} = \dfrac{23\sqrt{58}}{58}$

(c) Length of the base AC: $b = \sqrt{(3 + 4)^2 + (3 - 0)^2} = \sqrt{58}$

Area of the triangle: $A = \dfrac{1}{2}bh$

$$= \frac{1}{2}\left(\sqrt{58}\right)\left(\frac{23}{\sqrt{58}}\right) = \frac{23}{2} \text{ units}^2$$

75. (a)



(b) Slope of the line AC: $m = \dfrac{3 - 0}{2 + 3} = \dfrac{3}{5}$

Equation of the line AC: $y - 0 = \dfrac{3}{5}(x + 3)$

$$3x - 5y + 9 = 0$$

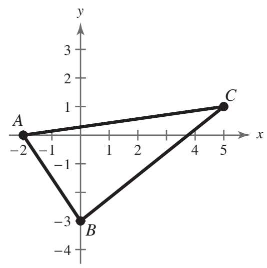
Altitude from $B = (0, -2)$: $h = \dfrac{\left|3(0) + (-5)(-2) + (9)\right|}{\sqrt{3^2 + (-5)^2}} = \dfrac{19}{\sqrt{34}} = \dfrac{19\sqrt{34}}{34}$

(c) Length of the base AC: $b = \sqrt{(2 + 3)^2 + (3 - 0)^2} = \sqrt{34}$

Area of the triangle: $A = \dfrac{1}{2}bh$

$$= \frac{1}{2}\left(\sqrt{34}\right)\left(\frac{19}{\sqrt{34}}\right) = \frac{19}{2} \text{ units}^2$$

76. (a)



(b) Slope of the line AC: $m = \dfrac{1 - 0}{5 + 2} = \dfrac{1}{7}$

Equation of the line AC: $y - 0 = \dfrac{1}{7}(x + 2)$

$$x - 7y + 2 = 0$$

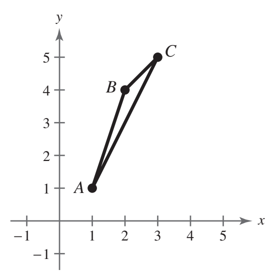
Altitude from $B = (0, -3)$: $h = \dfrac{|(1)(0) + (-7)(-3) + (2)|}{\sqrt{1^2 + (-7)^2}} = \dfrac{23}{\sqrt{50}} = \dfrac{23\sqrt{2}}{10}$

(c) Length of the base AC: $b = \sqrt{(5 + 2)^2 + (1 - 0)^2} = \sqrt{50} = 5\sqrt{2}$

Area of the triangle: $A = \dfrac{1}{2}bh$

$$= \frac{1}{2}\left(5\sqrt{2}\right)\left(\frac{23\sqrt{2}}{10}\right) = \frac{23}{2}\text{ units}^2$$

77. (a)



(b) Slope of the line AC: $b = \sqrt{(5 + 2)^2 + (1 - 0)^2} = \sqrt{50} = 5\sqrt{2}$

Equation of the line AC: $y - 1 = 2(x - 1)$

$$2x - y - 1 = 0$$

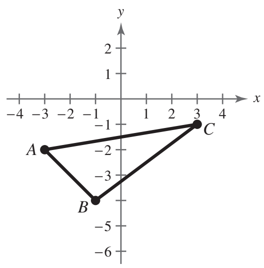
Altitude from $B = (2, 4)$: $h = \dfrac{|(2)(2) + (-1)(4) + (-1)|}{\sqrt{2^2 + (-1)^2}} = \dfrac{1}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}$

(c) Length of the base AC: $b = \sqrt{(3 - 1)^2 + (5 - 1)^2} = \sqrt{20} = 2\sqrt{5}$

Area of the triangle: $A = \dfrac{1}{2}bh$

$$= \frac{1}{2}\left(2\sqrt{5}\right)\left(\frac{\sqrt{5}}{5}\right) = 1\text{ unit}^2$$

78. (a)



(b) Slope of the line AC: $m = \dfrac{-1 + 2}{3 + 3} = \dfrac{1}{6}$

Equation of the line AC: $y + 2 = \dfrac{1}{6}(x + 3)$

$x - 6y - 9 = 0$

Altitude from $B = (-1, -4)$: $h = \dfrac{|(1)(-1) + (-6)(-4) + (-9)|}{\sqrt{1^2 + (-6)^2}} = \dfrac{14}{\sqrt{37}} = \dfrac{14\sqrt{37}}{37}$

(c) Length of the base AC: $b = \sqrt{(3 + 3)^2 + (-1 + 2)^2} = \sqrt{37}$

Area of the triangle: $A = \dfrac{1}{2}bh$

$= \dfrac{1}{2}\left(\sqrt{37}\right)\left(\dfrac{14}{\sqrt{37}}\right) = 7 \text{ units}^2$

79. $x + y = 1 \Rightarrow (0, 1)$ is a point on the line $\Rightarrow x_1 = 0$ and $y_1 = 1$

$x + y = 5 \Rightarrow A = 1, B = 1,$ and $C = -5$

$$d = \frac{|1(0) + 1(1) + (-5)|}{\sqrt{1^2 + 1^2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

80. $3x - 4y = 1$

$3x - 4y = 10$

A point on $3x - 4y = 10$ is $\left(0, -\dfrac{5}{2}\right)$. The distance between $\left(0, -\dfrac{5}{2}\right)$ and $3x - 4y = 1$ is:

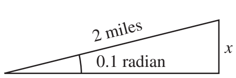
$A = 3, B = -4, C = -1, x_1 = 0, y_1 = -\dfrac{5}{2}$

$$d = \frac{|3(0) + (-4)(-5/2) - 1|}{\sqrt{3^2 + (-4)^2}} = \frac{9}{5}$$

81. Slope: $m = \tan 0.1 \approx 0.1003$

Change in elevation: $\sin 0.1 = \dfrac{x}{2(5280)}$

$x \approx 1054$ feet



Not drawn to scale

82. Slope: $m = \tan 0.2 \approx 0.2027$

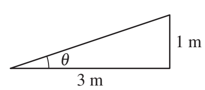
Change in elevation:

$\sin 0.20 = \dfrac{x}{5280} \Rightarrow x = 5280 \sin 0.20 \approx 1049$ feet

83. Slope $= \frac{3}{5}$

Inclination $= \tan^{-1} \frac{3}{5} \approx 31.0°$

84. (a)



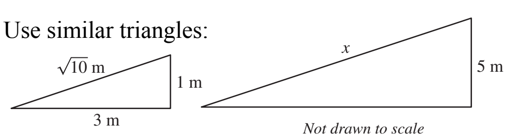
$m = \dfrac{1}{3}$

$\dfrac{1}{3} = \tan\theta$

$\tan^{-1}\left(\dfrac{1}{3}\right) = \theta$

(b) or $\theta \approx 18.4°$

(c) Use similar triangles:



Not drawn to scale

$\dfrac{x}{5} = \dfrac{\sqrt{10}}{1}$

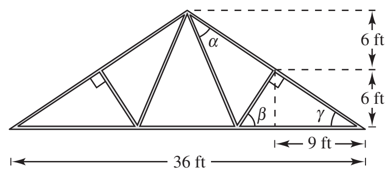
$x = 5\sqrt{10} \approx 15.8$ m

85. $\tan \gamma = \frac{6}{9}$

$\gamma = \arctan\left(\frac{2}{3}\right) \approx 33.69°$

$\beta = 90 - \gamma \approx 56.31°$

Also, because the right triangles containing α and β are equal, $\alpha = \gamma \approx 33.69°$



86. (a) $m = \tan \theta$

$0.36 = \tan \theta$

$\tan^{-1} 0.36 = \theta$

$\theta \approx 0.3456$ radian, or 19.8°

(b) Let z be the change in elevation.

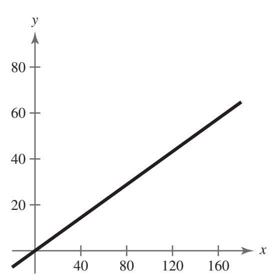
$\sin \theta = \frac{z}{170}$

$170 \sin \theta = z$

$z = 170 \sin(19.8°) \approx 57.6$ ft

(c) The slope is $m = 0.36$ and the y-intercept is $(0, 0)$. So, an equation is $y = 0.36x$.

(d)



87. True. The inclination of a line is related to its slope by $m = \tan \theta$. If the line has an inclination of 0 radians, then the slope is 0 radians.

88. True. The inclination of a line is related to its slope by $m = \tan \theta$. If the angle is greater than $\pi/2$ but less than π, then the angle is in the second quadrant where the tangent function is negative.

89. False. Substitute $m_1 = \tan \theta_1$ and $m_2 = \tan \theta_2$ into the formula for the angle between two lines.

90. False. The inclination is the positive angle measured counterclockwise from the x-axis.

91. False. By definition, the inclination of a nonhorizontal line is the positive angle θ measured counter clockwise from the x-axis to the line. So, the angle θ can be acute, right or obtuse. The angle θ between two lines is less than $\pi/2$ because, if $\theta > \frac{\pi}{2}$, then $\tan \theta < 0$.

Because the formula for the angle between two lines involves absolute value, then $\tan \theta$ will always be positive. So, θ cannot be larger than $\pi/2$.

92. (a) Partition the pentagon into triangles and find the areas of the triangles using the method from Example 5.

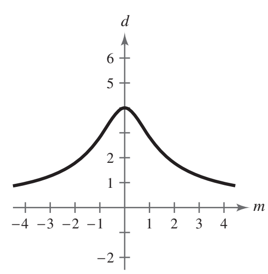
(b) For acute angles, apply the formula directly. For obtuse angles, subtract the angle of intersection from 180°.

93. (a) $(0, 0) \Rightarrow x_1 = 0$ and $y_1 = 0$

$y = mx + 4 \Rightarrow 0 = mx - y + 4$

$$d = \frac{|m(0) + (-1)(0) + 4|}{\sqrt{m^2 + (-1)^2}} = \frac{4}{\sqrt{m^2 + 1}}$$

(b)



(c) The maximum distance of 4 occurs when the slope m is 0 and the line through $(0, 4)$ is horizontal.

(d) The graph has a horizontal asymptote at $d = 0$. As the slope becomes larger, the distance between the origin and the line, $y = mx + 4$, becomes smaller and approaches 0.

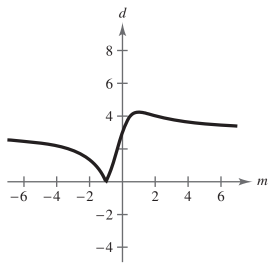
94. Slope m and y-intercept $(0, 4)$

(a) $(x_1, y_1) = (3, 1)$ and line: $y = mx + 4$

$A = -m, B = 1, C = -4$

$$d = \frac{|(-m)(3) + (1)(1) + (-4)|}{\sqrt{(-m)^2 + 1^2}} = \frac{3|m + 1|}{\sqrt{m^2 + 1}}$$

(b)



(c) From the graph it appears that the maximum distance is obtained when $m = 1$.

(d) Yes. From the graph it appears that the distance is 0 when $m = -1$.

(e) The asymptote of the graph in part (b) is $d = 3$. As the line approaches the vertical, the distance approaches 3.

Section 10.2 Introduction to Conics: Parabolas

1. conic

2. degenerate conic

3. locus

4. parabola; directrix; focus

5. axis

6. vertex

7. focal chord

8. tangent

9. $y^2 = 4x$

Vertex: $(0, 0)$

$p = 1 > 0$

The graph opens to the right because p is positive. So, the equation matches graph (c).

10. $x^2 = 2y$

Vertex: $(0, 0)$

$p = \frac{1}{2} > 0$

The graph opens upward because p is positive. So, the equation matches graph (a).

11. $x^2 = -8y$

Vertex: $(0, 0)$

$p = -2 < 0$

The graph opens downward because p is negative. So, the equation matches graph (b).

12. $y^2 = -12x$

Vertex: $(0, 0)$

$p = -3 < 0$

The graph opens to the left because p is negative. So, the equation matches graph (d).

13. Vertex: $(0, 0) \Rightarrow h = 0, k = 0$

Graph opens upward.

$x^2 = 4py$

Focus: $(0, 1)$

$p = 1$

$x^2 = 4(1)y$

$x^2 = 4y$

14. Vertex: $(0, 0) \Rightarrow h = 0, k = 0$

Focus: $\left(-\frac{9}{2}, 0\right) \Rightarrow p = -\frac{9}{2}$

$y^2 = 4px$

$y^2 = 4\left(-\frac{9}{2}\right)x$

$y^2 = -18x$

15. Vertex: $(0, 0) \Rightarrow h = 0, k = 0$

Focus: $\left(0, \frac{1}{2}\right) \Rightarrow p = \frac{1}{2}$

$x^2 = 4py$

$x^2 = 4\left(\frac{1}{2}\right)y$

$x^2 = 2y$

16. Vertex: $(0, 0) \Rightarrow h = 0, k = 0$

Focus: $\left(\frac{3}{2}, 0\right) \Rightarrow p = \frac{3}{2}$

$$y^2 = 4px$$
$$y^2 = 4\left(\frac{3}{2}\right)x$$
$$y^2 = 6x$$

17. Focus: $(-2, 0) \Rightarrow p = -2$

$$y^2 = 4px$$
$$y^2 = 4(-2)x$$
$$y^2 = -8x$$

18. Vertex: $(0, 0) \Rightarrow h = 0, k = 0$

Focus: $(0, -1) \Rightarrow p = -1$

$$x^2 = 4px$$
$$x^2 = 4(-1)y$$
$$x^2 = -4y$$

19. Vertex: $(0, 0) \Rightarrow h = 0, k = 0$

Directrix: $y = 2 \Rightarrow p = -2$

$$x^2 = 4py$$
$$x^2 = 4(-2)y$$
$$x^2 = -8y$$

20. Vertex: $(0, 0) \Rightarrow h = 0, k = 0$

Directrix: $y = -4 \Rightarrow p = 4$

$$x^2 = 4py$$
$$x^2 = 4(4)y$$
$$x^2 = 16y$$

21. Vertex: $(0, 0) \Rightarrow h = 0, k = 0$

Directrix: $x = -1 \Rightarrow p = 1$

$$y^2 = 4px$$
$$y^2 = 4(1)x$$
$$y^2 = 4x$$

22. Vertex: $(0, 0) \Rightarrow h = 0, k = 0$

Directrix: $x = 3 \Rightarrow p = -3$

$$y^2 = 4px$$
$$y^2 = 4(-3)x$$
$$y^2 = -12x$$

23. Vertex: $(0, 0) \Rightarrow h = 0, k = 0$

Vertical axis

Passes through: $(4, 6)$

$$x^2 = 4py$$
$$4^2 = 4p(6)$$
$$16 = 24p$$
$$p = \tfrac{2}{3}$$
$$x^2 = 4\left(\tfrac{2}{3}\right)y$$
$$x^2 = \tfrac{8}{3}y$$

24. Vertical axis

Passes through: $(-3, -3)$

$$x^2 = 4py$$
$$(-3)^2 = 4p(-3)$$
$$9 = -12p$$
$$p = -\tfrac{3}{4}$$
$$x^2 = -3y$$

25. Vertex: $(0, 0) \Rightarrow h = 0, k = 0$

Horizontal axis

Passes through: $(-2, 5)$

$$y^2 = 4px$$
$$5^2 = 4p(-2)$$
$$25 = -8p$$
$$p = -\tfrac{25}{8}$$
$$y^2 = 4\left(-\tfrac{25}{8}\right)x$$
$$y^2 = -\tfrac{25}{2}x$$

26. Vertex: $(0, 0) \Rightarrow h = 0, k = 0$

Horizontal axis

Passes through: $(3, -2)$

$$y^2 = 4px$$
$$(-2)^2 = 4p(3)$$
$$4 = 12p$$
$$p = \tfrac{1}{3}$$
$$y^2 = 4\left(\tfrac{1}{3}\right)x$$
$$y^2 = \tfrac{4}{3}x$$

27. Vertex: $(2, 6) \Rightarrow h = 2, k = 6$

Focus: $(2, 4) \Rightarrow p = -2$

$$(x - h)^2 = 4p(y - k)$$
$$(x - 2)^2 = 4(-2)(y - 6)$$
$$(x - 2)^2 = -8(y - 6)$$

28. Vertex: $(-4, 0) \Rightarrow h = -4, k = 0$

Focus: $(-3, 0) \Rightarrow p = 1$

$$(y - k)^2 = 4p(x - h)$$
$$\left[y - (0)\right]^2 = 4(1)\left[x - (-4)\right]$$
$$y^2 = 4(x + 4)$$

29. Vertex: $(6, 3) \Rightarrow h = 6, k = 3$

Focus: $(4, 3) \Rightarrow p = -2$

$$(y - k)^2 = 4p(x - h)$$
$$(y - 3)^2 = 4(-2)(x - 6)$$
$$(y - 3)^2 = -8(x - 6)$$

30. Vertex: $(1, -8) \Rightarrow h = 1, k = -8$

Focus: $(3, -8) \Rightarrow p = 2$

$$(y - k)^2 = 4p(x - h)$$
$$\left[y - (-8)\right]^2 = 4(2)(x - 1)$$
$$(y + 8)^2 = 8(x - 1)$$

31. Vertex: $(0, 2)$

Directrix: $y = 4$

Vertical axis

$$p = 2 - 4 = -2$$
$$(x - 0)^2 = 4(-2)(y - 2)$$
$$x^2 = -8(y - 2)$$

32. Vertex: $(1, 2) \Rightarrow h = 1, k = 2$

Directrix: $y = -1$

Vertical axis

$$p = 2 - (-1) = 3$$
$$(x - 1)^2 = 4(3)(y - 2)$$
$$(x - 1)^2 = 12(y - 2)$$

33. Focus: $(2, 2)$

Directrix: $x = -2$

Horizontal axis

Vertex: $(0, 2)$

$$p = 2 - 0 = 2$$
$$(y - 2)^2 = 4(2)(x - 0)$$
$$(y - 2)^2 = 8x$$

34. Focus: $(0, 0)$

Directrix: $y = 8 \Rightarrow p = -4$

$\Rightarrow h = 0, k = 4$

$$(x - h)^2 = 4p(y - k)$$
$$x^2 = 4(-4)(y - 4)$$
$$x^2 = -16(y - 4)$$

35. Vertex: $(3, -3) \Rightarrow h = 3, k = -3$

Vertical Axis; Passes through $(0, 0)$

$$(x - h)^2 = 4p(y - k)$$
$$(x - 3)^2 = 4p(y + 3)$$
$$(0 - 3)^2 = 4p(0 + 3)$$
$$9 = 12p$$
$$p = \frac{3}{4}$$
$$(x - 3)^2 = 3(y + 3)$$

36. Vertex: $(-1, 6) \Rightarrow h = -1, k = 6$

Horizontal Axis; Passes through $(-9, 2)$

$$(y - 6)^2 = 4p(x + 1)$$
$$(2 - 6)^2 = 4p(-9 + 1)$$
$$16 = -32p$$
$$p = -\frac{1}{2}$$
$$(y - 6)^2 = 4\left(-\frac{1}{2}\right)(x + 1)$$
$$(y - 6)^2 = -2(x + 1)$$

37. $y = \frac{1}{2}x^2$

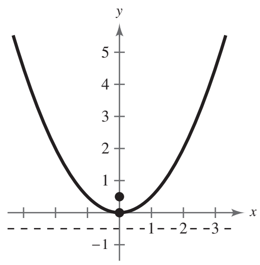
$x^2 = 2y$

$x^2 = 4\left(\frac{1}{2}\right)y \Rightarrow h = 0, k = 0, p = \frac{1}{2}$

Vertex: $(0, 0)$

Focus: $\left(0, \frac{1}{2}\right)$

Directrix: $y = -\frac{1}{2}$



38. $y = -4x^2$

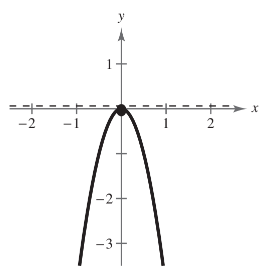
$x^2 = -\frac{1}{4}y$

$x^2 = 4\left(-\frac{1}{16}\right)y$

Vertex: $(0, 0)$

Focus: $\left(0, -\frac{1}{16}\right)$

Directrix: $y = \frac{1}{16}$



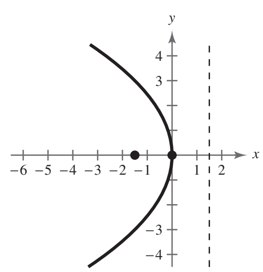
39. $y^2 = -6x$

$y^2 = 4\left(-\frac{3}{2}\right)x \Rightarrow h = 0, k = 0, p = -\frac{3}{2}$

Vertex: $(0, 0)$

Focus: $\left(-\frac{3}{2}, 0\right)$

Directrix: $x = \frac{3}{2}$

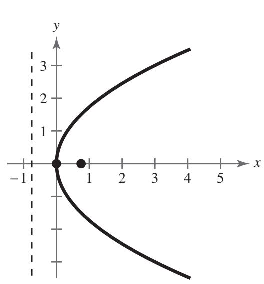


40. $y^2 = 3x \Rightarrow 4\left(\frac{3}{4}\right)x$

Vertex: $(0, 0)$

Focus: $\left(\frac{3}{4}, 0\right)$

Directrix: $x = -\frac{3}{4}$



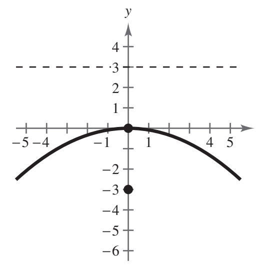
41. $x^2 + 12y = 0$

$x^2 = -12y = 4(-3)y \Rightarrow h = 0, k = 0, p = -3$

Vertex: $(0, 0)$

Focus: $(0, -3)$

Directrix: $y = 3$



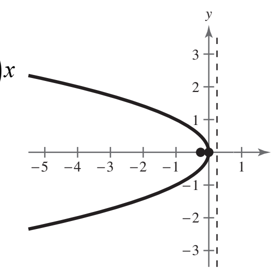
42. $x + y^2 = 0$

$y^2 = -x = 4\left(-\frac{1}{4}\right)x$

Vertex: $(0, 0)$

Focus: $\left(-\frac{1}{4}, 0\right)$

Directrix: $x = \frac{1}{4}$



43. $(x - 1)^2 + 8(y + 2) = 0$

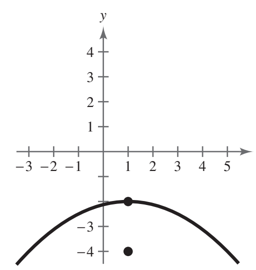
$(x - 1)^2 = 4(-2)(y + 2)$

$h = 1, k = -2, p = -2$

Vertex: $(1, -2)$

Focus: $(1, -4)$

Directrix: $y = 0$



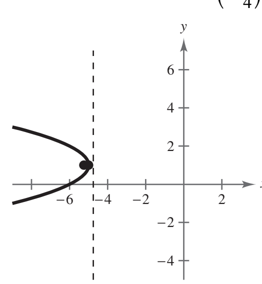
44. $(x + 5) + (y - 1)^2 = 0$

$(y - 1)^2 = 4\left(-\frac{1}{4}\right)(x + 5)$

Vertex: $(-5, 1)$

Focus: $\left(-5 + \left(-\frac{1}{4}\right), 1\right) \Rightarrow \left(-\frac{21}{4}, 1\right)$

Directrix: $x = -5 - \left(-\frac{1}{4}\right) = -\frac{19}{4}$



45. $(y+7)^2 = 4\left(x - \dfrac{3}{2}\right)$

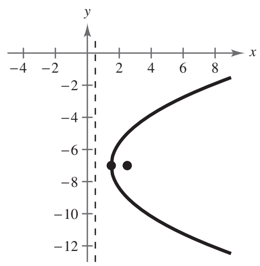
$(y+7)^2 = 4(1)\left(x - \dfrac{3}{2}\right)$

$h = \dfrac{3}{2}, k = -7, p = 1$

Vertex: $\left(\dfrac{3}{2}, -7\right)$

Focus: $\left(\dfrac{5}{2}, -7\right)$

Directrix: $x = \dfrac{1}{2}$

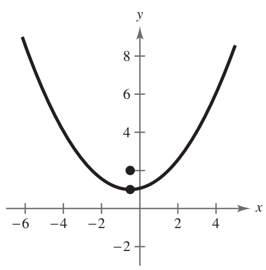


46. $\left(x + \frac{1}{2}\right)^2 = 4(y-1) = 4(1)(y-1)$

Vertex: $\left(-\frac{1}{2}, 1\right)$

Focus: $\left(-\frac{1}{2}, 1+1\right) \Rightarrow \left(-\frac{1}{2}, 2\right)$

Directrix: $y = 1 - 1 = 0$



47. $y = \frac{1}{4}(x^2 - 2x + 5)$

$4y = x^2 - 2x + 5$

$4y - 5 + 1 = x^2 - 2x + 1$

$4y - 4 = (x-1)^2$

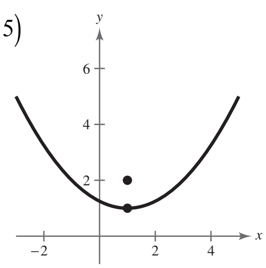
$(x-1)^2 = 4(1)(y-1)$

$h = 1, k = 1, p = 1$

Vertex: $(1, 1)$

Focus: $(1, 2)$

Directrix: $y = 0$



48. $x = \frac{1}{4}(y^2 + 2y + 33)$

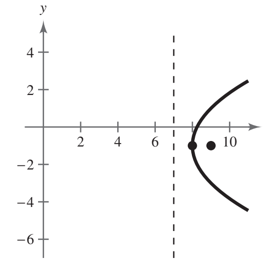
$4x = y^2 + 2y + 1 - 1 + 33 = (y+1)^2 + 32$

$(y+1)^2 = 4(1)(x-8)$

Vertex: $(8, -1)$

Focus: $(9, -1)$

Directrix: $x = 7$



49. $y^2 + 6y + 8x + 25 = 0$

$y^2 + 6y + 9 = -8x - 25 + 9$

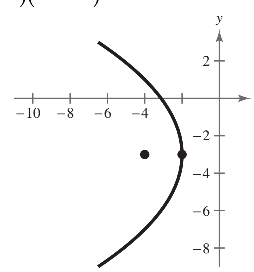
$(y+3)^2 = 4(-2)(x+2)$

$h = -2, k = -3, p = -2$

Vertex: $(-2, -3)$

Focus: $(-4, -3)$

Directrix: $x = 0$



50. $x^2 - 4x - 4y = 0$

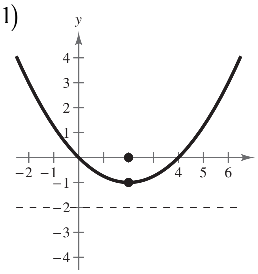
$x^2 - 4x + 4 = 4y + 4$

$(x-2)^2 = 4(1)(y+1)$

Vertex: $(2, -1)$

Focus: $(2, 0)$

Directrix: $y = -2$



51. $x^2 + 4x - 6y = -10$

$x^2 + 4x + 4 = 6y - 10 + 4$

$x^2 + 4x + 4 = 6y - 6$

$(x+2)^2 = 6(y-1)$

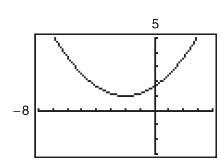
$(x+2)^2 = 4\left(\dfrac{3}{2}\right)(y-1)$

$h = -2, k = 1, p = \dfrac{3}{2}$

Vertex: $(-2, 1)$

Focus: $\left(-2, \dfrac{5}{2}\right)$

Directrix: $y = -\dfrac{1}{2}$



52. $x^2 - 2x + 8y + 9 = 0$

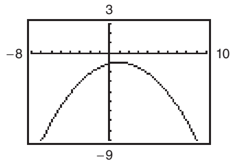
$$x^2 - 2x + 1 = -8y - 9 + 1$$

$$(x - 1)^2 = -8(y + 1) = 4(-2)(y + 1)$$

Vertex: $(1, -1)$

Focus: $(1, -3)$

Directrix: $y = 1$



53. $y^2 + x + y = 0$

$$y^2 + y + \frac{1}{4} = -x + \frac{1}{4}$$

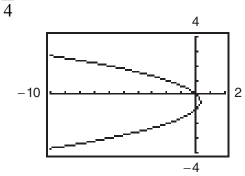
$$\left(y + \frac{1}{2}\right)^2 = 4\left(-\frac{1}{4}\right)\left(x - \frac{1}{4}\right)$$

$h = \frac{1}{4}, k = -\frac{1}{2}, p = -\frac{1}{4}$

Vertex: $\left(\frac{1}{4}, -\frac{1}{2}\right)$

Focus: $\left(0, -\frac{1}{2}\right)$

Directrix: $x = \frac{1}{2}$



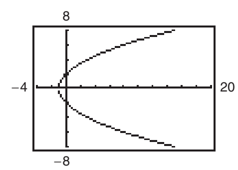
54. $y^2 - 4x - 4 = 0$

$$y^2 = 4x + 4 = 4(1)(x + 1)$$

Vertex: $(-1, 0)$

Focus: $(0, 0)$

Directrix: $x = -2$



55. $x^2 = 8y$

$x^2 = 4(2)y \Rightarrow p = 2$

Focus: $(0, 2)$

$$d_1 = 2 - b$$

$$d_2 = \sqrt{(6 - 0)^2 + \left(\frac{9}{2} - 2\right)^2} = \sqrt{36 + \frac{25}{4}}$$

$$2 - b = \frac{13}{2}$$

$$b = -\frac{9}{2}$$

$$m = \frac{-(9/2) - (9/2)}{0 - 6} = \frac{3}{2}$$

Tangent line: $y = \frac{3}{2}x - \frac{9}{2}$

56. $x^2 = -4y$

$x^2 = 4(-1)y \Rightarrow p = -1$

Focus: $(0, -1)$

$$d_1 = 1 + b$$

$$d^2 = \sqrt{(-6 - 0)^2 + (-9 - (-1))^2} = \sqrt{36 + 64} = 10$$

$$1 + b = 10$$

$$b = 9$$

$$m = \frac{9 - (-9)}{0 - (-6)} = 3$$

Tangent line: $y = 3x + 9$

57. $x^2 = 2y \Rightarrow p = \frac{1}{2}$

Point: $(4, 8)$

Focus: $\left(0, \frac{1}{2}\right)$

$$d_1 = \frac{1}{2} - b$$

$$d_2 = \sqrt{(4 - 0)^2 + \left(8 - \frac{1}{2}\right)^2}$$

$$= \frac{17}{2}$$

$$d_1 = d_2 \Rightarrow b = -8$$

$$m = \frac{8 - (-8)}{4 - 0} = 4$$

Tangent line: $y = 4x - 8$

58. $x^2 = 2y$

$x^2 = 4\left(\frac{1}{2}\right)y \Rightarrow p = \frac{1}{2}$

Focus: $\left(0, \frac{1}{2}\right)$

$d_1 = \frac{1}{2} - b$

$d_2 = \sqrt{(-3 - 0)^2 + \left(\frac{9}{2} - \frac{1}{2}\right)^2} = 5$

$\frac{1}{2} - b = 5$

$b = -\frac{9}{2}$

$m = \frac{-(9/2) - (9/2)}{0 + 3} = -3$

Tangent line: $y = -3x - \frac{9}{2}$

59. $y = -2x^2$

$x^2 = -\frac{1}{2}y \Rightarrow p = -\frac{1}{8}$

Point: $(-1, -2)$

Focus: $\left(0, -\frac{1}{8}\right)$

$d_1 = b - \left(-\frac{1}{8}\right) = b + \frac{1}{8}$

$d_2 = \sqrt{(-1 - 0)^2 + \left(-2 - \left(-\frac{1}{8}\right)\right)^2}$

$= \frac{17}{8}$

$d_1 = d_2 \Rightarrow b = 2$

$m = \frac{-2 - 2}{-1 - 0} = 4$

$y = 4x + 2$

60. $y = -2x^2$

$x^2 = -\frac{1}{2}y \Rightarrow p = -\frac{1}{8}$

Focus: $\left(0, -\frac{1}{8}\right)$

$d_1 = \frac{1}{8} + b$

$d^2 = \sqrt{(2 - 0)^2 + \left(-8 - \left(-\frac{1}{8}\right)\right)^2} = \frac{65}{8}$

$\frac{1}{8} + b = \frac{65}{8}$

$b = \frac{64}{8} = 8$

$m = \frac{-8 - 8}{2 - 0} = -8$

Tangent line: $y = -8x + 8$

61. $y^2 = 4px,\ p = 1.5$

$y^2 = 4(1.5)x$

$y^2 = 6x$

62. The receiver is located at the focus of the parabola.

$x^2 = 4py,\ p = 3.5$

$x^2 = 4(3.5)y$

$x^2 = 14y$

$y = \frac{1}{14}x^2$

63. Vertex: $(0, 0)$

$(y - 0)^2 = 4p(x - 0)$

$y^2 = 4px$

At $(1000, 800)$: $800^2 = 4p(1000) \Rightarrow p = 160$

$y^2 = 4(160)x$

$y^2 = 640x$

64. (a) Vertex: $(0, 0) \Rightarrow h = 0, k = 0$

Points on the parabola: $(\pm 16, -0.4)$

$$x^2 = 4py$$
$$(\pm 16)^2 = 4p(-0.4)$$
$$256 = -1.6p$$
$$-160 = p$$
$$x^2 = 4(-160y)$$
$$x^2 = -640y$$
$$y = -\tfrac{1}{640}x^2$$

(b) When $y = -0.1$ we have

$$-0.1 = -\tfrac{1}{640}x^2$$
$$64 = x^2$$
$$\pm 8 = x.$$

So, 8 feet away from the center of the road, the road surface is 0.1 foot lower than in the middle.

65. (a)
$$x^2 = 4py$$
$$32^2 = 4p\left(\frac{1}{12}\right)$$
$$1024 = \frac{1}{3}p$$
$$3072 = p$$
$$x^2 = 4(3072)y$$
$$x^2 = 12{,}288y \text{ (in feet)}$$

(b)
$$\frac{1}{24} = \frac{x^2}{12{,}288}$$
$$\frac{12{,}288}{24} = x^2$$
$$512 = x^2$$
$$x \approx 22.6 \text{ feet}$$

66. (a)
$$x^2 = 4py$$
$$18^2 = 4p\left(\frac{2}{12}\right)$$
$$324 = \frac{2}{3}p$$
$$486 = p$$
$$x^2 = 4(486)y$$
$$x^2 = 1944y \text{ (in feet)}$$

(b)
$$\frac{1}{24} = \frac{x^2}{1944}$$
$$81 = x^2$$
$$9 = x \Rightarrow 9 \text{ feet}$$

67. Vertex: $(0, 48) \Rightarrow h = 0, k = 48$

Passes through $\left(10\sqrt{3}, 0\right)$

Vertical axis

$$(x - 0)^2 = 4p(y - 48)$$
$$\left(10\sqrt{3} - 0\right)^2 = 4p(0 - 48)$$
$$300 = -192p$$
$$-\tfrac{25}{16} = p$$
$$x^2 = 4\left(-\tfrac{25}{16}\right)(y - 48)$$
$$x^2 = -\tfrac{25}{4}(y - 48)$$

68. Parabola: Vertex: $(0, 4)$

Passes through $(\pm 4, 0)$

$$x^2 = 4p(y - 4)$$
$$16 = 4p(0 - 4)$$
$$16 = -16p$$
$$-1 = p$$
$$x^2 = -4(y - 4)$$

69. $x^2 = 4p(y - 12)$

$(4, 10)$ on curve:

$$16 = 4p(10 - 12) = -8p \Rightarrow p = -2$$
$$x^2 = 4(-2)(y - 12) = -8y + 96$$
$$y = \frac{-x^2 + 96}{8}$$
$$y = 0 \text{ if } x^2 = 96 \Rightarrow x = 4\sqrt{6}$$

So, the width is about $2\left(4\sqrt{6}\right) \approx 19.6$ meters.

70. Vertex: $(0, 8) \Rightarrow h = 0, k = 8$

Points on the parabola: $(\pm 2, 4)$

Vertical axis

$$(x - 0)^2 = 4p(y - 8)$$
$$(2 - 0)^2 = 4p(4 - 8)$$
$$4 = 4p(-4)$$
$$4 = -16p$$
$$-\tfrac{1}{4} = p$$
$$x^2 = 4\left(-\tfrac{1}{4}\right)(y - 8)$$
$$x^2 = -(y - 8)$$

When $y = 0$, the lattice arch is at ground level.

$$x^2 = -(0 - 8)$$
$$x^2 = 8$$
$$x = \pm 2\sqrt{2}$$

So, the lattice arch is about $2\left(2\sqrt{2}\right) \approx 5.66$ feet wide at ground level.

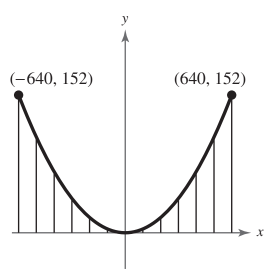
71. (a) $x^2 = 4py$

$60^2 = 4p(20) \Rightarrow p = 45$

Focus: $(0, 45)$

(b) $x^2 = 4(45)y$ or $y = \dfrac{1}{180}x^2$

72. (a)



(b) Vertex: $(0, 0) \Rightarrow h = 0, k = 0$

Points on the parabola: $(\pm 640, 152)$

Vertical axis

$$(x - 0)^2 = 4p(y - 0)$$
$$(640 - 0)^2 = 4p(152 - 0)$$
$$409{,}600 = 608p \Rightarrow p = \frac{12{,}800}{19}$$
$$x^2 = 4py$$
$$x^2 = 4\left(\frac{12{,}800}{19}\right)y$$
$$x^2 = \frac{51{,}200}{19}y \text{ or } y = \frac{19}{51{,}200}x^2$$

(c)

Distance, x	Height, y
0	0
100	3.71
250	23.19
400	59.38
500	92.77

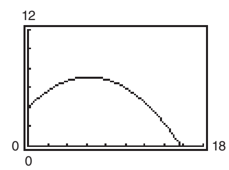
73. (a) $V = 17{,}500\sqrt{2}$ mi/h

$\approx 24{,}750$ mi/h

(b) $p = -4100, (h, k) = (0, 4100)$

$$(x - 0)^2 = 4(-4100)(y - 4100)$$
$$x^2 = -16{,}400(y - 4100)$$

74. (a)



(b) Highest point: $(6.25, 7.125)$

Range: About 15.69 feet

75. (a) $x^2 = -\dfrac{v^2}{16}(y - s)$

$$x^2 = -\frac{(28)^2}{16}(y - 100)$$
$$x^2 = -49(y - 100)$$

(b) The ball bits the ground when $y = 0$.

$$x^2 = -49(0 - 100)$$
$$x^2 = 4900$$
$$x = 70$$

The ball travels 70 feet.

76. $v = \dfrac{255 \text{ mi}}{1 \text{ hr}} \cdot \dfrac{5280 \text{ ft}}{1 \text{ mi}} \cdot \dfrac{1 \text{ hr}}{60 \text{ min}} \cdot \dfrac{1 \text{ min}}{60 \text{ sec}} = 374 \text{ ft/sec}$

$s = 500$ ft

The crate hits the ground when $y = 0$.

$$x^2 = \frac{-v^2}{16}(y - s)$$
$$x^2 = -\frac{(374)^2}{16}(0 - 500)$$
$$x^2 = 4{,}371{,}125$$
$$x \approx 2090.72$$

The distance is about 2090.7 feet.

77. False. It is not possible for a parabola to intersect its directrix. If the graph crossed the directrix there would exist points closer to the directrix than the focus.

78. False. The tangent line at the vertex is parallel to the directrix.

79. True. If the axis (line connecting the vertex and focus) is horizontal, then the directrix must be vertical.

80. $y - y_1 = \frac{x_1}{2p}(x - x_1)$

Slope: $m = \frac{x_1}{2p}$

81. Both (a) and (b) are parabolas with vertical axes, while (c) is a parabola with a horizontal axis.

So, equations (a) and (b) are equivalent when $p = \frac{1}{4a}$.

(a) $y = a(x - h)^2 + k$

(b)
$$\begin{aligned}(x - h)^2 &= 4p(y - k)\\ (x - h)^2 &= 4py - 4pk\\ (1/4p)\left((x - h)^2 + 4pk\right) &= 4py(1/4p)\\ (1/4p)(x - h)^2 + k &= y = a(x - h)^2 + k\\ a &= \left(\frac{1}{4p}\right)\\ 4a &= (1/p)\\ p &= \frac{1}{4a}\end{aligned}$$

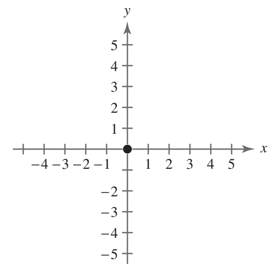
82. (a) A circle is formed when a plane intersects the top or bottom half of a double-napped cone and is perpendicular to the axis of the cone.

(b) An ellipse is formed when a plane intersects only the top or bottom half of a double-napped cone but is not parallel or perpendicular to the axis of the cone, is not parallel to the side of the cone, and does not intersect the vertex.

(c) A parabola is formed when a plane intersects the top or bottom half of a double-napped cone, is parallel to the side of the cone, and does not intersect the vertex.

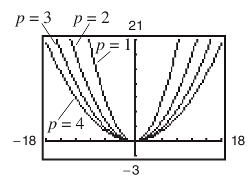
(d) A hyperbola is formed when a plane intersects both halves of a double-napped cone, is parallel to the axis of the cone, and does not intersect the vertex.

83. The graph of $x^2 + y^2 = 0$ is a single point, $(0, 0)$.



The plane intersects the double-napped cone at the vertices of the cones.

84. (a)



As p increases, the graph becomes wider.

(b) $(0, 1), (0, 2), (0, 3), (0, 4)$

(c) 4, 8, 12, 16. The latus rectum passing through the focus and parallel to the directrix has length $|4p|$.

(d) This provides an easy way to determine two additional points on the graph, each of which is $|2p|$ units away from the focus on the latus rectum.

85. (a)
$$\begin{aligned}A &= \frac{8}{3}(2)^{1/2}(4)^{3/2}\\ &= \frac{8}{3}\left(\sqrt{2}\right)(8)\\ &= \frac{64\sqrt{2}}{3} \text{ square units}\end{aligned}$$

(b) As p approaches zero, the parabola becomes narrower and narrower, so the area becomes smaller and smaller.

Section 10.3 Ellipses

1. ellipse; foci

2. major axis, center

3. minor axis

4. eccentricity

5. $\frac{x^2}{4} + \frac{y^2}{9} = 1$

   Center: $(0, 0)$

   $a = 3, b = 2$

   Vertical major axis
   Matches graph (b).

6. $\frac{x^2}{9} + \frac{y^2}{4} = 1$

   Center: $(0, 0)$

   $a = 3, b = 2$

   Horizontal major axis
   Matches graph (c).

7. $\frac{(x - 2)^2}{16} + (y + 1)^2 = 1$

   Center: $(2, -1)$

   $a = 4, b = 1$

   Horizontal major axis
   Matches graph (a).

8. $\frac{(x + 2)^2}{9} + \frac{(y + 2)^2}{4} = 1$

   Center: $(-2, -2)$

   $a = 3, b = 2$

   Horizontal major axis
   Matches graph (d).

9. Center: $(0, 0)$

   $a = 4, b = 2$

   Vertical major axis

   $$\frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1$$
   $$\frac{x^2}{4} + \frac{y^2}{16} = 1$$

10. Vertices: $(\pm 2, 0) \Rightarrow a = 2$

    Endpoints of minor axis:

    $\left(0, \pm\frac{3}{2}\right) \Rightarrow b = \frac{3}{2}$

    $$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
    $$\frac{x^2}{2^2} + \frac{y^2}{(3/2)^2} = 1$$
    $$\frac{x^2}{4} + \frac{4y^2}{9} = 1$$

11. Center: $(0, 0)$

    Vertices: $(\pm 7, 0) \Rightarrow a = 7$

    Foci: $(\pm 2, 0) \Rightarrow c = 2$

    $b^2 = a^2 - c^2 = 49 - 4 = 45$

    $$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
    $$\frac{x^2}{49} + \frac{y^2}{45} = 1$$

12. Vertices: $(0, \pm 8) \Rightarrow a = 8$

    Foci: $(0, \pm 4) \Rightarrow c = 4$

    $b^2 = a^2 - c^2 = 64 - 16 = 48$

    $$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$
    $$\frac{x^2}{48} + \frac{y^2}{64} = 1$$

13. Center: $(0, 0)$

    Foci: $(\pm 4, 0) \Rightarrow c = 4$

    Length of horizontal major axis: $10 \Rightarrow a = 5$

    $b^2 = a^2 - c^2 = 25 - 16 = 9$

    $$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
    $$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

14. Center: $(0, 0)$

Foci: $(0, \pm 3) \Rightarrow c = 3$

Length of vertical major axis: $8 \Rightarrow a = 4$

$b^2 = a^2 - c^2 = 16 - 9 = 7$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{7} + \frac{y^2}{16} = 1$$

15. Major axis vertical

Passes through: $(0, 6)$ and $(3, 0)$

$a = 6, b = 3$

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$$\frac{x^2}{9} + \frac{y^2}{36} = 1$$

16. Major axis horizontal

Passes through: $(5, 0)$ and $(0, 2)$

$a = 5, b = 2$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{25} + \frac{y^2}{4} = 1$$

17. Vertices: $(\pm 6, 0) \Rightarrow a = 6$

Major axis horizontal

Passes through: $(4, 1)$

$$\frac{x^2}{36} + \frac{y^2}{b^2} = 1$$

$$\frac{4^2}{36} + \frac{1^2}{b^2} = 1$$

$$16b^2 + 36 = 36b^2$$

$$36 = 20b^2$$

$$\frac{9}{5} = b^2$$

$$\frac{x^2}{36} + \frac{y^2}{\frac{9}{5}} = 1 \text{ or } \frac{x^2}{36} + \frac{5y^2}{9} = 1$$

18. Vertices: $(0, \pm 8) \Rightarrow a = 8$

Center: $(0, 0)$

Vertical major axis

$$\frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1$$

$$\frac{x^2}{b^2} + \frac{y^2}{64} = 1$$

Passes through: $(3, 4)$

$$\frac{3^2}{b^2} + \frac{4^2}{64} = 1$$

$$\frac{9}{b^2} + \frac{1}{4} = 1$$

$$\frac{9}{b^2} = 1 - \frac{1}{4}$$

$$\frac{9}{b^2} = \frac{3}{4}$$

$$36 = 3b^2$$

$$12 = b^2$$

$$\frac{x^2}{12} + \frac{y^2}{64} = 1$$

19. Center: $(2, 3)$

$a = 3, b = 1$

Vertical major axis

$$\frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1$$

$$\frac{(x - 2)^2}{1} + \frac{(y - 3)^2}{9} = 1$$

20. Vertices: $(0, -1), (4, -1) \Rightarrow a = 2$

Center: $(2, -1) \Rightarrow h = 2, k = -1$

Endpoints of minor axis: $(2, 0), (2, -2) \Rightarrow b = 1$

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

$$\frac{(x - 2)^2}{4} + \frac{(y + 1)^2}{1} = 1$$

21. Vertices: $(2, 0), (10, 0) \Rightarrow a = 4$

Horizontal major axis

Length of minor axis: $4 \Rightarrow b = 2$

Center: $(6, 0) \Rightarrow h = 6, k = 0$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$
$$\frac{(x-6)^2}{16} + \frac{(y-0)^2}{4} = 1$$
$$\frac{(x-6)^2}{16} + \frac{y^2}{4} = 1$$

22. Vertices: $(3, 1), (3, 11) \Rightarrow a = 5$

Vertical major axis

Center: $(3, 6) \Rightarrow h = 3, k = 6$

Length of minor axis: $2 \Rightarrow b = 1$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$
$$\frac{(x-3)^2}{1} + \frac{(y-6)^2}{25} = 1$$

23. Foci: $(0, 0), (4, 0) \Rightarrow c = 2$

Length of major axis: $6 \Rightarrow a = 3$

Center: $(2, 0) = (h, k)$

$b^2 = a^2 - c^2 = 9 - 4 = 5$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$
$$\frac{(x-2)^2}{9} + \frac{y^5}{5} = 1$$

24. Foci: $(0, 0), (0, 8) \Rightarrow c = 4$

Major axis of length: $16 \Rightarrow a = 8$

$b^2 = a^2 - c^2 = 64 - 16 = 48$

Center: $(0, 4) = (h, k)$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$
$$\frac{x^2}{48} + \frac{(y-4)^2}{64} = 1$$

25. Center: $(1, 3)$

Vertex: $(-2, 3) \Rightarrow a = 3$

Major axis horizontal

Length of minor axis: $4 \Rightarrow b = 2$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$
$$\frac{(x-1)^2}{9} + \frac{(y-3)^2}{4} = 1$$

26. Center: $(2, -1) \Rightarrow h = 2, k = -1$

Vertex: $\left(2, \frac{1}{2}\right) \Rightarrow a = \frac{3}{2}$

Minor axis length: $2 \Rightarrow b = 1$

$$\frac{(x-h)}{b^2} + \frac{(y-k)^2}{a^2} = 1$$
$$\frac{(x-2)^2}{1} + \frac{(y+1)^2}{\left(\frac{3}{2}\right)^2} = 1$$
$$(x-2)^2 + \frac{4(y+1)^2}{9} = 1$$

27. Center: $(1, 4)$

Vertices: $(1, 0)$ and $(1, 8) \Rightarrow a = 4$

Major axis vertical

$$a = 2c$$
$$4 = 2c$$
$$c = 2$$
$$b^2 = a^2 - c^2$$
$$b^2 = 16 - 4 = 12$$
$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$
$$\frac{(x-1)^2}{12} + \frac{(y-4)^2}{16} = 1$$

28. Center: $(3, 2) \Rightarrow h = 3, k = 2$

$a = 3c$

Foci: $(1, 2), (5, 2) \Rightarrow c = 2, a = 6$

$b^2 = a^2 - c^2 = 36 - 4 = 32$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$
$$\frac{(x-3)^2}{36} + \frac{(y-2)^2}{32} = 1$$

29. Vertices: $(0, 2), (4, 2) \Rightarrow a = 2$

Center: $(2, 2)$

Endpoints of the minor axis: $(2, 3), (2, 1) \Rightarrow b = 1$

Horizontal major axis:

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

$$\frac{(x - 2)^2}{4} + \frac{(y - 2)^2}{1} = 1$$

30. Vertices: $(5, 0), (5, 12) \Rightarrow a = 6$

Endpoints of the minor axis: $(1, 6), (9, 6) \Rightarrow b = 4$

Center: $(5, 6) \Rightarrow h = 5, k = 6$

$$\frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1$$

$$\frac{(x - 5)^2}{16} + \frac{(y - 6)^2}{36} = 1$$

31. $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$

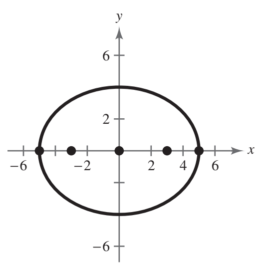
$a = 5, b = 4, c = 3$

Center: $(0, 0)$

Vertices: $(\pm 5, 0)$

Foci: $(\pm 3, 0)$

Eccentricity: $e = \dfrac{3}{5}$



32. $\dfrac{x^2}{16} + \dfrac{y^2}{81} = 1$

$a = 9, b = 4$

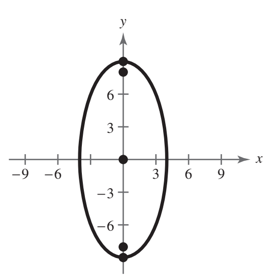
$e^2 = a^2 - b^2 = 81 - 16 = 65 \Rightarrow c = \sqrt{65}$

Center: $(0, 0)$

Vertices: $(0, \pm 9)$

Foci: $\left(0, \pm\sqrt{65}\right)$

Eccentricity: $e = \dfrac{\sqrt{65}}{9}$



33. $9x^2 + y^2 = 36$

$\dfrac{x^2}{4} + \dfrac{y^2}{36} = 1$

$a = 6, b = 2$

$c^2 = a^2 - b^2$

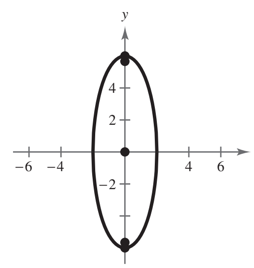
$= 36 - 4 = 32$

Center: $(0, 0)$

Vertices: $(0, \pm 6)$

Foci: $\left(0, \pm 4\sqrt{2}\right)$

Eccentricity: $e = \dfrac{4\sqrt{2}}{6} = \dfrac{2\sqrt{2}}{3}$



34. $x^2 + 16y^2 = 64$

$\dfrac{x^2}{64} + \dfrac{y^2}{4} = 1$

$a = 8, b = 2$

$c^2 = a^2 - b^2$

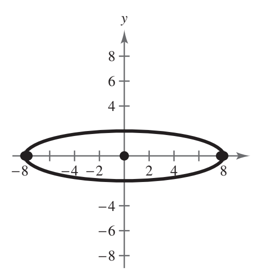
$= 64 - 4 = 60$

Center: $(0, 0)$

Vertices: $(\pm 8, 0)$

Foci: $\left(\pm 2\sqrt{15}, 0\right)$

Eccentricity: $e = \dfrac{2\sqrt{15}}{8} = \dfrac{\sqrt{15}}{4}$



35. $\dfrac{(x-4)^2}{16} + \dfrac{(y+1)^2}{25} = 1$

$a = 5, b = 4$

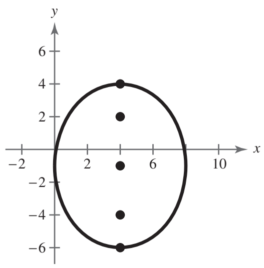
$c^2 = a^2 - b^2 = 25 - 16 = 9 \Rightarrow c = 3$

Center: $(4, -1)$

Vertices: $(4, 4), (4, -6)$

Foci: $(4, 2), (4, -4)$

Eccentricity: $e = \dfrac{3}{5}$



36. $\dfrac{(x+3)^2}{12} + \dfrac{(y-2)^2}{16} = 1$

$a = 4, b = 2\sqrt{3}$

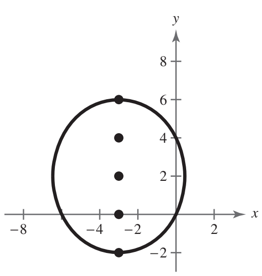
$c^2 = a^2 - b^2 = 16 - 12 = 4 \Rightarrow c = 2$

Center: $(-3, 2)$

Vertices: $(-3, 6), (-3, -2)$

Foci: $(-3, 4), (-3, 0)$

Eccentricity: $e = \dfrac{1}{2}$



37. $\dfrac{(x+5)^2}{9/4} + (y-1)^2 = 1$

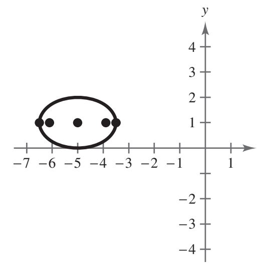
$a = \dfrac{3}{2}, b = 1, c = \dfrac{\sqrt{5}}{2}$

Center: $(-5, 1)$

Vertices: $\left(-\dfrac{7}{2}, 1\right), \left(-\dfrac{13}{2}, 1\right)$

Foci: $\left(-5 + \dfrac{\sqrt{5}}{2}, 1\right), \left(-5 - \dfrac{\sqrt{5}}{2}, 1\right)$

Eccentricity: $e = \dfrac{\sqrt{5}}{3}$



38. $\dfrac{(x+2)^2}{1} + \dfrac{(y+4)^2}{1/4} = 1$

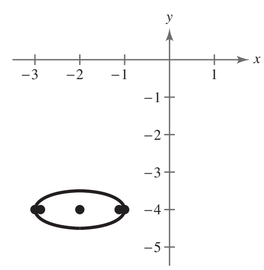
$a = 1, b = \dfrac{1}{2}, c = \dfrac{\sqrt{3}}{2}$

Center: $(-2, -4)$

Vertices: $(-1, -4), (-3, -4)$

Foci: $\left(-2 \pm \dfrac{\sqrt{3}}{2}, -4\right) = \left(\dfrac{-4 \pm \sqrt{3}}{2}, -4\right)$

Eccentricity: $e = \dfrac{\sqrt{3}}{2}$



39.

$$\begin{aligned} 9x^2 + 4y^2 + 36x - 24y + 36 &= 0 \\ 9(x^2 + 4x + 4) + 4(y^2 - 6y + 9) &= -36 + 36 + 36 \\ 9(x+2)^2 + 4(y-3)^2 &= 36 \\ \frac{(x+2)^2}{4} + \frac{(y-3)^2}{9} &= 1 \end{aligned}$$

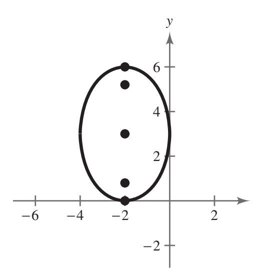
$a = 3, b = 2, c = \sqrt{5}$

Center: $(-2, 3)$

Vertices: $(-2, 6), (-2, 0)$

Foci: $\left(-2, 3 \pm \sqrt{5}\right)$

Eccentricity: $e = \dfrac{\sqrt{5}}{3}$



40. $$9x^2 + 4y^2 - 54x + 40y + 37 = 0$$
$$9(x^2 - 6x + 9) + 4(y^2 + 10y + 25) = -37 + 81 + 100$$
$$\frac{(x-3)^2}{16} + \frac{(y+5)^2}{36} = 1$$

$a = 6, b = 4,$

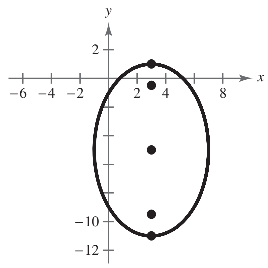
$c = \sqrt{20} = 2\sqrt{5}$

Center: $(3, -5)$

Vertices: $(3, 1), (3, -11)$

Foci: $\left(3, -5 \pm 2\sqrt{5}\right)$

Eccentricity: $e = \frac{\sqrt{5}}{3}$



41. $$x^2 + 5y^2 - 8x - 30y - 39 = 0$$
$$(x^2 - 8x + 16) + 5(y^2 - 6y + 9) = 39 + 16 + 45$$
$$(x-4)^2 + 5(y-3)^2 = 100$$
$$\frac{(x-4)^2}{100} + \frac{(y-3)^2}{20} = 1$$

$a = 10, b = \sqrt{20} = 2\sqrt{5},$

$c = \sqrt{80} = 4\sqrt{5}$

Center: $(4, 3)$

Foci: $\left(4 \pm 4\sqrt{5}, 3\right)$

Vertices: $(14, 3), (-6, 3)$

Eccentricity: $e = \frac{4\sqrt{5}}{10} = \frac{2\sqrt{5}}{5}$

42. $$3x^2 + y^2 + 18x - 2y - 8 = 0$$
$$3(x^2 + 6x + 9) + (y^2 - 2y + 1) = 8 + 27 + 1$$
$$3(x+3)^2 + (y-1)^2 = 36$$
$$\frac{(x+3)^2}{12} + \frac{(y-1)^2}{36} = 1$$

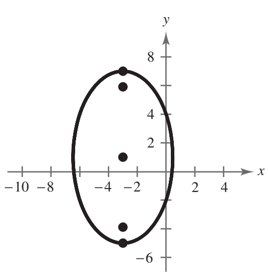
$a = 6, b = \sqrt{12} = 2\sqrt{3}, c = \sqrt{24} = 2\sqrt{6}$

Center: $(-3, 1)$

Vertices: $(-3, 7), (-3, -5)$

Foci: $\left(-3, 1 \pm 2\sqrt{6}\right)$

Eccentricity: $e = \frac{\sqrt{6}}{3}$



43. $$6x^2 + 2y^2 + 18x - 10y + 2 = 0$$
$$6\left(x^2 + 3x + \frac{9}{4}\right) + 2\left(y^2 - 5y + \frac{25}{4}\right) = -2 + \frac{27}{2} + \frac{25}{2}$$
$$6\left(x + \frac{3}{2}\right)^2 + 2\left(y - \frac{5}{2}\right)^2 = 24$$
$$\frac{\left(x + \frac{3}{2}\right)^2}{4} + \frac{\left(y - \frac{5}{2}\right)^2}{12} = 1$$

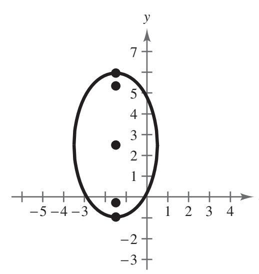
$a = \sqrt{12} = 2\sqrt{3}, b = 2, c = \sqrt{8} = 2\sqrt{2}$

Center: $\left(-\frac{3}{2}, \frac{5}{2}\right)$

Vertices: $\left(-\frac{3}{2}, \frac{5}{2} \pm 2\sqrt{3}\right)$

Foci: $\left(-\frac{3}{2}, \frac{5}{2} \pm 2\sqrt{2}\right)$

Eccentricity: $e = \frac{2\sqrt{2}}{2\sqrt{3}} = \frac{\sqrt{6}}{3}$



44. $$x^2 + 4y^2 - 6x + 20y - 2 = 0$$
$$(x^2 - 6x + 9) + 4\left(y^2 + 5y + \frac{25}{4}\right) = 2 + 9 + 25$$
$$(x-3)^2 + 4\left(y + \frac{5}{2}\right)^2 = 36$$
$$\frac{(x-3)^2}{36} + \frac{\left(y + \frac{5}{2}\right)^2}{9} = 1$$

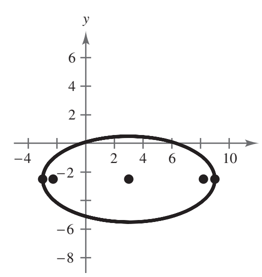
$a = 6 =, b = 3, c = \sqrt{27} = 3\sqrt{3}$

Center: $\left(3, -\frac{5}{2}\right)$

Vertices: $\left(9, -\frac{5}{2}\right), \left(-3, -\frac{5}{2}\right)$

Foci: $\left(3 \pm 3\sqrt{3}, -\frac{5}{2}\right)$

Eccentricity: $e = \frac{\sqrt{3}}{2}$



45. $12x^2 + 20y^2 - 12x + 40y - 37 = 0$

$$12\left(x^2 - x + \frac{1}{4}\right) + 20\left(y^2 + 2y + 1\right) = 37 + 3 + 20$$

$$12\left(x - \frac{1}{2}\right)^2 + 20(y + 1)^2 = 60$$

$$\frac{\left(x - \frac{1}{2}\right)^2}{5} + \frac{(y + 1)^2}{3} = 1$$

$a^2 = 5, b^2 = 3$

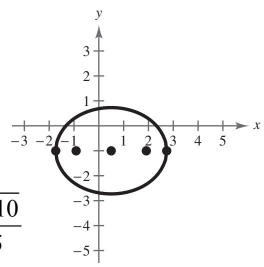
$c^2 = 5 - 3 = 2$

Center: $\left(\frac{1}{2}, -1\right)$

Vertices: $\left(\frac{1}{2} \pm \sqrt{5}, -1\right)$

Foci: $\left(\frac{1}{2} \pm \sqrt{2}, -1\right)$

Eccentricity: $e = \frac{\sqrt{2}}{\sqrt{5}} = \frac{\sqrt{10}}{5}$



46. $36x^2 + 9y^2 + 48x - 36y + 43 = 0$

$$36\left(x^2 + \frac{4}{3}x + \frac{4}{9}\right) + 9\left(y^2 - 4y + 4\right) = -43 + 16 + 36$$

$$36\left(x + \frac{2}{3}\right)^2 + 9(y - 2)^2 = 9$$

$$\frac{\left(x + \frac{2}{3}\right)^2}{\frac{1}{4}} + \frac{(y - 2)^2}{1} = 1$$

$a = 1, b = \frac{1}{2}$

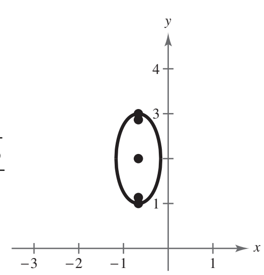
$c^2 = 1 - \frac{1}{4} = \frac{3}{4}$

Center: $\left(-\frac{2}{3}, 2\right)$

Vertices: $\left(-\frac{2}{3}, 3\right), \left(-\frac{2}{3}, 1\right)$

Foci: $\left(-\frac{2}{3}, 2 \pm \frac{\sqrt{3}}{2}\right)$

Eccentricity: $e = \frac{\frac{\sqrt{3}}{2}}{1} = \frac{\sqrt{3}}{2}$



47. $5x^2 + 3y^2 = 15$

$$\frac{x^2}{3} + \frac{y^2}{5} = 1$$

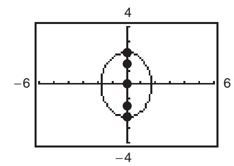
$a = \sqrt{5}, b = \sqrt{3}, c = \sqrt{2}$

Center: $(0, 0)$

Vertices: $\left(0, \pm\sqrt{5}\right)$

Foci: $\left(0, \pm\sqrt{2}\right)$

Eccentricity: $e = \frac{\sqrt{10}}{5}$



48. $3x^2 + 4y^2 = 12$

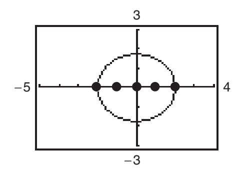
$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

$a^2 = 4, b^2 = 3, c^2 = 1$

Center: $(0, 0)$

Vertices: $(\pm 2, 0)$

Foci: $(\pm 1, 0)$



49. $x^2 + 9y^2 - 10x + 36y + 52 = 0$

$$\left(x^2 - 10x + 25\right) + 9\left(y^2 + 4y + 4\right) = -52 + 25 + 36$$

$$(x - 5)^2 + 9(y + 2)^2 = 9$$

$$\frac{(x - 5)^2}{9} + \frac{(y + 2)}{1} = 1$$

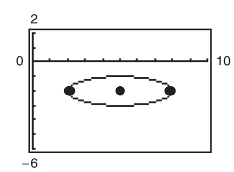
$a = 3, b = 1, c = 2\sqrt{2}$

Center: $(5, -2)$

Vertices: $(8, -2), (2, -2)$

Foci: $\left(5 \pm 2\sqrt{2}, -2\right)$

Eccentricity: $e = \frac{2\sqrt{2}}{3}$



50. $4x^2 + 3y^2 - 8x + 18y + 19 = 0$

$$4\left(x^2 - 2x + 1\right) + 3\left(y^2 + 6y + 9\right) = -19 + 4 + 27$$

$$4(x - 1)^2 + 3(y + 3)^2 = 12$$

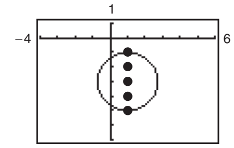
$$\frac{(x - 1)^2}{3} + \frac{(y + 3)^2}{4} = 1$$

$a^2 = 4, b^2 = 3, c^2 = 1$

Center: $(1, -3)$

Vertices: $(1, -1), (1, -5)$

Foci: $(1, -4), (1, -2)$



51. Vertices: $(\pm 5, 0) \Rightarrow a = 5$

$$e = \frac{3}{5} \Rightarrow c = \frac{3}{5}a = 3$$

$$b^2 = a^2 - c^2 = 25 - 9 = 16$$

Center: $(0, 0) = (h, k)$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

52. Vertices: $(0, \pm 8) \Rightarrow a = 8$

$$e = \frac{1}{2} \Rightarrow \frac{c}{a} = \frac{1}{2}, c = 4$$

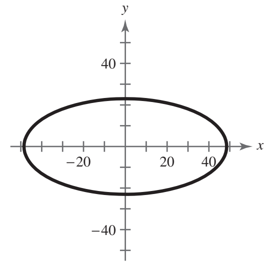
$$b^2 = a^2 - c^2 = 64 - 16 = 48$$

Center: $(0, 0)$

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$$\frac{x^2}{48} + \frac{y^2}{64} = 1$$

53. (a)



$$\frac{x^2}{2352.25} + \frac{y^2}{23^2} = 1 \text{ or } \frac{x^2}{529} + \frac{y^2}{2352.25} = 1$$

$$a = \frac{97}{2}, b = 23, c = \sqrt{\left(\frac{97}{2}\right)^2 - (23)^2} \approx 4.7$$

(b) Distance between foci: $2(4.7) \approx 85.4$ feet

54. The tacks should be placed at the foci and the length of the string is the length of the major axis, $2a$.

Center: $(0, 0)$

$a = 3, b = 2, c = \sqrt{5}$

Foci (Positions of the tacks): $(\pm\sqrt{5}, 0)$

Length of string: 6 feet

55. The length of the major axis and minor axis are 280 millimeters and 160 millimeters, respectively. Therefore,

$$2a = 280 \Rightarrow a = 140 \text{ and } 2b = 160 \Rightarrow b = 80.$$

$$a^2 = b^2 + c^2$$

$$140^2 = 80^2 + c^2$$

$$13{,}200 = c^2$$

$$\sqrt{13{,}200} = c$$

$$20\sqrt{33} = c$$

The kidney stone and spark plug are each located at a focus, therefore they are 2c millimeters apart, or $2(20\sqrt{33}) = 40\sqrt{33} \approx 229.8$ millimeters apart.

56. (a) $a = \dfrac{35.88}{2} = 17.94$

$$e = \frac{c}{a} = 0.967$$

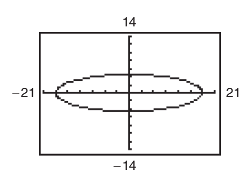
$$c = ea \approx 17.35$$

$$b^2 = a^2 - c^2 \approx 20.89$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{321.84} + \frac{y^2}{20.89} = 1$$

(b)



(c) The sun's center is at a focus of the orbit, 17.35 astronomical units from the center of the orbit.

Aphelion $\approx 17.35 + \frac{1}{2}(35.88) = 35.29$ astronomical units

Perihelion $\approx \frac{1}{2}(35.88) - 17.35 = 0.59$ astronomical unit

57. $a + c = 6378 + 939 = 7317$

$a - c = 6378 + 215 = 6593$

Solving this system for a and c yields

$$a + c = 7317$$
$$a - c = 6593$$
$$2a = 13{,}910$$
$$a = 6955$$
$$6955 + c = 7317$$
$$c = 362$$

Eccentricity: $e = \frac{c}{a} = \frac{362}{6955} \approx 0.0520$

58. For $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, you have $c^2 = a^2 - b^2$.

When $x = c$: $\frac{c^2}{a^2} + \frac{y^2}{b^2} = 1$

$$y^2 = b^2\left(1 - \frac{a^2 - b^2}{a^2}\right) = \frac{b^4}{a^2}$$

$$y = \frac{b^2}{a}$$

Length of latus rectum: $2y = \frac{2b^2}{a}$

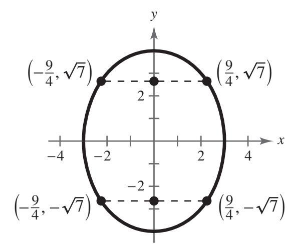
59. $\frac{x^2}{9} + \frac{y^2}{16} = 1$

$a = 4, b = 3, c = \sqrt{7}$

Points on the ellipse: $(\pm 3, 0), (0, \pm 4)$

Length of latus recta: $\frac{2b^2}{a} = \frac{2(3)^2}{4} = \frac{9}{2}$

Additional points: $\left(\pm\frac{9}{4}, -\sqrt{7}\right), \left(\pm\frac{9}{4}, \sqrt{7}\right)$



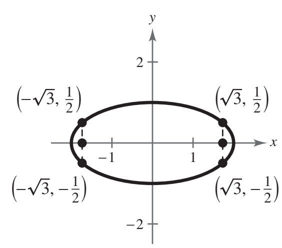
60. $\frac{x^2}{4} + \frac{y^2}{1} = 1$

$a = 2, b = 1, c = \sqrt{3}$

Points on the ellipse: $(\pm 2, 0), (0, \pm 1)$

Length of latera recta: $\frac{2b^2}{a} = \frac{2(1)^2}{2} = 1$

Additional points: $\left(-\sqrt{3}, \pm\frac{1}{2}\right), \left(\sqrt{3}, \pm\frac{1}{2}\right)$



61. $5x^2 + 3y^2 = 15$

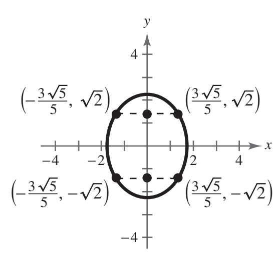
$$\frac{x^2}{3} + \frac{y^2}{5} = 1$$

$a = \sqrt{5}, b = \sqrt{3}, c = \sqrt{2}$

Points on the ellipse: $\left(\pm\sqrt{3}, 0\right), \left(0, \pm\sqrt{5}\right)$

Length of latus recta: $\frac{2b^2}{a} = \frac{2 \cdot 3}{\sqrt{5}} = \frac{6\sqrt{5}}{5}$

Additional points: $\left(\pm\frac{3\sqrt{5}}{5}, \pm\sqrt{2}\right)$



62. $9x^2 + 4y^2 = 36$

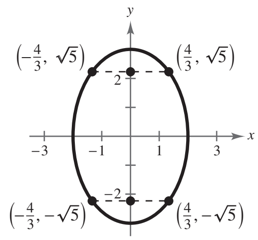
$\frac{x^2}{4} + \frac{y^2}{9} = 1$

$a = 3, b = 2, c = \sqrt{5}$

Points on the ellipse: $(\pm 2, 0), (0, \pm 3)$

Length of latera recta: $\frac{2b^2}{a} = \frac{2 \cdot 2^2}{3} = \frac{8}{3}$

Additional points: $\left(\pm\frac{4}{3}, -\sqrt{5}\right), \left(\pm\frac{4}{3}, \sqrt{5}\right)$



63. False. The graph of $\frac{x^2}{4} + y^4 = 1$ is not an ellipse.

The degree of y is 4, not 2.

64. True. When e is close to 1, the ellipse is elongated and the foci are close to the vertices.

65. *Sample answer:* Foci: $(2, 2), (10, 2) \Rightarrow c = 4$

Center: $(6, 2)$

Let $a^2 = 324$ and $b^2 = 308$

So that $c^2 = a^2 - b^2$.

$\frac{(x - 6)^2}{324} + \frac{(y - 2)^2}{308} = 1$

66. (a) Length of string $= 2a$

(b) By keeping the string taut, the sum of the distances from the two fixed points is constant (equal to the length of the string).

67. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(a) $a + b = 20 \Rightarrow b = 20 - a$

$A = \pi ab = \pi a(20 - a)$

(b) $264 = \pi a(20 - a)$

$0 = -\pi a^2 + 20\pi a - 264$

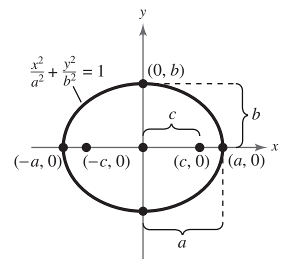
$0 = \pi a^2 - 20\pi a + 264$

By the Quadratic Formula: $a \approx 14$ or $a \approx 6$. Choosing the larger value of a, you have $a \approx 14$ and $b \approx 6$.

The equation of an ellipse with an area of 264 is $\frac{x^2}{196} + \frac{y^2}{36} = 1$.

68. Because the eccentricity, $e = \frac{c}{a}$, describes the ovalness of an ellipse and the closer to 0 the more circular the ellipse, $e_B < e_A < e_C$.

69.



The length of half the major axis is a and the length of half the minor axis is b.

Find the distance between $(0, b)$ and $(c, 0)$ and $(0, b)$ and $(-c, 0)$.

$d_1 = \sqrt{(0 - c)^2 + (b - 0)^2} = \sqrt{c^2 + b^2}$

$d_2 = \sqrt{(0 - (-c))^2 + (b - 0)^2} = \sqrt{c^2 + b^2}$

The sum of the distances from any point on the ellipse to the two foci is constant. Using the vertex $(a, 0)$, the constant sum is $(a + c) + (a - c) = 2a$.

So, the sum of the distances from $(0, b)$ to the two foci is

$\sqrt{c^2 + b^2} + \sqrt{c^2 + b^2} = 2a$

$2\sqrt{c^2 + b^2} = 2a$

$\sqrt{c^2 + b^2} = a$

$c^2 + b^2 = a^2$

So, $a^2 = b^2 + c^2$ for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > 0, b > 0$.

Section 10.4 Hyperbolas

1. hyperbola; foci

2. branches

3. transverse axis; center

4. asymptotes

5. $\dfrac{y^2}{9} - \dfrac{x^2}{25} = 1$

Center: $(0, 0)$

$a = 3, b = 5$

Vertical transverse axis
Matches graph (b).

6. $\dfrac{x^2}{9} - \dfrac{y^2}{25} = 1$

Center: $(0, 0)$

$a = 3, b = 5$

Horizontal transverse axis
Matches graph (d).

7. $\dfrac{x^2}{25} - \dfrac{(y + 2)^2}{9} = 1$

Center: $(0, -2)$

$a = 5, b = 3$

Horizontal transverse axis
Matches graph (c).

8. $\dfrac{(y + 4)^2}{25} - \dfrac{(x - 2)^2}{9} = 1$

Center: $(2, -4)$

$a = 5, b = 3$

Vertical transverse axis
Matches graph (a).

9. Vertices: $(0, \pm 2) \Rightarrow a = 2$

Foci: $(0, \pm 4) \Rightarrow c = 4$

$b^2 = c^2 - a^2 = 16 - 4 = 12$

Center: $(0, 0) = (h, k)$

$$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$$
$$\frac{y^2}{4} - \frac{x^2}{12} = 1$$

10. Vertices: $(\pm 4, 0) \Rightarrow a = 4$

Foci: $(\pm 6, 0) \Rightarrow c = 6$

$b^2 = c^2 - a^2 = 36 - 16 = 20 \Rightarrow b = 2\sqrt{5}$

Center: $(0, 0) = (h, k)$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$
$$\frac{x^2}{16} - \frac{y^2}{20} = 1$$

11. Vertices: $(2, 0), (6, 0) \Rightarrow a = 2$

Foci: $(0, 0), (8, 0) \Rightarrow c = 4$

$b^2 = c^2 - a^2 = 16 - 4 = 12$

Center: $(4, 0) = (h, k)$

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$
$$\frac{(x - 4)^2}{4} - \frac{y^2}{12} = 1$$

12. Vertices: $(2, 3), (2, -3) \Rightarrow a = 3$

Center: $(2, 0) = (h, k)$

Foci: $(2, 6), (2, -6) \Rightarrow c = 6$

$b^2 = c^2 - a^2 = 36 - 9 = 27$

$$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$$
$$\frac{y^2}{9} - \frac{(x - 2)^2}{27} = 1$$

13. Vertices: $(4, 1), (4, 9) \Rightarrow a = 4$

Foci: $(4, 0), (4, 10) \Rightarrow c = 5$

$b^2 = c^2 - a^2 = 25 - 16 = 9$

Center: $(4, 5) = (h, k)$

$$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$$
$$\frac{(y - 5)^2}{16} - \frac{(x - 4)^2}{9} = 1$$

14. Vertices: $(-1, 1), (3, 1) \Rightarrow a = 2$

Center: $(1, 1) = (h, k)$

Foci: $(-2, 1), (4, 1) \Rightarrow c = 3$

$$b^2 = c^2 - a^2 = 9 - 4 = 5$$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-1)^2}{4} - \frac{(y-1)^2}{5} = 1$$

15. Vertices: $(2, 3), (2, -3) \Rightarrow a = 3$

Passes through the point: $(0, 5)$

Center: $(2, 0) = (h, k)$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$\frac{y^2}{9} - \frac{(x-2)^2}{b^2} = 1$$

$$\frac{(x-2)^2}{b^2} = \frac{y^2}{9} - 1 = \frac{y^2 - 9}{9}$$

$$b^2 = \frac{9(x-2)^2}{y^2 - 9} = \frac{9(-2)^2}{25 - 9}$$

$$= \frac{36}{16} = \frac{9}{4}$$

$$\frac{y^2}{9} - \frac{(x-2)^2}{9/4} = 1$$

$$\frac{y^2}{9} - \frac{4(x-2)^2}{9} = 1$$

16. Vertices: $(-2, 1), (2, 1) \Rightarrow a = 2$

Center: $(0, 1) = (h, k)$

Point on curve: $(5, 4)$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{x^2}{4} - \frac{(y-1)^2}{b^2} = 1$$

$$\frac{25}{4} - \frac{9}{b^2} = 1$$

$$b^2 = \frac{12}{7}$$

$$\frac{x^2}{4} - \frac{(y-1)^2}{12/7} = 1$$

$$\frac{x^2}{4} - \frac{7(y-1)^2}{12} = 1$$

17. Vertices: $(0, -3), (4, -3) \Rightarrow a = 2$

Center: $(2, -3)$

Passes through: $(-4, 5)$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-2)^2}{4} - \frac{(y+3)^2}{b^2} = 1$$

$$\frac{(-4-2)^2}{4} - \frac{(5+3)^2}{b^2} = 1$$

$$9 - \frac{64}{b^2} = 1 \Rightarrow b^2 = 8$$

$$\frac{(x-2)^2}{4} - \frac{(y+3)^2}{8} = 1$$

18. Vertices: $(1, -3), (1, -7) \Rightarrow a = 2$

Center: $(1, -5)$

Passes through: $(5, -11)$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$\frac{(y+5)^2}{4} - \frac{(x-1)^2}{b^2} = 1$$

$$\frac{(-11+5)^2}{4} - \frac{(5-1)^2}{b^2} = 1$$

$$9 - \frac{16}{b^2} = 1 \Rightarrow b^2 = 2$$

$$\frac{(y+5)^2}{4} - \frac{(x-1)^2}{2} = 1$$

19. $x^2 - y^2 = 1$

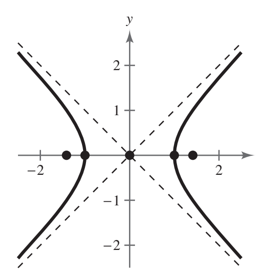
$a = 1, b = 1, c = \sqrt{2}$

Center: $(0, 0)$

Vertices: $(\pm 1, 0)$

Foci: $(\pm\sqrt{2}, 0)$

Asymptotes: $y = \pm x$



20. $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$

$a = 3, b = 4$

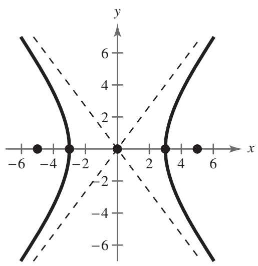
$c = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

Center: $(0, 0)$

Vertices: $(\pm 3, 0)$

Foci: $(\pm 5, 0)$

Asymptotes: $y = \pm\dfrac{4}{3}x$



21. $\dfrac{y^2}{36} - \dfrac{x^2}{100} = 1$

$a = 6, b = 10$

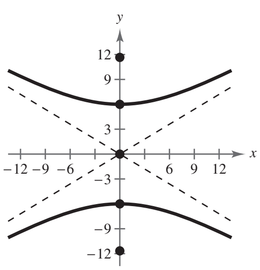
$c^2 = a^2 + b^2 = 136 \Rightarrow c = 2\sqrt{34}$

Center: $(0, 0)$

Vertices: $(0, \pm 6)$

Foci: $\left(0, \pm 2\sqrt{34}\right)$

Asymptotes: $y = \pm\dfrac{3}{5}x$



22. $\dfrac{x^2}{144} - \dfrac{y^2}{169} = 1$

$a = 12, b = 13$

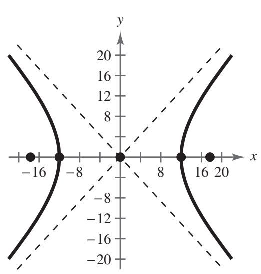
$c^2 = a^2 + b^2 = 313 \Rightarrow c = \sqrt{313}$

Center: $(0, 0)$

Vertices: $(\pm 12, 0)$

Foci: $\left(\pm\sqrt{313}, 0\right)$

Asymptotes: $y = \pm\dfrac{13}{12}x$



23. $2y^2 - \dfrac{x^2}{2} = 2$

$y^2 - \dfrac{x^2}{4} = 1$

$a = 1, b = 2,$

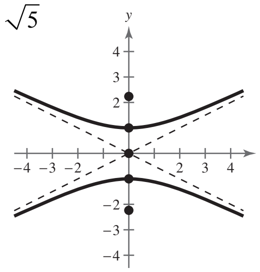
$c^2 = a^2 + b^2 = 5 \Rightarrow c = \sqrt{5}$

Center: $(0, 0)$

Vertices: $(0, \pm 1)$

Foci: $\left(0, \pm\sqrt{5}\right)$

Asymptotes: $y = \pm\dfrac{1}{2}x$



24. $\dfrac{y^2}{3} - 3x^2 = 3$

$\dfrac{y^2}{9} - \dfrac{x^2}{1} = 1$

$a = 3, b = 1$

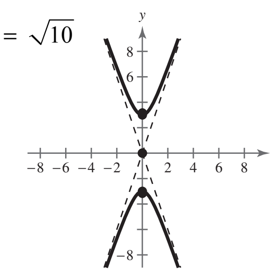
$c^2 = a^2 + b^2 = 10 \Rightarrow c = \sqrt{10}$

Center: $(0, 0)$

Vertices: $(0, \pm 3)$

Foci: $\left(0, \pm\sqrt{10}\right)$

Asymptotes: $y = \pm 3x$



25. $\dfrac{(x-1)^2}{4} - \dfrac{(y+2)^2}{1} = 1$

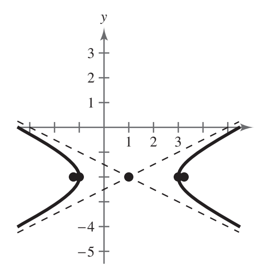
$a = 2, b = 1, c = \sqrt{5}$

Center: $(1, -2)$

Vertices: $(-1, -2), (3, -2)$

Foci: $\left(1 \pm \sqrt{5}, -2\right)$

Asymptotes: $y = -2 \pm \dfrac{1}{2}(x - 1)$



26. $\dfrac{(x+3)^2}{144} - \dfrac{(y-2)^2}{25} = 1$

$a = 12, b = 5,$

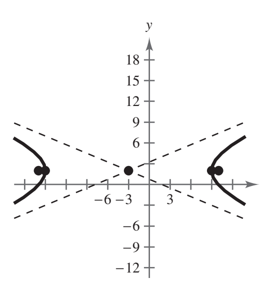
$c = \sqrt{144 + 25} = 13$

Center: $(-3, 2)$

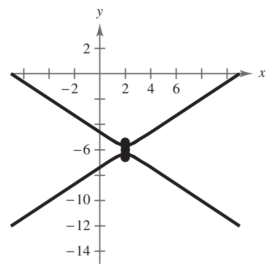
Vertices: $(9, 2), (-15, 2)$

Foci: $(10, 2), (-16, 2)$

Asymptotes: $y = 2 \pm \dfrac{5}{12}(x + 3)$



27. $\dfrac{(y+6)^2}{1/9} - \dfrac{(x-2)^2}{1/4} = 1$



$a = \dfrac{1}{3}, b = \dfrac{1}{2},$

$c = \dfrac{\sqrt{13}}{6}$

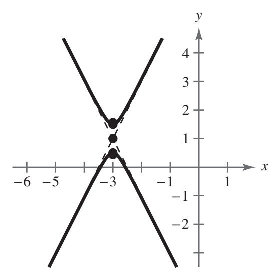
Center: $(2, -6)$

Vertices: $\left(2, -\dfrac{17}{3}\right), \left(2, -\dfrac{19}{3}\right)$

Foci: $\left(2, -6 \pm \dfrac{\sqrt{13}}{6}\right)$

Asymptotes: $y = -6 \pm \dfrac{2}{3}(x - 2)$

28. $\dfrac{(y-1)^2}{1/4} - \dfrac{(x+3)^2}{1/16} = 1$



$a = \dfrac{1}{2}, b = \dfrac{1}{4},$

$c = \sqrt{\dfrac{1}{4} + \dfrac{1}{16}} = \dfrac{\sqrt{5}}{4}$

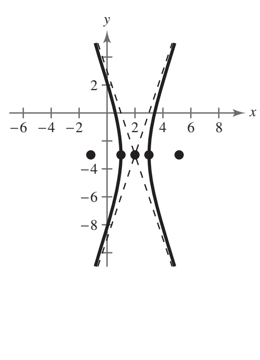
Center: $(-3, 1)$

Vertices: $\left(-3, \dfrac{3}{2}\right), \left(-3, \dfrac{1}{2}\right)$

Foci: $\left(-3, 1 \pm \dfrac{\sqrt{5}}{4}\right)$

Asymptotes: $y = 1 \pm 2(x + 3)$

29.
$$9x^2 - y^2 - 36x - 6y + 18 = 0$$
$$9(x^2 - 4x + 4) - (y^2 + 6y + 9) = -18 + 36 - 9$$
$$9(x - 2)^2 - (y + 3)^2 = 9$$
$$\frac{(x-2)^2}{1} - \frac{(y+3)^2}{9} = 1$$



$a = 1, b = 3, c = \sqrt{10}$

Center: $(2, -3)$

Vertices: $(1, -3), (3, -3)$

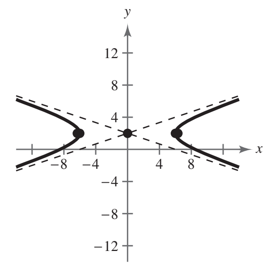
Foci: $(2 \pm \sqrt{10}, -3)$

Asymptotes:
$y = -3 \pm 3(x - 2)$

30.
$$x^2 - 9y^2 + 36y - 72 = 0$$
$$x^2 - 9(y^2 - 4y + 4) = 72 - 36$$
$$x^2 - 9(y - 2)^2 = 36$$
$$\frac{x^2}{36} - \frac{(y-2)^2}{4} = 1$$

$a = 6, b = 2,$

$c = \sqrt{36 + 4} = 2\sqrt{10}$



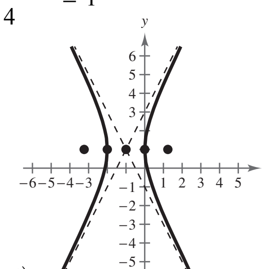
Center: $(0, 2)$

Vertices: $(\pm 6, 2)$

Foci: $(\pm 2\sqrt{10}, 2)$

Asymptotes: $y = 2 \pm \dfrac{1}{3}x$

31.
$$4x^2 - y^2 + 8x + 2y - 1 = 0$$
$$4(x^2 + 2x + 1) - (y^2 - 2y + 1) = 1 + 4 - 1$$
$$4(x + 1)^2 - (y - 1)^2 = 4$$
$$\frac{(x+1)^2}{1} - \frac{(y-1)^2}{4} = 1$$



$a = 1, b = 2, c = \sqrt{5}$

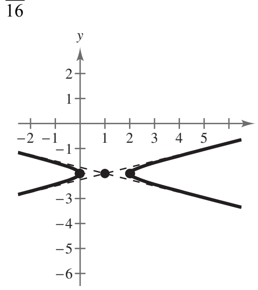
Center: $(-1, 1)$

Vertices: $(-2, 1), (0, 1)$

Foci: $(-1 \pm \sqrt{5}, 1)$

Asymptotes: $y = 1 \pm 2(x + 1)$

32.
$$16y^2 - x^2 + 2x + 64y + 64 = 0$$
$$16(y^2 + 4y + 4) - (x^2 - 2x + 1) = -64 + 64 - 1$$
$$16(y + 2)^2 - (x - 1)^2 = -1$$
$$\frac{(x-1)^2}{1} - \frac{(y+2)^2}{\frac{1}{16}} = 1$$



$a = 1, b = \dfrac{1}{4}, c = \dfrac{\sqrt{17}}{4}$

Center: $(1, -2)$

Vertices: $(2, -2), (0, -2)$

Foci: $\left(1 \pm \dfrac{\sqrt{17}}{4}, -2\right)$

Asymptotes: $y = -2 \pm \dfrac{1}{4}(x - 1)$

33. $2x^2 - 3y^2 = 6$

$$\frac{x^2}{3} - \frac{y^2}{2} = 1$$

$a = \sqrt{3}, b = \sqrt{2}, c = \sqrt{5}$

Center: $(0, 0)$

Vertices: $\left(\pm\sqrt{3}, 0\right)$

Foci: $\left(\pm\sqrt{5}, 0\right)$

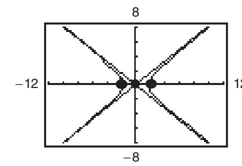
Asymptotes: $y = \pm\sqrt{\frac{2}{3}}x = \pm\frac{\sqrt{6}}{3}x$

To use a graphing utility, solve for y first.

$$y^2 = \frac{2x^2 - 6}{3}$$

$$\left.\begin{aligned} y_1 &= \sqrt{\frac{2x^2 - 6}{3}} \\ y_2 &= -\sqrt{\frac{2x^2 - 6}{3}} \end{aligned}\right\} \text{Hyperbola}$$

$$\left.\begin{aligned} y_3 &= \frac{\sqrt{6}}{3}x \\ y_4 &= -\frac{\sqrt{6}}{3}x \end{aligned}\right\} \text{Asymptotes}$$



34. $6y^2 - 3x^2 = 18$

$$\frac{y^2}{3} - \frac{x^2}{6} = 1$$

$a = \sqrt{3}, b = \sqrt{6}, c = \sqrt{3 + 6} = 3$

Center: $(0, 0)$

Vertices: $\left(0, \pm\sqrt{3}\right)$

Foci: $(0, \pm 3)$

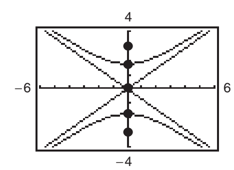
Asymptotes: $y = \pm\frac{\sqrt{2}}{2}x$

To use a graphing utility, solve for y first.

$$y^2 = \frac{x^2 + 6}{2}$$

$$\left.\begin{aligned} y_1 &= \sqrt{\frac{x^2 + 6}{2}} \\ y_2 &= -\sqrt{\frac{x^2 + 6}{2}} \end{aligned}\right\} \text{Hyperbola}$$

$$\left.\begin{aligned} y_3 &= \frac{\sqrt{2}}{2}x \\ y_4 &= -\frac{\sqrt{2}}{2}x \end{aligned}\right\} \text{Asymptotes}$$



35. $25y^2 - 9x^2 = 225$

$$\frac{y^2}{9} - \frac{x^2}{25} = 1$$

$a = 3, b = 5,$

$c^2 = a^2 + b^2 = 9 + 25 = 34 \Rightarrow c = \sqrt{34}$

Center: $(0, 0)$

Vertices: $(0, \pm 3)$

Foci: $\left(0, \pm\sqrt{34}\right)$

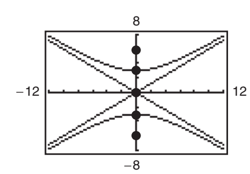
Asymptotes: $y = \pm\frac{3}{5}x$

To use a graphing utility, solve for y first.

$$y^2 = \frac{225 + 9x^2}{25}$$

$$\left.\begin{aligned} y_1 &= \sqrt{\frac{9x^2 + 225}{25}} \\ y_2 &= -\sqrt{\frac{9x^2 + 225}{25}} \end{aligned}\right\} \text{Hyperbola}$$

$$\left.\begin{aligned} y_3 &= \frac{3}{5}x \\ y_4 &= -\frac{3}{5}x \end{aligned}\right\} \text{Asymptotes}$$



36. $25x^2 - 4y^2 = 100$

$$\frac{x^2}{4} - \frac{y^2}{25} = 1$$

$a = 2, b = 5,$

$c^2 = a^2 + b^2 = 4 + 25 = 29 \Rightarrow c = \sqrt{29}$

Center: $(0, 0)$

Vertices: $(\pm 2, 0)$

Foci: $(\pm\sqrt{29}, 0)$

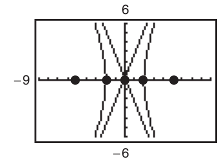
Asymptotes: $y = \pm\frac{5}{2}x$

To use a graphing utility, solve for y first.

$$y^2 = \frac{25x^2 - 100}{4}$$

$$\left.\begin{aligned} y_1 &= \sqrt{\frac{25x^2 - 100}{4}} \\ y_2 &= -\sqrt{\frac{25x^2 - 100}{4}} \end{aligned}\right\} \text{Hyperbola}$$

$$\left.\begin{aligned} y_3 &= \frac{5}{2}x \\ y_4 &= -\frac{5}{2}x \end{aligned}\right\} \text{Asymptotes}$$



37. $9y^2 - x^2 + 2x + 54y + 62 = 0$

$9(y^2 + 6y + 9) - (x^2 - 2x + 1) = -62 - 1 + 81$

$9(y + 3)^2 - (x - 1)^2 = 18$

$$\frac{(y + 3)^2}{2} - \frac{(x - 1)^2}{18} = 1$$

$a = \sqrt{2}, b = 3\sqrt{2}, c = 2\sqrt{5}$

Center: $(1, -3)$

Vertices: $(1, -3 \pm \sqrt{2})$

Foci: $(1, -3 \pm 2\sqrt{5})$

Asymptotes: $y = -3 \pm \frac{1}{3}(x - 1)$

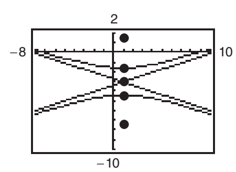
To use a graphing utility, solve for y first.

$9(y + 3)^2 = 18 + (x - 1)^2$

$$y = -3 \pm \sqrt{\frac{18 + (x - 1)^2}{9}}$$

$$\left.\begin{aligned} y_1 &= -3 + \frac{1}{3}\sqrt{18 + (x - 1)^2} \\ y_2 &= -3 - \frac{1}{3}\sqrt{18 + (x - 1)^2} \end{aligned}\right\} \text{Hyperbola}$$

$$\left.\begin{aligned} y_3 &= -3 + \frac{1}{3}(x - 1) \\ y_4 &= -3 - \frac{1}{3}(x - 1) \end{aligned}\right\} \text{Asymptotes}$$



38. $9x^2 - y^2 + 54x + 10y + 55 = 0$

$9(x^2 + 6x + 9) - (y^2 - 10y + 25) = -55 + 81 - 25$

$$\frac{(x + 3)^2}{1/9} - \frac{(y - 5)^2}{1} = 1$$

$a = \frac{1}{3}, b = 1, c = \frac{\sqrt{10}}{3}$

Center: $(-3, 5)$

Vertices: $\left(-3 \pm \frac{1}{3}, 5\right) \Rightarrow \left(-\frac{10}{3}, 5\right), \left(-\frac{8}{3}, 5\right)$

Foci: $\left(-3 \pm \frac{\sqrt{10}}{3}, 5\right)$

Asymptotes: $y = 5 \pm 3(x + 3)$

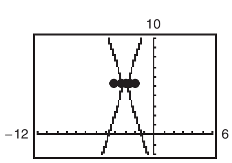
To use a graphing utility, solve for y first.

$$(y - 5)^2 = \frac{(x + 3)^2}{1/9} - 1$$

$$y = 5 \pm \sqrt{9(x + 3)^2 - 1}$$

$$\left.\begin{aligned} y_1 &= 5 + \sqrt{9(x + 3)^2 - 1} \\ y_2 &= 5 - \sqrt{9(x + 3)^2 - 1} \end{aligned}\right\} \text{Hyperbola}$$

$$\left.\begin{aligned} y_3 &= 5 + 3(x + 3) \\ y_4 &= 5 - 3(x + 3) \end{aligned}\right\} \text{Asymptotes}$$



39. Vertices: $(\pm 1, 0) \Rightarrow a = 1$

Asymptotes: $y = \pm 5x \Rightarrow \dfrac{b}{a} = 5, b = 5$

Center: $(0, 0) = (h, k)$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$
$$\frac{x^2}{1} - \frac{y^2}{25} = 1$$

40. Vertices: $(0, \pm 3) \Rightarrow a = 3$

Asymptotes: $y = \pm 3x \Rightarrow \dfrac{a}{b} = 3, b = 1$

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$
$$\frac{y^2}{9} - \frac{x^2}{1} = 1$$

41. Foci: $(0, \pm 8) \Rightarrow c = 8$

Asymptotes: $y = \pm 4x \Rightarrow \dfrac{a}{b} = 4 \Rightarrow a = 4b$

Center: $(0, 0) = (h, k)$

$$c^2 = a^2 + b^2 \Rightarrow 64 = 16b^2 + b^2$$
$$\frac{64}{17} = b^2 \Rightarrow a^2 = \frac{1024}{17}$$
$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$
$$\frac{y^2}{1024/17} - \frac{x^2}{64/17} = 1$$
$$\frac{17y^2}{1024} - \frac{17x^2}{64} = 1$$

42. Foci: $(\pm 10, 0) \Rightarrow c = 10$

Asymptotes: $y = \pm \dfrac{3}{4}x \Rightarrow \dfrac{b}{a} = \dfrac{3m}{4m}$

$$c^2 = a^2 + b^2 \Rightarrow 100 = (3m)^2 + (4m)^2$$
$$100 = 25m^2$$
$$2 = m$$

$a = 4(2) = 8$

$b = 3(2) = 6$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$
$$\frac{x^2}{64} - \frac{y^2}{36} = 1$$

43. Vertices: $(1, 2), (3, 2) \Rightarrow a = 1$

Asymptotes: $y = x, y = 4 - x$

$\dfrac{b}{a} = 1 \Rightarrow \dfrac{b}{1} = 1 \Rightarrow b = 1$

Center: $(2, 2) = (h, k)$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$
$$\frac{(x-2)^2}{1} - \frac{(y-2)^2}{1} = 1$$

44. Vertices: $(3, 0), (3, 6) \Rightarrow a = 3$

Center: $(3, 3)$

Asymptotes: $y = 6 - x, y = x$

$\dfrac{a}{b} = 1 \Rightarrow b = 3$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$
$$\frac{(y-3)^2}{9} - \frac{(x-3)^2}{9} = 1$$

45. Vertices: $(3, 0), (3, 4) \Rightarrow a = 2$

Asymptotes: $y = \dfrac{2}{3}x, y = 4 - \dfrac{2}{3}x$

$\dfrac{a}{b} = \dfrac{2}{3} \Rightarrow b = 3$

Center: $(3, 2) = (h, k)$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$
$$\frac{(y-2)^2}{4} - \frac{(x-3)^2}{9} = 1$$

46. Vertices: $(-4, 1), (0, 1) \Rightarrow a = 2$

Asymptotes: $y = x + 3, y = -x - 1$

$\dfrac{a}{b} = \dfrac{2}{2} \Rightarrow b = 2$

Center: $(-2, 1) = (h, k)$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$
$$\frac{(x+2)^2}{4} - \frac{(y-1)^2}{4} = 1$$

47. Foci: $(-1, -1), (9, -1) \Rightarrow c = 5$

Asymptotes: $y = \frac{3}{4}x - 4,\ y = -\frac{3}{4}x + 2$

$\frac{b}{a} = \frac{3}{4} \Rightarrow b = 3, a = 4$

Center: $(4, -1) = (h, k)$

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$

$$\frac{(x - 4)^2}{16} - \frac{(y + 1)^2}{9} = 1$$

48. Foci: $\left(9, \pm 2\sqrt{10}\right) \Rightarrow c = 2\sqrt{10} \Rightarrow c^2 = 40$

Asymptotes: $y = 3x - 27,\ y = -3x + 27$

$c^2 = a^2 + b^2$

$40 = a^2 + b^2$

$\frac{a}{b} = 3 \Rightarrow a = 3b$

$40 = 9b^2 + b^2$

$40 = 10b^2$

$b^2 = 4 \Rightarrow b = 2$

$a = 6$

Vertical transverse axis

Center: $(9, 0) = (h, k)$

$$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$$

$$\frac{(y - 0)^2}{36} - \frac{(x - 9)^2}{4} = 1$$

$$\frac{y^2}{36} - \frac{(x - 9)^2}{4} = 1$$

49. (a) Vertices: $(\pm 1, 0) \Rightarrow a = 1$

Horizontal transverse axis

Center: $(0, 0)$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Point on the graph: $(2, 13)$

$$\frac{2^2}{1^2} - \frac{13^2}{b^2} = 1$$

$$4 - \frac{169}{b^2} = 1$$

$$3b^2 = 169$$

$$b^2 = \frac{169}{3}$$

So, $\frac{x^2}{1} - \frac{y^2}{169/3} = 1.$

(b) When $y = 5$: $x^2 = 1 + \frac{5^2}{56.33}$

$$x = \sqrt{1 + \frac{25}{56.33}} \approx 1.2016$$

So, the width is about $2x \approx 2.403$ feet.

50. (a) Center: $(0, 0)$

Vertices: $(\pm 1, 0) \Rightarrow a = 1$

Passes through: $(2, 9) \Rightarrow (x, y)$

Horizontal transverse axis

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{2^2}{1^2} - \frac{9^2}{b^2} = 1$$

$$4 - \frac{81}{b^2} = 1$$

$$-\frac{81}{b^2} = -3$$

$$-81 = -3b^2$$

$$27 = b^2$$

$$\frac{x^2}{1} - \frac{y^2}{27} = 1$$

(b) When $y = -8\frac{1}{3}$: $x^2 = 1 + \frac{\left(-8\frac{1}{3}\right)^2}{27}$

$$x = \sqrt{\frac{868}{243}} \approx 1.89 \text{ units}$$

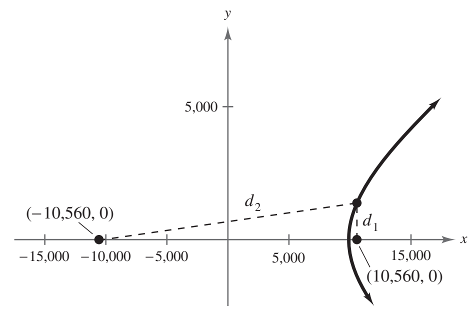
Width: $2x \approx 3.78$ units.

Each unit is $\frac{1}{2}$ foot, so the width is about 1.89 feet.

51. $$2c = 4 \text{ mi} = 21{,}120 \text{ ft}$$
$$c = 10{,}560 \text{ ft}$$
$$(1100 \text{ ft/s})(18 \text{ s}) = 19{,}800 \text{ ft}$$

The lightning occurred 19,800 feet further from B than from A:

$$d_2 - d_1 = 2a = 19{,}800 \text{ ft}$$
$$a = 9900 \text{ ft}$$
$$b^2 = c^2 - a^2 = (10{,}560)^2 - (9900)^2$$
$$b^2 = 13{,}503{,}600$$
$$\frac{x^2}{(9900)^2} - \frac{y^2}{13{,}503{,}600} = 1$$
$$\frac{x^2}{98{,}010{,}000} - \frac{y^2}{13{,}503{,}600} = 1$$



52. (a) Because listening station C heard the explosion 4 seconds after listening station A, and because listening station B heard the explosion one second after listening station A, and sound travels 1100 feet per second, the explosion is located in Quadrant IV on the line $x = 3300$. The locus of all points 4400 feet closer to A than C is one branch of the hyperbola.

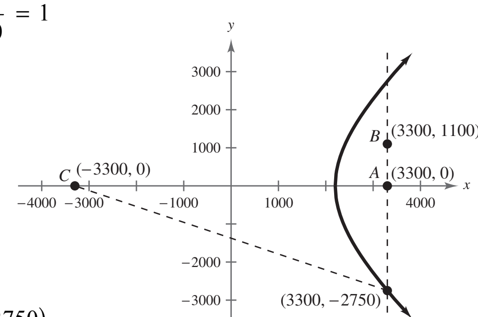
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ where $c = 3300$ feet and $a = \frac{4400}{2} = 2200$ feet, $b^2 = c^2 - a^2 = 6{,}050{,}000$.

So, the explosion occurred along the right branch of $\frac{x^2}{484{,}000} - \frac{y^2}{6{,}050{,}000} = 1$

(b) When $x = 3300$ you have $\frac{3300^2}{2200^2} - \frac{y^2}{6{,}050{,}000} = 1$.

Solving for y: $$y^2 = 6{,}050{,}000\left(\frac{3300^2}{2200^2} - 1\right)$$
$$= 7{,}562{,}500$$
$$y = \pm 2750$$

Because the explosion is in Quadrant IV, its coordinates are $(3300, -2750)$.



53. (a) Foci: $(\pm 150, 0) \Rightarrow c = 150$

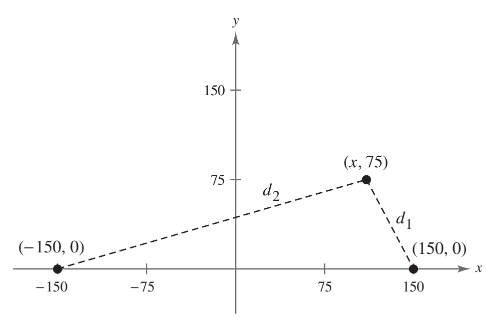
Center: $(0, 0) = (h, k)$

$$\frac{d_2}{186{,}000} - \frac{d_1}{186{,}000} = 0.001 \Rightarrow 2a = 186, a = 93$$
$$b^2 = c^2 - a^2 = 150^2 - 93^2 = 13{,}851$$
$$\frac{x^2}{93^2} - \frac{y^2}{13{,}851} = 1$$
$$x^2 = 93^2\left(1 + \frac{75^2}{13{,}851}\right) \approx 12{,}161$$
$$x \approx 110.3 \text{ miles}$$

(b) $c - a = 150 - 93 = 57$ miles

(c) Using the asymptote with positive slope,

$$y = k \pm \frac{b}{a}(x - h)$$
$$y = \frac{\sqrt{13{,}851}}{\sqrt{8694}}x$$
$$y = \frac{27\sqrt{19}}{93}x$$



54. Center: $(0, 0) = (h, k)$

Focus: $(24, 0) \Rightarrow c = 24$

Solution point: $(24, 24)$

$24^2 = a^2 + b^2 \Rightarrow b^2 = 24^2 - a^2$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{x^2}{a^2} - \frac{y^2}{24^2 - a^2} = 1 \Rightarrow \frac{24^2}{a^2} - \frac{24^2}{24^2 - a^2} = 1$$

Solving yields $a = 12\sqrt{2\left(3 - \sqrt{5}\right)}$ OR

$12\left(\sqrt{5} - 1\right) \approx 14.83$ and $b^2 \approx 355.9876$.

Thus, we have $\dfrac{x^2}{220.0124} - \dfrac{y^2}{355.9876} = 1$.

The right vertex is at $(a, 0) \approx (14.83, 0)$.

55. $9x^2 + 4y^2 - 18x + 16y - 119 = 0$

$A = 9, C = 4$

$AC = (9)(4) = 36 > 0 \Rightarrow$ Ellipse

56. $x^2 + y^2 - 4x - 6y - 23 = 0$

$A = 1, C = 1$

$A = C \Rightarrow$ Circle

57. $4x^2 - y^2 - 4x - 3 = 0$

$A = 4, C = -1$

$AC = (4)(-1) = -4 < 0 \Rightarrow$ Hyperbola

58. $y^2 - 6y - 4x + 21 = 0$

$A = 0, C = 1$

$AC = (0)(1) = 0 \Rightarrow$ Parabola

59. $y^2 - 4x^2 + 4x - 2y - 4 = 0$

$A = -4, C = 1$

$AC = (-4)(1) = -4 < 0 \Rightarrow$ Hyperbola

60. $y^2 + 12x + 4y + 28 = 0$

$A = 0, C = 1$

$AC = (0)(1) = 0 \Rightarrow$ Parabola

61. $4x^2 + 25y^2 + 16x + 250y + 541 = 0$

$A = 4, C = 25$

$AC = (4)(25) = 100 > 0 \Rightarrow$ Ellipse

62. $4y^2 - 2x^2 - 4y - 8x - 15 = 0$

$AC = (-2)(4) < 0 \Rightarrow$ Hyperbola

63. $25x^2 - 10x - 200y - 119 = 0$

$A = 25, C = 0$

$AC = 25(0) = 0 \Rightarrow$ Parabola

64. $4y^2 + 4x^2 - 24x + 35 = 0$

$A = C = 4 \Rightarrow$ Circle

65. $100x^2 + 100y^2 - 100x + 400y + 409 = 0$

$A = 100, C = 100$

$A = C \Rightarrow$ Circle

66. $9x^2 + 4y^2 - 90x + 8y + 228 = 0$

$A = 9, C = 4$

$AC = (9)(4) = 36 > 0 \Rightarrow$ Ellipse

67. True. For a hyperbola, $c^2 = a^2 + b^2$ or

$$e^2 = \frac{c^2}{a^2} = 1 + \frac{b^2}{a^2}.$$

The larger the ratio of b to a, the larger the eccentricity $e = c/a$ of the hyperbola.

68. True. For two lines to intersect at right anges, the slopes of the lines are negative reciprocals of each other. The asymptotes of the hyperbola are $y_1 = \dfrac{b}{a}x$ and

$y_2 = -\dfrac{b}{a}x.$

If they intersect at right angles, then

$\dfrac{b}{a} = \dfrac{-1}{-b/a} = \dfrac{a}{b}$ which implies $a = b$.

69. False. The graph is two intersecting lines.

$$x^2 - y^2 + 4x - 4y = 0$$
$$\left(x^2 + 4x + 4\right) - \left(y^2 + 4y + 4\right) = 4 - 4$$
$$(x-2)^2 - (y+2)^2 = 0$$
$$(x-2)^2 = (y+2)^2$$
$$x - 2 = \pm(y-2)$$
$$y = x \text{ and } y = -x + 4$$

70.

$$9x^2 - 54x - 4y^2 + 8y + 41 = 0$$
$$9(x^2 - 6x + 9) - 4(y^2 - 2y + 1) = -41 + 81 - 4$$
$$9(x - 3)^2 - 4(y - 1)^2 = 36$$
$$\frac{(x - 3)^2}{4} - \frac{(y - 1)^2}{9} = 1$$
$$\frac{(y - 1)^2}{9} = \frac{(x - 3)^2}{4} - 1$$
$$(y - 1)^2 = 9\left[\frac{(x - 3)^2}{4} - 1\right]$$

The bottom half of the hyperbola is:

$$y - 1 = -\sqrt{9\left[\frac{(x - 3)^2}{4} - 1\right]}$$
$$y = 1 - 3\sqrt{\frac{(x - 3)^2}{4} - 1}$$

71. Draw a rectangle through the vertices and the endpoints of the conjugate axis.
Sketch the asymptotes by drawing lines through the opposite corners of the rectangle.

72. (a) $4x^2 - y^2 - 8x - 2y - 13 = 0$ matches (iv) because $AC = 4(-1) < 0$, the graph is a hyperbola.

(b) $x^2 + y^2 - 2x - 8 = 0$ matches (ii) because $A = C = 1$, the graph is a circle.

(c) $2x^2 - 4x - 3y - 3 = 0$ matches (i) because $AC = 2(0) = 0$, the graph is a parabola.

(d) $x^2 + 6y^2 - 2x - 5 = 0$ matches (iii) because $AC = (1)(6) > 0$, the graph is an ellipse.

73. Because the transverse axis is vertical, $\frac{(y + 5)^2}{9} - \frac{(x - 3)^2}{4} = 1$, where $a = 3, b = 2, h = 3,$ and $k = -5$ the equations of the asymptotes should be $y = k \pm \frac{a}{b}(x - h)$

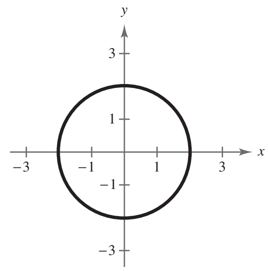
$$y = -5 \pm \frac{3}{2}(x - 3)$$
$$y = \frac{3}{2}x - \frac{19}{2} \text{ and } y = -\frac{3}{2}x - \frac{1}{2}.$$

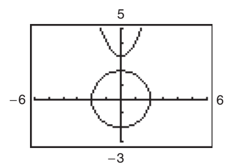
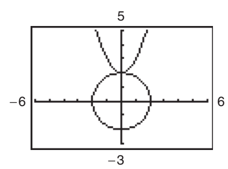
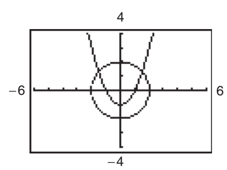
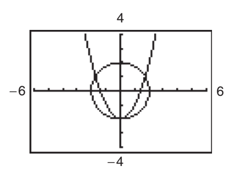
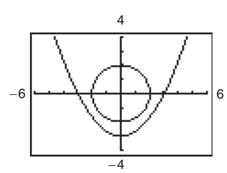
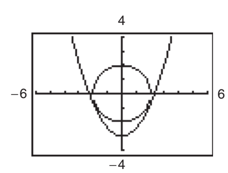
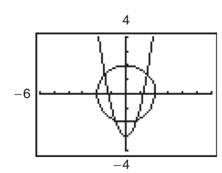
74. Let (x, y) be such that the difference of the distances from $(c, 0)$ and $(-c, 0)$ is $2a$ (again only deriving one of the forms).

$$2a = \left|\sqrt{(x + c)^2 + y^2} - \sqrt{(x - c)^2 + y^2}\right|$$
$$2a + \sqrt{(x - c)^2 + y^2} = \sqrt{(x + c)^2 + y^2}$$
$$4a^2 + 4a\sqrt{(x - c)^2 + y^2} + (x - c)^2 + y^2 = (x + c)^2 + y^2$$
$$4a\sqrt{(x - c)^2 + y^2} = 4cx - 4a^2$$
$$a\sqrt{(x - c)^2 + y^2} = cx - a^2$$
$$a^2(x^2 - 2cx + c^2 + y^2) = c^2x^2 - 2a^2cx + a^4$$
$$a^2(c^2 - a^2) = (c^2 - a^2)x^2 - a^2y^2$$

Let $b^2 = c^2 - a^2$. Then $a^2b^2 = b^2x^2 - a^2y^2 \Rightarrow 1 = \frac{x^2}{a^2} - \frac{y^2}{b^2}.$

75.



Value of C	Possible number of points of intersection
$C > 2$	
$C = 2$	
$-2 < C < 2$	
$C = -2$	
$C < -2$	 or  or 

For $C \le -2$, analyze the two curves to determine the number of points of intersection.

$C = -2$: $x^2 + y^2 = 4$ and $y = x^2 - 2$

$$x^2 = y + 2$$

Substitute: $(y+2)+y^2=4$

$$y^2+y-2=0$$
$$(y+2)(y-1)=0$$
$$y=-2, 1$$

$x^2=y+2$	$x^2=y+2$
$x^2=-2+2$	$x^2=1+2$
$x^2=0$	$x^2=3$
$x=0$	$x=\pm\sqrt{3}$
$(0,-2)$	$(-\sqrt{3},1), (\sqrt{3},1)$

There are three points of intersection when $C=-2$.

$C<-2$: $x^2+y^2=4$ and $y=x^2+C$

$$x^2=y-C$$

Substitute: $(y-C)+y^2=4$

$$y^2+y-4-C=0$$
$$y=\frac{-1\pm\sqrt{(1)^2-(4)(1)(-C-4)}}{2}$$
$$y=\frac{-1\pm\sqrt{1+4(C+4)}}{2}$$

If $1+4(C+4)<0$, there are no real solutions (no points of intersection):

$$1+4C+16<0$$
$$4C<-17$$
$$C<\frac{-17}{4}, \text{ no points of intersection}$$

If $1+4(C+4)=0$, there is one real solution (two points of intersection):

$$1+4C+16=0$$
$$4C=-17$$
$$C=\frac{-17}{4}, \text{ two points of intersection}$$

If $1+4(C+4)>0$, there are two real solutions (four points of intersection):

$$1+4C+16>0$$
$$4C>-17$$
$$C>\frac{-17}{4}, \text{ (but } C<-2\text{), four points of intersection}$$

Summary:

a. no points of intersection: $C>2$ or $C<\frac{-17}{4}$

b. one point of intersection: $C=2$

c. two points of intersection: $-2<C<2$ or $C=\frac{-17}{4}$

d. three points of intersection: $C=-2$

e. four points of intersection: $\frac{-17}{4}<C<-2$